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PRIMITIVE IDEMPOTENTS OF THE GROUP ALGEBRA CSL(2,q)

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Abstract

In this paper a full system of primitive idempotents of the group algebra CSL(2,q) has been found using normed Gaussian sums over the finite field $F_q = GF(q)$ and a result of G.J. Janusz.

INTRODUCTION

Let p be an odd prime number and $F_q = GF(q)$ be a finite field of order $q=p^s$ for some $s\in\mathbf{N}$. Then $F_q = F(\theta)$ with $f(\theta)=0$, where $F= F_p$ and

$$f(x)= \mathrm{Irr}(\theta,x,F)=x^s-a_s x^{s-1} -...-a_2x-a_1 \in F[x]$$

is the minimal polynomial of θ over F. Thus

$$F_q = F\oplus F\theta \oplus...\oplus F\theta^{s-1}$$

becomes an additive elementary abelian group. On the other hand $F^*_q=F_q\backslash\{0\}$ the multiplicative group of the field F_q, is cyclic of order q-1 and $F^*_q = <\rho>$ for some generator ρ. Let $K: = <p^2>$ then

$$F^*_q= K\cup\rho K \quad \text{(disjoint)}$$

Let $\Psi=\Psi_{h_1,...,h_s}$ be a nontrivial irreducible additive character of the additive group F_q such that $0\le h_1,...,h_s \le p-1$; $(h_1,...,h_s) \neq (0,\dots,0)$ and

$$\Psi(\beta)=\varepsilon^{k_1h_1+......+k_sh_s},$$

where $\beta = k_1.1_F + k_2\theta+...+k_s\theta^{s-1}$; $0\le k_i \le p-1$, $i=1,...,s$;

$\varepsilon=\cos(2\pi/p)+i\sin(2\pi/p)$ and by abuse of notation we may also write $k_1,...,k_s\in F$. Let ζ be the irreducible multiplicative character of the multiplicative group F^*_q with

$$\zeta(\rho^i) = (-1)^i \quad \text{for any} \quad i \in \mathbf{Z}.$$

Now define

$$\tau_{(s)}(\zeta;\Psi) := \sum_{0\neq\beta\in F_q}\zeta(\beta)\Psi(\beta);\ \ x_{(s)}(\Psi) = \sum_{\beta\in K}\Psi(\beta);\ \ y_{(s)}(\Psi) = \sum_{\beta\in\rho K}\Psi(\beta)$$

and write $\tau_{(s)}$; $x_{(s)}$; $y_{(s)}$ instead of $\tau_{(s)}(\zeta;\Psi)$; $x_{(s)}(\Psi)$; $y_{(s)}(\Psi)$ for $\Psi = \Psi_{1,0,\ldots,0}$ and we call $\tau_{(s)}$ the normed Gaussian sum over the finite field F_q.

Let G = GL(2,q) denote the group of all non-singular 2x2 matrices over F_q and S = SL(2,q) denote the group of 2x2 matrices over Fq with determinant unity. S is a normal subgroup of G.

In this paper we will use the following properties to obtain the primitive idempotents of **C**SL(2,q) which correspond to the irreducible **C**SL(2,q) characters.

Property 1 ([6]). 1. $F_q = F(\rho) = F(\rho^2)$; i.e. ρ and ρ^2 are primitive elements of F_q over F, namely, θ can be chosen as ρ and ρ^2 for any $s \in \mathbf{N}$.

2.a) If s=2n+1, $n \in \mathbf{N} \cup \{0\}$ then $\tau_{(s)}$, $x_{(s)}$ and $y_{(s)}$ are independent of the choice of the primitive element θ.

b) If s=2n, $n \in \mathbf{N}$, then

$$\tau_{(s)} = -\sqrt{q};\ \ x_{(s)} = -\frac{1}{2}(1+\sqrt{q});\ \ y_{(s)} = -\frac{1}{2}(1-\sqrt{q})$$

for any primitive element $\theta \in \rho K$.

