İstanbul Üniv. Fen Fak. Mat. Dergisi 55 - 56 (1996 - 1997), 35-41

ON PSEUDO CONCIRCULAR SYMMETRIC MANIFOLD ADMITTING A TYPE OF QUARTER SYMMETRIC METRIC CONNECTION

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Abstract: The object of this paper is to study a type of non-flat Riemannian manifold called concircular symmetric manifold admits a special kind of quarter symmetric metric connection.

INTRODUCTION

In a recent paper the author and **u.c.De.** [1] introduced and studied a new type of non-flat Riemannian manifold (M^n, g) $(n > 2)$ whose concircular curvature tensor $\cap$ satisfies the condition

$$(\nabla_X \cap)(Y, Z)W = 2A(X)\cap(Y,Z)W + A(Y)\cap(X,Z)W + A(Z)\cap(Y,X)W + A(W)\cap(Y,Z)X + g[\cap(Y,Z)W, X]\rho \qquad (1)$$

where $\cap$ is given by

$$\cap(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] \qquad (2)$$

A is a non-zero 1-form such that

$$g(X, \rho) = A(X) \qquad (3)$$

For any vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric g. Such a manifold was called a pseudo concircular symmetric manifold, the 1-form A was called its associated 1-form and an n-dimensional manifold of this kind was denoted by $(P\cap S)_n$.

This paper deals with a type of $(P\cap S)_n$ admitting a special kind of quarter symmetric metric connection. The idea of such a connection was introduced by **S.Golab[2]**. The object of introducing a quarter symmetric metric connection in a $(P\cap S)_n$ is as follows.

In case of a $(P\cap S)_n$ it is known that the scalar curvature r is constant, but no further information about the nature of the form of r in terms of the associated vector field ρ is known. But, introducing a particular type of quarter symmetric metric connection in a $(P\cap S)_n$ it is possible to obtain the nature of the associated 1-form and also a particular form of r in term of the associated vector field ρ. It is shown that if a $(P\cap S)_n$ admits a particular type of quarter symmetric metric connection, then the 1-form A is closed and the scalar curvature of the manifold is of the form

$$r = n\left[1 - \frac{1}{(n-1)A(\rho)}\right]$$

1. PRELIMINARIES

Here we consider a type of quarter symmetric metric connection ∇ on a $(P\cap S)_n$ whose torsion tensor **T** is given by

$$T(X,Y) = A(Y)LX - A(X)LY \qquad (1.1)$$

Where **A** is the associated 1-form of $(P\cap S)_n$ and **X,Y** are any vector fields in (M^n, g) and **L** is given by

$$G(LX,Y) = S(X,Y), \qquad (1.2)$$

S being the Ricci tensor. Further we take the curvature tensor **K** of and torsion tensor **T** of satisfy the conditions

$$K(X,Y)Z = 0 \qquad (1.3)$$

And

$$(\ _X T)(Y,Z) = A(X)T(Y,Z) \qquad (1.4)$$

It is known [3] that if is a quarter symmetric metric connection with associated 1-form A and a (1,1) tensor L then

$$_X Y = \nabla_X Y + A(Y)LX - S(X,Y)\rho \qquad (1.5)$$

where ρ is given by (3) .

If the curvature tensor of the quarter symmetric metric connection be denoted by **K** and that of Levi-Civita connection ∇ by **R** then it is also known [3] that

$$\begin{aligned} K(X,Y)Z = R(X,Y)Z - \alpha(Y,Z)LX + \alpha(X,Z)LY - S(Y,Z)QX \\ + S(X,Z)QY + [(\nabla_X L)(Y) - (\nabla_Y L)(X)]A(Z) \\ - [(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)]\rho \end{aligned} \qquad (1.6)$$

where α is a tensor field of type **(0 , 2)** defined by

$$\alpha(X,Y) = (\nabla_X Y)(Y) - A(Y)A(LX) + \tfrac{1}{2}A(\rho)S(X,Y) \qquad (1.7)$$

and where **Q** is a tensor field of type (1 ,1) defined by

$$QX = \nabla_X \rho - A(LX)\rho + \tfrac{1}{2} A(\rho) LX \qquad (1.8)$$

2. $(P \cap S)_n$ ADMITTING A TYPE OF QUARTER SYMMETRIC METRIC CONNECTION

From (1.1) we get

$$(C\ \ T)(Y) = r\, g(Y, \rho) - S(Y, \rho) \qquad (2.1)$$

where **C** denotes the operation of contraction. It is known **[1]** that in a $(P \cap S)_n$ the following relations holds:

$$S(X, \rho) = \frac{r}{n} g(X, \rho) \qquad (2.2)$$

and

$$dr(X) = 0 \qquad (2.3)$$

Now, using (3) and (2.2) it follows from (2.1) that

$$(C\ \ T)(Y) = \frac{n-1}{n} r\, A(Y) \qquad (2.4)$$

From (2.3) and (2.4) we get

$$(\ \ {}_X C\ \ T)(Y) = \frac{n-1}{n} [\, r(\ \ {}_X A)(Y)\,] \qquad (2.5)$$

Again from (1.4) we get

$$(\ \ {}_X C\ \ T)(Y) = A(X)(C\ \ T)(Y) \qquad (2.6)$$

Hence using (2.4) it follows from (2.5) and (2.6) that

$$(\ \ {}_X A)(Y) = A(X) A(Y) \quad [\, r \# 0\,] \qquad (2.7)$$

and where **Q** is a tensor field of type (1 ,1) defined by

$$QX = \nabla_X \rho - A(LX)\rho + \tfrac{1}{2} A(\rho) LX \qquad (1.8)$$

2. $(P\cap S)_n$ ADMITTING A TYPE OF QUARTER SYMMETRIC METRIC CONNECTION

From (1.1) we get

$$(C\ T)(Y) = r\ g(Y,\rho) - S(Y,\rho) \qquad (2.1)$$

where **C** denotes the operation of contraction. It is known **[1]** that in a $(P\cap S)_n$ the following relations holds:

