THE 2'-R GROUPS WITH ONE CONJUGACY CLASS OF INVOLUTIONS

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Summary : In this note we shall study the structure of the Sylow 2-subgroups of the solvable groups all whose irreducible characters are rational valued on the 2-regular elements.

BİR EŞLENİK İNVOLÜSYONLAR SINIFINA SAHİP 2'-R GRUPLAR

Özet : Bu çalışmada, 2-regüler elemanlar üzerinde bütün indirgenemez karakterleri rasyonel değerli olan çözülebilir grupların 2-Sylow alt gruplarının yapısı incelenmektedir.

The notations and terminology are standard (see [2]). All groups will be finite.

Definition. A 2'-r group is a finite group all whose characters are rational valued on the 2-regular elements.

Proposition 1. A group G is a 2'-r group if and only if for every 2-regular $x \in G$, $N_G(\langle x \rangle)/C_G(\langle x \rangle) \simeq \mathrm{Aut}(\langle x \rangle)$ (see [1]).

Theorem 2 (Thompson, see [3], p. 511). Suppose that G is a solvable group of even order and that the Sylow 2-subgroup of G contains more than one involution. Suppose that all the involutions in G are conjugate. Then the Sylow 2-subgroups of G are either homocyclic or Suzuki 2-groups.

Theorem 3. Suppose that G is a solvable 2'-r group with one conjugacy class of involutions. Then, the Sylow 2-subgroups of G are either cyclic or generalized quaternion groups.

Proof. Clearly we can suppose that $O_{2'}(G)$ is trivial. Let $I(G)=\{x \in G \mid x^2=1\}$. Let A be a minimal normal subgroup of G. Since G is solvable and $O_{2'}(G)$ is trivial, then A is an elementary abelian 2-subgroup, hence $A = I(G)$.

Let S be a Sylow 2-subgroup of G. We shall prove now by induction on $|G|$ that $N_G(S)=S$. If $|G|=1$ the statement is trivial. Now G/A is a solvable 2'-r group (see [1]) having S/A as a Sylow 2-subgroup. If follows by induction that $N_{G/A}(S/A) = S/A$, so that if $x \in N_G(S)$, then $xA \in S/A$. Set $xA = yA$ with $y \in S$. Then $y^{-1} x \in A$, therefore $x \in S$.

By Theorem 2, if G contains more than one involution, then the Sylow 2-subgroups of G are either homocyclic or Suzuki 2-groups. If a Sylow 2-subgroup S is homocyclic then it is trivial that $A \subseteq Z(S)$. If S is a Suzuki 2-group, then (see [3], pg. 313) $S' = Z(S) = A = I(S)$. By Burnside theorem (see [2], pg. 240) if $x, y \in Z(S)$ and $x^2 = y$, with $z \in G$, then there is $t \in N_G(S)$ such that $x^t = y$. Since $N_G(S) = S$ it follows that for every $x, y \in A \subseteq S$, there is a $t \in S$ such that $x^t = y$. This is a contradiction since $A \subseteq Z(S)$. Therefore G contains only one involution, so that S is either a cyclic group or a generalized quaternion group.

REFERENCES

[1] ARMEANU, I. : *The structure of the groups all whose characters are rational valued on the 2'-elements* (to appear).

[2] GORENSTEIN, D. : Finite Groups, Harper and Row, 1968.

[3] HUPPERT, B. and BLACKBURN, N. : Fiuite Groups, Vol. 2. Springer-Verlag, 1982.