c) For any $s \in \mathbf{N}$ and for any primitive element $\theta \in \langle\rho^2\rangle = K$ we always have

$$\tau_{(s)} = \eta\sqrt{q};\ \ x_{(s)} = -\frac{1}{2}(1-\eta\sqrt{q});\ \ y_{(s)} = -\frac{1}{2}(1+\eta\sqrt{q}),$$

where

$$\eta = \begin{cases} +1, \text{if } q \equiv 1 \pmod 4 \\ +i, \text{if } q \equiv 3 \pmod 4 \end{cases} \qquad ; i = \sqrt{-1}$$

Property 2. G be a finite group of order n and **K** be an algebraically closed field with characteristic not dividing n. If χ is an irreducible **K**G-character affording the central idempotent e_χ of **K**G, then

$$e_\chi = \chi(1)\, n^{-1} \sum_{g\in G} \chi(g^{-1})g$$

where $\chi(1)$ is the degree χ.

Property 3 ([2], [3]). Let H be a subgroup of G, ψ be an irreducible **K**H-character of degree 1 and η an irreducible **K**G-character. Assume that the multiplicity of η in the induced **K**G-character ψ^G is one. If η and ψ afford the central idempotents e_η and e_ψ respectively, then $e_\eta e_\psi$ is a primitive idempotent of **KG** which corresponds to η.

Method and Results

Let θ be a primitive element of F_q; i.e. $F_q=F(\theta)$. Consider the following elements of S= SL(2,q):

$$\mathbf{a}_i = \begin{pmatrix} 1 & 0 \\ \theta^{i-1} & 1 \end{pmatrix};\quad \mathbf{a}_i^p = I;\quad i=1,\ldots,s.$$

Then S has the elementary abelian subgroup

$$H= \langle \mathbf{a}_1\rangle x \ldots x \langle \mathbf{a}_s\rangle .$$

The order of H is q. Since $|S|= q(q^2-1)$, then H is a Sylow p-subgroup of S and

$$H= \left\{ \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \Big|\ \beta \in F_q \right\}$$

All the characters of $\langle \mathbf{a}_i \rangle$ are linear and with the form

$$\varphi_{h_i}(\mathbf{a}_i) = \varepsilon^{h_i},$$

where $\varepsilon=\cos(2\pi/p)+i\sin(2\pi/p)$; $0\le h_i\le p-1$; $i=1,\ldots,s$.

Thus all the characters of H are linear and with the form

$$(\varphi_{h_1} \cdots \varphi_{h_s})(a_i)=\varepsilon^{h_i} \qquad (0 \le h_i \le p-1;\ \ i=1,\ldots,s).$$

Let us denote them by the symbol $\varphi_{h_1,\ldots,h_s}$.

If $x = \begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix}$, $\beta = k_1 + k_2\theta + \ldots + k_s\theta^{s-1}$,, $(0\le k_i\le p-1;\ (k_1,\ldots k_s) \ne (0,\ldots,0))$

is an element of H, then

$$\varphi_{h_1,\ldots,h_s}(x) = \varepsilon^{k_1 h_1+\ldots+k_s h_s} = \Psi_{h_1,\ldots,h_s}(\beta) = \Psi(\beta).$$

Thus we have all irreducible CH-characters which are given by table 1.

$\beta = k_1 + k_2\theta + \ldots + k_s\theta^{s-1}$, ;$k_i$=0,1,...,p-1; $(k_1,\ldots,k_s)\neq(0,\ldots,0)$, h_i=0,1,...,p-1; i=1,...,s; $\varepsilon^p=1$, $\varepsilon\neq1$			
Element	Number of Conjugacy Classes	Number of elements in the conj. class	$\varphi_{h_1,\ldots,h_s}$
$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	1	1	1
$\begin{pmatrix}1 & 0\\\beta & 1\end{pmatrix}$	q-1	1	$\varepsilon^{k_1h_1+\ldots+k_sh_s}$

Table 1

The irreducible CG-characters are given by table II in [5] and are as follows:

Irreducible CG-Characters	Degree	Frequency
$\chi_1^{(n)}$	1	$q-1$
$\chi_q^{(n)}$	q	$q-1$
$\chi_{q+1}^{(m,n)}$	q+1	$\frac{1}{2}(q-1)(q-2)$
$\chi_{q-1}^{(n)}$	q-1	$\frac{1}{2}q(q-1)$

Table 2

All the elements of H-{1} are conjugate in G and each of them are similar to the matrix

$$\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}.$$

We thus have the values of irreducible CG-characters on H. They are shown in Table 3.