$$S(X,\rho) = \frac{r}{n} g(X,\rho) \qquad (2.2)$$

and

$$dr(X) = 0 \qquad (2.3)$$

Now, using (3) and (**2.2**) it follows from (**2.1**) that

$$(C\ T)(Y) = \frac{n-1}{n}\, r\ A(Y) \qquad (2.4)$$

From (**2.3**) and (**2.4**) we get

$$(\ {}_X C\ T)(Y) = \frac{n-1}{n} [\, r(\ {}_X A)(Y)\,] \qquad (2.5)$$

Again from (**1.4**) we get

$$(\ {}_X C\ T)(Y) = A(X)(C\ T)(Y) \qquad (2.6)$$

Hence using (**2.4**) it follows from (**2.5**) and (**2.6**) that

$$(\ {}_X A)(Y) = A(X)A(Y) \qquad [\, r \# 0\,] \qquad (2.7)$$

Now from (3), (2.3), (1.5) and (2.7) we get

$$(\nabla_X A)(Y) = \left(1 + \frac{r}{n}\right) A(X)A(Y) - A(\rho)S(X,Y) \qquad (2.8)$$

This leads to the following theorem :

Theorem 1. If a $(P\cap S)_n$ with non –zero scaler curvature admits a quacter symmetric metric connection whose torsion tensor T is given by (**1.1**) and whose curvature tensor **K** and torsion tensor **T** satisfy the conditions (**1.3**) and (**1.4**) respectively, then the associated 1-form **A** is closed.

Now using (**2.8**) the equation (**1.7**) can be written as follows:

$$\alpha(X,Y) = A(X)A(Y) - \tfrac{1}{2} A(\rho)S(X,Y) \qquad (2.9)$$

From the above we also have

$$QX = A(X)\rho - \tfrac{1}{2} A(\rho)LX \qquad (2.10)$$

In virtue of (**1.3**) the equation (**1.6**) takes the form

$$\begin{aligned} {}'R(X,Y,Z,W) = {} & \alpha(Y,Z)S(X,W) - \alpha(X,Z)S(Y,W) \\ & + S(Y,Z)\alpha(X,W) - S(X,Z)\alpha(Y,W) \\ & - [(\nabla_X S)(Y,W) - (\nabla_Y S)(X,W)]A(Z) \\ & + [(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z)]A(W) \end{aligned} \qquad (2.11)$$

where $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$

Next it is known [1] that in a $(P\cap S)_n$

$$(\nabla_X S)(Y,Z) - (\nabla_Y S)(X,Z) - \frac{1}{n(n-1)}[dr(X)g(Y,Z) - dr(Y)g(X,Z)]$$

$$= 3A(\cap(X,Y)Z) + A(X)\left[S(Y,Z) - \frac{r}{n} g(Y,Z)\right]$$

$$+ A(Y)\left[-S(X,Z) + \frac{r}{n} g(X,Z)\right] \qquad (2.12)$$

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Thus using (**2.2**) , (**2.3**) and (**2.12**) we get from (**2.11**) on confraction

$$S(Y,Z) = rA(Y)A(Z) - rS(Y,Z)A(\rho) - \frac{r}{n}A(Y)A(Z)$$

$$+ A(\rho)S(LY,Z) + A(\rho)S(Y,Z) - \frac{r}{n}A(Y)A(Z)$$

$$+ A(\rho)\left[S(Y,Z) - \frac{r}{n}g(Y,Z)\right] + 3A(\cap(\rho,Z)Y) \qquad (2.13)$$

Now putting $Z = \rho$ in (**2.13**) we get

$$\frac{r}{n}A(Y) = rA(\rho)A(Y) - \frac{r^2 A(Y)A(\rho)}{n} - \frac{rA(Y)A(\rho)}{n} + \frac{r^2 A(Y)A(\rho)}{n^2} \qquad (2.14)$$

Since A(Y) # **0** and r # **0** in ($P \cap S$)$_n$ it follows from above equation

$$r = n\left[1 - \frac{1}{(n-1)A(\rho)}\right].$$

This leads to the following theorem:

<u>**Theorem 2 :**</u> If a ($P \cap S$)$_n$ with non-zero scalar curvature admits a quarter symmetric metric connection whose torsion tensor **T** is given by (**1.1**) and whose curvature tensor K and torsion tensor T satisfy the conditions (**1.3**) and (**1.4**) respectively then the scalar curvature of ($P \cap S$)$_n$ is of the form

$$r = n\left[1 - \frac{1}{(n-1)A(\rho)}\right].$$

<u>**Acknowledgement:**</u> The author is grateful to Prof. U.C.De. for his valuable suggestions.

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