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RATIONAL GROUPS WITH NORMAL PRIME ORDER SUBGROUPS

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Summary : In this note we shall study the rational groups all whose odd prime order subgroups are normal.

ASAL MERTEBELİ NORMAL ALT GRUPLARI HAİZ RASYONEL GRUPLAR

Özet : Bu çalışmada, tek asal mertebeli bütün alt grupları normal olan rasyonel gruplar incelenmektedir.

All groups will be finite. The definitions and notations will be those of [3].

Definition (see [3]). A rational group (or a **Q**-group) is a group all whose irreducible characters are rational valued.

Proposition 1 (see [3]). A group G is rational iff for every $x \in G$, $N(\langle x \rangle)/(C(x) \simeq \mathrm{Aut}(\langle x \rangle))$.

Proposition 2 (see [3]). Let G be a rational group. Then:

i) G/G' is an elementary abelian 2-group.

ii) G' contains all odd order elements of G.

Theorem 4. Let G be a rational group such that every odd prime order subgroup is normal. Then G is a solvable group, $|G| = 2^a 3^b$ and G' is 3-nilpotent.

Proof. Let p be an odd prime and $A \leq G$ of order p. Then A is normal in G and $G/C(A)$ is abelian. It follows that $G' \leq C(A)$. Hence in G' every odd prime order subgroup is in $Z(G')$.

We prove now that G' is p-nilpotent for every odd prime divisor of $|G'|$.

Let H be a group of minimal order such that every odd prime order element of H is in $Z(H)$ and H is not p-nilpotent for p odd prime. Since the hypothesis remains valid for every subgroup, then every proper subgroup of H is p-nilpotent but H is not. By Ito's Theorem [2] H has a normal Sylow p-subgroup P and

P has $\exp(P) = p$. Then $P \leq Z(H)$ anh H is p-nilpotent. Therefore G' is p-nilpotent for every odd prime p.

Let P be a Sylow p-subgroup of G for p, an odd prime divisor of $|G|$. By Prop. 2 $P \leq G'$. Since G' is p-nilpotent, it follows that P is p-rational. Clearly G' is solvable. Since G is a rational group, for every element $x \in G$ of odd prime order $C_G(x) \neq G$, by Prop. 1. Therefore $G' \neq G$ and G is solvable.

By Gow Theorem [1] $|G| = 2^a 3^b 5^c$. Let $P \in \mathrm{Syl}_5(G)$. Then $P \leq G'$ and since G' is 5-nilpotent it follows that for an element x of order 5 of P, no even order automorphism of $\langle x \rangle$ belongs to G'. By Prop. 2 G/G' is an elementary abelian 2-group and by Prop. 1 $\mathrm{Aut}(\langle x \rangle)$ is cyclic of order 4. It follows that G cannot contain elements of order 5. Hence $|G| = 2^a 3^b$.

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