Elements	$\chi_1^{(n)}$	$\chi_q^{(n)}$	$\chi_{q+1}^{(m,n)}$	$\chi_{q-1}^{(n)}$
$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$	1	q	q+1	q-1
$\begin{pmatrix}1 & 0\\\beta & 1\end{pmatrix}$	1	0	1	-1

Table 3

If $\varphi=\varphi_{h_1,\ldots,h_s}$ is a nontrivial irreducible character of H, we have

$$\sum_{x \in H-\{1\}} \varphi(x) = -1 \qquad (1)$$

If $\chi \neq \chi_1^{(n)}$ is an irreducible **C**G-character, by

Table 1, Table 3 and (1)

$$(\varphi, \chi_H) = 1, \qquad (2)$$

where χ_H is the restriction of χ to H. By Frobenius Theorem

$$(\varphi^G, \chi)_G = (\varphi, \chi_H)_H = 1. \qquad (3)$$

Let $S = SL(2,q)$. The conjugacy classes and character table of S is given in [1]. The irreducible **C**S-characters are as follows:

Irreducible CS-charac.	Degree	Frequency
1_S	1	1
ϕ	q	1
χ_i	q+1	$\frac{1}{2}(q-3)$
θ_j	q-1	$\frac{1}{2}(q-1)$
ξ_1	$\frac{1}{2}(q+1)$	1
ξ_2	$\frac{1}{2}(q+1)$	1
η_1	$\frac{1}{2}(q-1)$	1
η_2	$\frac{1}{2}(q-1)$	1

Table 4

Let $1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $c = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $d = \begin{pmatrix} 1 & 0 \\ \rho & 1 \end{pmatrix}$.

For any $x \in S$, let (x) denote the conjugacy class of S containing x.

If $\beta \in K$ then $\begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \in (c)$; if $\beta \in \rho K$ then $\begin{pmatrix} 1 & 0 \\ \beta & 1 \end{pmatrix} \in (d)$.

Thus we have the values of the irreducible CS-characters on H. They are shown in Table 5.

The values of irreducible CS-characters on H.

	1_S	ϕ	χ_i	θ_j	ξ_1	ξ_2	η_1	η_2
1	1	q	q+1	q-1	$\frac{1}{2}(q+1)$	$\frac{1}{2}(q+1)$	$\frac{1}{2}(q-1)$	$\frac{1}{2}(q-1)$
c	1	0	1	-1	$\frac{1}{2}(1+\eta\sqrt{q})$	$\frac{1}{2}(1-\eta\sqrt{q})$	$\frac{1}{2}(-1+\eta\sqrt{q})$	$\frac{1}{2}(-1-\eta\sqrt{q})$
d	1	0	1	-1	$\frac{1}{2}(1-\eta\sqrt{q})$	$\frac{1}{2}(1+\eta\sqrt{q})$	$\frac{1}{2}(-1-\eta\sqrt{q})$	$\frac{1}{2}(-1+\eta\sqrt{q})$

Table 5

If $q\equiv 1 \pmod 4$, the element (-1) is a square in F^*_q. If $q\equiv 3 \pmod 4$, the element (-1) is not a square in F^*_q, so that c^{-1} and c are not conjugate and d^{-1} and d are not conjugate in S, forcing $c^{-1}\in(d)$, $d^{-1}\in(c)$. Thus we have the following Lemma:

Lemma. 1. For $q \equiv 1 \pmod 4$: Every element of H is conjugate to its inverse in S.

