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ON A SEMI-SYMMETRIC NON-METRIC CONNECTION IN AN SP-SASAKIAN MANIFOLD

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Summary : In the present paper we have studied the properties of curvature tensor of a semi-symmetric non-metric connection in an sp-Sasakian manifold.

BİR SP-SASAKIAN MANİFOLDDA METRİK OLMAYAN BİR YARI SİMETRİK BAĞLANTI HAKKINDA

Özet : Bu çalışmada, bir sp-Sasakian manifoldda metrik olmayan bir yarı simetrik bağlantının eğrilik tensörünün özelikleri incelenmektedir.

1. Introduction. Let Mn be an n-dimensional C^∞ manifold. If there exist in Mn a tensor field F of type (1,1), a vector field T and 1-form A satisfying

$$\bar{\bar{X}} = X - A(X)\,T, \tag{1.1}$$

where $\bar{X} \overset{\text{def}}{=} F X$, then Mn is called an almost paracontact manifold. Let g be Riemannian metric satisfying

$$A(X) = g(X, T), \tag{1.2}$$

$$A(\bar{X}) = 0, \quad (\bar{T}) = 0, \quad \text{rank } (F) = n - 1 \tag{1.3}$$

$$g(\bar{X}, \bar{Y}) = g(X, Y) - A(X)\,A(Y). \tag{1.4}$$

The set (F, T, A, g) satisfying (1.1), (1.2), (1.3) and (1.4) is called an almost paracontact Riemannian structure and the manifold with such a structure is called an almost paracontact Riemannian manifold [2].

Moreover, if $'F(X, Y) \overset{\text{def}}{=} g(\bar{X}, Y)$ then in addition to the above relation the following are satisfied

$$'F(X, Y) = {}'F(Y, X), \tag{1.5a}$$

$$'F(\bar{X}, \bar{Y}) = {}'F(X, Y). \tag{1.5b}$$

Now, we consider an n-dimensional differentiable manifold with a positive definite metric g which admits a 1-form A satisfying

$$(D_X A)(Y) - (D_Y A)(X) = 0 \tag{1.6}$$

and

$$(D_X D_Y A)(Z) = (-g(X, Z) + A(X) A(Z)) A(Y) + + (-g(X, Y) + A(X) A(Y)) A(Z). \tag{1.7}$$

Furthermore,

$$A(X) = g(X, T), D_X T = \bar{X} \tag{1.8}$$

then it is easily verified that the manifold in consideration becomes an almost paracontact Riemannian manifold. Such a manifold is called a p-Sasakian manifold and the following relations hold [1]:

$$A(K(X, Y, Z)) = g(X, Z) A(Y) - g(Y, Z) A(X), \tag{1.9a}$$

$$\text{Ric}(X, T) = -(n-1) A(X), \tag{1.9b}$$

where K and Ric are the curvature tensor and Ric tensor respectively.

Let us consider an n-dimensional differentiable manifold with a Riemannian metric g which admits a 1-form A satisfying

$$(D_X A)(Y) = -g(X, Y) + A(X) A(Y). \tag{1.10}$$

Such a manifold is called an sp-Sasakian manifold [1].

Thus in an sp-Sasakian manifold, we have

$$'F(X, Y) = -g(X, Y) + A(X) A(Y). \tag{1.11}$$

A semi-symmetric non-metric connection B in an almost paracontact metric manifold is given by

$$B_X Y = D_X Y + A(Y) X \tag{1.12}$$

where D is a Riemannian connection with respect to metric g [3].

2. Curvature tensor. Let us denote the curvature tensor of the connection B by R and curvature tensor of the connection D by K. By straight forward calculation, we find

$$R(X, Y, Z) = K(X, Y, Z) + (B_X A)(Z) Y - (B_Y A)(Z) X. \tag{2.1}$$

In consequence of (1.10) and (1.12), (2.1) reduces to

$$R(X, Y, Z) = K(X, Y, Z) - g(X, Z) Y + g(Y, Z) X. \tag{2.2}$$

Theorem 2.1. If in an sp-Sasakian manifold the curvature tensor of a semi-symmetric non-metric connection vanishes, then the manifold is projectively flat.

Proof. Since $R = 0$, then (2.2) gives

$$K(X, Y, Z) = g(X, Z)Y - g(Y, Z)X. \tag{2.3}$$

Contracting the above equations, we get

$$\text{Ric}\ (Y, Z) = -(n-1)\, g(Y, Z). \tag{2.4}$$

Hence (2.3) and (2.4) give

$$K(X, Y, Z) - \frac{1}{n-1}(\text{Ric}\ (Y, Z)K - \text{Ric}\ (X, Z)Y) = 0 \tag{2.5}$$

or $W = 0$ where W is the projective curvature tensor of the manifold.

Theorem 2.2. If in an sp-Sasakian manifold the Ric tensor of a semi-symmetric non-metric connection B vanishes, then the curvature tensor of B is equal to the projecting curvature tensor of the manifold.

Proof. From (2.2), we have

$$R(X, Y, Z) = K(X, Y, Z) - g(X, Z)Y + g(Y, Z)X. \tag{2.6}$$

Contracting the above equation, we get

$$\text{Ric}'\ (Y, Z) = \text{Ric}\ (Y, Z) + (n-1)\, g(Y, Z). \tag{2.7}$$

Since $\text{Ric}' = 0$, we have

$$g(Y, Z) = -\frac{1}{n-1}\ \text{Ric}\ (Y, Z). \tag{2.8}$$

From (2.6) and (2.8), we have

$$R = W. \tag{2.9}$$

Theorem 2.3. In an sp-Sasakian manifold the projective curvature tensor of a semi-symmetric non-metric connection B is equal to the projective curvature tensor of the manifold.

Proof. From (2.6) and (2.7), we get

$$R(X, Y, Z) = K(X, Y, Z) + \frac{1}{n-1}[\text{Ric}'(Y, Z) - \text{Ric}\ (Y, Z)]X -$$

$$- \frac{1}{n-1}[\text{Ric}'\ (X, Z) - \text{Ric}\ (X, Z)]\, Y$$

or

$$R(X, Y, Z) - \frac{1}{n-1}[\text{Ric}'\ (Y, Z)X - \text{Ric}'(X, Z)Y] =$$

$$= K(X, Y, Z) - \frac{1}{n-1}[\text{Ric}\ (Y, Z)X - \text{Ric}\ (X, Z)Y]$$

or $W' = W$ where W' is the projective curvature tensor of the semi-symmetric non-metric connection.

Theorem 2.4. In an sp-Sasakian manifold with semi-symmetric non-metric connection B, we have

$$R(X, Y, Z) + R(Y, Z, X) + R(Z, X, Y) = 0 \tag{2.10a}$$

$$'R(X, Y, Z, U) + 'R(X, Y, U, Z) = 0 \tag{2.10b}$$

$$'R(X, Y, Z, T) = 0 \tag{2.10c}$$

$$R(X, Y, T) = 0 \tag{2.10d}$$

$$\text{Rie}\ (Y, T) = 0. \tag{2.10e}$$

Proof. Using Bianchi's first identity (2.6) gives (2.10a). From (2.6) we get (2.10b). Similarly other results can also be obtained.

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