2. For $q \equiv 3 \pmod 4$: $c^{-1}\in(d)$, $d^{-1}\in(c)$.

Let $\varphi=\varphi_{h_1,\ldots,h_s}$ be a nontrivial irreducible character of H. By Frobenius Theorem

$$(\varphi^S)^G=\varphi^G \text{ and } ((\varphi^S)^G,\chi)_O=(\varphi^S,\chi_S)_S .$$

Thus we have by (3)

$$(\varphi^S,\chi_S)_S = (\varphi^G,\chi)_G=1$$

and by [4]

$$(\chi_S,\chi_S)=1 \text{ or } 2$$

Then it is easy to see that

$$(\varphi^S,\phi)_S=1,\ (\varphi^S,\chi_i)_S=1,\ (\varphi^S,\theta_j)_S=1\ , \qquad (4)$$

where $1 \le i \le (q-3)/2$; $1 \le j \le (q-1)/2$.

1) If $q\equiv 1 \pmod 4$: Then $\eta=+1$ and

$(\varphi^S, \xi_j)_S = (\varphi, \xi_{j|H})_H =$

$$= q^{-1}\left\{\frac{1}{2}(q+1) + \left(\sum_{\beta\in K}\Psi(\beta)\right)\frac{1}{2}(1+(-1)^{j-1}\sqrt{q}) + \left(\sum_{\beta\in\rho K}\Psi(\beta)\right)\frac{1}{2}(1+(-1)^{j}\sqrt{q})\right\}$$

$= 1$ or 0.

(5)

$(\varphi^S, \eta_j)_S = (\varphi, \eta_{j|H})_H =$

$$= q^{-1}\left\{\frac{1}{2}(q-1) + \left(\sum_{\beta\in K}\Psi(\beta)\right)\frac{1}{2}(-1+(-1)^{j-1}\sqrt{q}) + \left(\sum_{\beta\in\rho K}\Psi(\beta)\right)\frac{1}{2}(-1+(-1)^{j}\sqrt{q})\right\}$$

$= 1$ or 0.

2) If $q \equiv 3 (\mathrm{mod}\ 4)$: Then $\eta = +i$ and

$(\varphi^S, \xi_j)_S = (\varphi, \xi_{j|H})_H =$

$$= q^{-1}\left\{\frac{1}{2}(q+1) + \left(\sum_{\beta\in K}\Psi(\beta)\right)\frac{1}{2}(1+(-1)^{j}\, i\sqrt{q}) + \left(\sum_{\beta\in\rho K}\Psi(\beta)\right)\frac{1}{2}(1+(-1)^{j-i}\, i\sqrt{q})\right\}$$

$= 1$ or 0,

(6)

$(\varphi^S, \eta_j)_S = (\varphi, \eta_{j|H})_H =$

$$= q^{-1}\left\{\frac{1}{2}(q-1) + \left(\sum_{\beta\in K}\Psi(\beta)\right)\frac{1}{2}(-1+(-1)^{j}\, i\sqrt{q}) + \left(\sum_{\beta\in\rho K}\Psi(\beta)\right)\frac{1}{2}(-1+(-1)^{j-i}\, i\sqrt{q})\right\}$$

$= 1$ or 0,

where $j=1,2$; $\beta = k_1 + k_2\theta + \ldots + k_s\theta^{s-1}$,; $\varphi = \varphi_{h_1,\ldots,h_s}$; $\Psi(\beta) = \varepsilon^{k_1h_1+\ldots+k_sh_s}$;

$\varepsilon^p = 1$, $\varepsilon \neq 1$; $0 \le k_i \le p-1$; $0 \le h_i \le p-1$, $i=1,\ldots,s$; $(h_1,\ldots,h_s) \neq (0,\ldots,0)$ and $i = \sqrt{-1}$.

If $(\varphi^S,\xi_1)_S=1$ then $(\varphi^S,\xi_2)_S=0$ and $(\varphi^S,\eta_1)_S=1$, $(\varphi^S,\eta_2)_S=0$.

(7)

If $(\varphi^S,\xi_1)_S=0$ then $(\varphi^S,\xi_2)_S=1$ and $(\varphi^S,\eta_1)_S=0$, $(\varphi^S,\eta_2)_S=1$.

Since $x_{(s)}(\Psi) = \sum_{\beta\in K}\psi(\beta)$, $y_{(s)}(\psi) = \sum_{\beta\in\rho K}\Psi(\beta)$, using (5), (6), (7) from the solution of a simple system of linear equations, we obtain the following result which we have already shown in [6] by another way.

Lemma 2. $\left\{ x_{(s)}(\Psi),\ y_{(s)}(\Psi) \right\} = \left\{ -\frac{1}{2}(1-\eta\sqrt{q}),\ -\frac{1}{2}(1+\eta\sqrt{q}) \right\}$

where $\eta = \begin{cases} +1 & \text{for } q\equiv 1\ (\mathrm{mod} 4) \\ +i & \text{for } q\equiv 3\ (\mathrm{mod} 4) \end{cases}$; $i = \sqrt{-1}$.

Proposition. 1) For any nontrivial irreducible character $\varphi=\varphi_{h_1,\ldots,h_s}$ of H,

$$(\varphi^S,\phi)_S=1,\ (\varphi^S,\chi_i)_S=1,\ (\varphi^S,\theta_j)_S=1$$

where $1 \le i \le (q-3)/2$; $1 \le j \le (q-1)/2$.

2) Let $\theta \in \rho K$ and $\theta^s = a_1+a_2\theta+\ldots+a_s\theta^{s-1}$, $a_1 \neq 0$. $q=p^s$.

a) If $s=2n+1$, $n\in \mathbf{N}\cup\{0\}$ then for $\varphi=\varphi_{1,0,\ldots,0}$

$$(\varphi^S,\xi_1)_S=1,\ (\varphi^S,\xi_2)_S=0,\ (\varphi^S,\eta_1)_S=1,\ (\varphi^S,\eta_2)_S=0$$

and for $\varphi=\varphi_{0,\ldots,0,a_1}$

$$(\varphi^S,\xi_1)_S=0,\ (\varphi^S,\xi_2)_S=1,\ (\varphi^S,\eta_1)_S=0,\ (\varphi^S,\eta_2)_S=1.$$

b) If $s=2n$, $n\in\mathbf{N}$ then for $\varphi=\varphi_{1,0,\ldots,0}$

$$(\varphi^S,\xi_1)_S=0,\ (\varphi^S,\xi_2)_S=1,\ (\varphi^S,\eta_1)_S=0,\ (\varphi^S,\eta_2)_S=1$$

and for $\varphi=\varphi_{0,\ldots,0,a_1}$

$$(\varphi^S,\xi_1)_S=1,\ (\varphi^S,\xi_2)_S=0,\ (\varphi^S,\eta_1)_S=1,\ (\varphi^S,\eta_2)_S=0.$$

3) Let $\theta\in K$ and $\varphi=\varphi_{1,0,\ldots,0}$ then for any $s\in\mathbf{N}$ $(q=p^s)$

$$(\varphi^S,\xi_1)_S=1,\ (\varphi^S,\xi_2)_S=0,\ (\varphi^S,\eta_1)_S=1,\ (\varphi^S,\eta_2)_S=0.$$

Proof. 1) See (4).

2) Since $\theta\in\rho K$ then by property 1

$$x_{(s)}=\begin{cases}-\frac{1}{2}(1-\eta\sqrt{q}) & \text{for } s=2n+1,\ n\in\mathbf{N}\cup\{0\}\\ -\frac{1}{2}(1+\sqrt{q}) & \text{for } s=2n,\ n\in\mathbf{N}\end{cases} \quad \text{and} \quad y_{(s)}=\begin{cases}-\frac{1}{2}(1+\eta\sqrt{q}) & \text{for } s=2n+1,\ n\in\mathbf{N}\cup\{0\}\\ -\frac{1}{2}(1-\sqrt{q}) & \text{for } s=2n,\ n\in\mathbf{N}\end{cases}$$

where $\eta=\begin{cases}+1, & q\equiv 1 \pmod 4\\ +i, & q\equiv 3 \pmod 4\end{cases}$; $i=\sqrt{-1}$.

a) If $s=2n+1$, $n\in\mathbf{N}\cup\{0\}$:

i) $q\equiv 1 \pmod 4$: $\eta=+1$ and if $\varphi=\varphi_{1,0,\ldots,0}$ then

$$\sum_{\beta\in K}\Psi(\beta)=x_{(s)};\quad \sum_{\beta\in\rho K}\Psi(\beta)=y_{(s)}. \tag{8}$$

Thus by (5) we have

$$(\varphi^S,\xi_1)_S=1,\ (\varphi^S,\xi_2)_S=0,\ (\varphi^S,\eta_1)_S=1,\ (\varphi^S,\eta_2)_S=0.$$

If $\theta^s=a_1+a_2\theta+\ldots+a_s\theta^{s-1}$, $a_1\neq 0$ and $\beta=k_1+k_2\theta+\ldots+k_s\theta^{s-1}$

then $\theta\beta = a_1k_s + k'_2\theta + ... + k'_s\theta^{s-1}$. Since $\theta \in \rho K$, if $\beta \in K$ then $\theta\beta \in \rho K$ and if $\beta \in \rho K$ then $\theta\beta \in K$. Thus we obtain

$$\sum_{\beta\in K} \varepsilon^{a_1k_s} = \sum_{\beta\in\rho K} \varepsilon^{k_1} = y_{(s)}; \quad \sum_{\beta\in\rho K} \varepsilon^{a_1k_s} = \sum_{\beta\in K} \varepsilon^{k_1} = x_{(s)}. \tag{9}$$

Let $\varphi = \varphi_{0,...,0,a_1}$ then by (5) and (9) we have

$$(\varphi^S, \xi_j)_S = q^{-1}\left\{\frac{1}{2}(q+1) + y_{(s)}\frac{1}{2}(1+(-1)^{j-1}\sqrt{q}) + x_{(s)}\frac{1}{2}(1+(-1)^j\sqrt{q})\right\}$$

$$(10)$$

$$(\varphi^S, \eta_j)_S = q^{-1}\left\{\frac{1}{2}(q-1) + y_{(s)}\frac{1}{2}(-1+(-1)^{j-1}\sqrt{q}) + x_{(s)}\frac{1}{2}(-1+(-1)^j\sqrt{q})\right\}$$

$$j = 1,2$$

and by (10)

$$(\varphi^S, \xi_1)_S=0, \quad (\varphi^S, \xi_2)_S=1, \quad (\varphi^S, \eta_1)_S=0, \quad (\varphi^S, \eta_2)_S=1$$

ii) $q \equiv 3 \pmod 4$: $\eta = +\mathbf{i}$ and if $\varphi = \varphi_{1,0,...,0}$ then by (8) and (6) we have

$$(\varphi^S, \xi_1)_S=1, \quad (\varphi^S, \xi_2)_S=0, \quad (\varphi^S, \eta_1)_S=1, \quad (\varphi^S, \eta_2)_S=0.$$

If $\varphi = \varphi_{0,...,0,a_1}$ then by (6) and (9) we have

$$(\varphi^S, \xi_j)_S = q^{-1}\left\{\frac{1}{2}(q+1) + y_{(s)}\frac{1}{2}(1+(-1)^j\,\mathbf{i}\sqrt{q}) + x_{(s)}\frac{1}{2}(1+(-1)^{j-1}\,\mathbf{i}\sqrt{q})\right\}$$

$$(11)$$

$$(\varphi^S, \eta_j)_S = q^{-1}\left\{\frac{1}{2}(q-1) + y_{(s)}\frac{1}{2}(-1+(-1)^j\,\mathbf{i}\sqrt{q}) + x_{(s)}\frac{1}{2}(-1+(-1)^{j-1}\,\mathbf{i}\sqrt{q})\right\}$$

$$j = 1,2$$

and by (11)

$$(\varphi^S, \xi_1)_S=0, \quad (\varphi^S, \xi_2)_S=1, \quad (\varphi^S, \eta_1)_S=0, \quad (\varphi^S, \eta_2)_S=1.$$

b) If $s=2n$, $n\in\mathbf{N}$: Then $q \equiv 1 \pmod 4$ and if

$\varphi=\varphi_{1,0,...,0}$ by (5) and (8) we have

$$(\varphi^S, \xi_1)_S=0, \quad (\varphi^S, \xi_2)_S=1, \quad (\varphi^S, \eta_1)_S=0, \quad (\varphi^S, \eta_2)_S=1.$$

If $\varphi = \varphi_{0,...,0,a_1}$ by (10)

$$(\varphi^S, \xi_1)_S=1, \quad (\varphi^S, \xi_2)_S=0, \quad (\varphi^S, \eta_1)_S=1, \quad (\varphi^S, \eta_2)_S=0.$$

3) If $\theta \in K$ by property 1 for any $s\in\mathbf{N}$

$$x_{(s)} = \begin{cases} -\frac{1}{2}(1-\sqrt{q}) \text{ for } q\equiv 1(\text{mod}4) \\ -\frac{1}{2}(1-i\sqrt{q}) \text{ for } q\equiv 3(\text{mod}4) \end{cases} ; \quad y_{(s)} = \begin{cases} -\frac{1}{2}(1+\sqrt{q}) \text{ for } q\equiv 1(\text{mod}4) \\ -\frac{1}{2}(1+i\sqrt{q}) \text{ for } q\equiv 3(\text{mod}4) \end{cases}$$

If $\varphi = \varphi_{1,0,\dots,0}$:

i) $q\equiv 1$ (mod 4): Then by (5) and (8) we have

$$(\varphi^S, \xi_1)_S=1, \quad (\varphi^S, \xi_2)_S=0, \quad (\varphi^S, \eta_1)_S=1, \quad (\varphi^S, \eta_2)_S=0.$$

ii) $q\equiv 3$ (mod 4): Then by (6) and (8) we have

$$(\varphi^S, \xi_1)_S=1, \quad (\varphi^S, \xi_2)_S=0, \quad (\varphi^S, \eta_1)_S=1, \quad (\varphi^S, \eta_2)_S=0.$$

Finally using the above proposition and property 3, we have the following theorem:

Theorem. If e_ϕ, e_{χ_i}, e_{θ_j}, e_{ξ_1}, e_{ξ_2}, e_{η_1}, e_{η_2}, are the central idempotents afforded by the irreducible **C**SL(2,q)-characters ϕ, χ_i, θ_j, ξ_1, ξ_2, η_1, η_2 respectively and $e_{h_1,\dots,h_s}$ is the central idempotent afforded by the irreducible **C**H-character $\varphi_{h_1,\dots,h_s}$ then:

1) The primitive idempotents of the group algebra **C**SL(2,q) which correspond to ϕ, χ_i, θ_j are as follows:

$e_\phi e_{h_1,\dots,h_s}$; $e_{\chi_i} e_{h_1,\dots,h_s}$; $e_{\theta_j} e_{h_1,\dots,h_s}$ respectively,

where $1 \le i \le (q-3)/2$; $1 \le j \le (q-1)/2$, $1 \le h_i \le p-1$, $i=1,\dots,s$; $(h_1,\dots,h_s)\neq(0,\dots 0)$.

2) If $\theta \in \rho K$ and $\theta^s = a_1+a_2\theta+\dots+a_s\,\theta^{s-1}$, $a_1\neq 0$, then the primitive idempotents of the group algebra **C**SL(2,q) which correspond to ξ_1, ξ_2, η_1, η_2 are as follows:

a-) If $s=2n+1$, $n\in\mathbf{N}\cup\{0\}$

$e_{\xi_1} e_{1,0,\dots,0}$; $e_{\xi_2} e_{0,\dots,0,a_1}$; $e_{\eta_1} e_{1,0,\dots,0}$; $e_{\eta_2} e_{0,\dots,0,a_1}$

, respectively.

b-) If $s=2n$, $n\in\mathbf{N}$

$e_{\xi_1} e_{0,\dots,0,a_1}$; $e_{\xi_2} e_{1,0,\dots,0}$; $e_{\eta_1} e_{0,\dots,0,a_1}$; $e_{\eta_2} e_{1,0,\dots,0}$

, respectively.

3-) If $\theta \in K$ and $u = \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix}$ then for any $s \in N$, the primitive idempotents of the group algebra CSL(2,q) which correspond to ξ_1, ξ_2, η_1, η_2 are as follows:

$$e_{\xi_1} e_{1,0,\ldots,0};\ u e_{\xi_1} e_{1,0,\ldots,0} u^{-1};\ e_{\eta_1} e_{1,0,\ldots,0};\ u e_{\eta_1} e_{1,0,\ldots,0} u^{-1}.$$

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