FLOW OF CONDUCTING BINGHAM PLASTICS BETWEEN TWO PARALLEL PLATES WITH SUCTION AND INJECTION UNDER A TRANSVERSE MAGNETIC FIELD

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In the present paper, we have investigated the flow of a conducting Bingham Plastic fluid between two parallel plates with constant suction and injection when one plate is moving with constant velocity, under a transverse magnetic field. We find that suction and injection has no effect on the thickness of a plug, but it shifts the plug towards the plate with suction.

1. Introduction. Recently Turgut Sarpkaya has investigated the flow of a conducting Bingham Plastic fluid between two parallel plates under a transverse magnetic field. Some of the expressions obtained by him are not correct. For example, the expression for P_{xy} the shearing stress for the fluid region has also been taken for the plug region where it does not hold. In the present paper the problem has been extended to include suction and injection and the motion of the boundary also. The shearing stress approaches the yield stress as the plug is approached and therefore the fluid coming from below becomes solid. The same quantity from the upper surface of the plug becomes liquid. Thus in the steady case we can assume a constant transverse velocity.

2. Basic equations and their integration. Let ϱ, μ, σ, μ_e and P_0 be the density, the viscosity, the electrical conductivity, the magnetic permeability and the yield stress of the fluid and $\mathbf{V}(U_x, v_0, 0)$, $\mathbf{H}(H_x, H_0, 0)$, $\mathbf{E}(0. 0, E_z)$, $\mathbf{J}(0, 0, J_z)$, P_{xy} and p be the velocity, the magnetic field, the electric field, the current density, the shearing stress and the pressure at any point. It is assumed that the flow is steady and does not depend upon x and z. In this case the pressure gradient aloug the x-axis comes out to be constant. The simplified equations for this case are

(1) $$v_0 \frac{\partial U_x}{\partial \bar{y}} = -\frac{1}{\varrho} \frac{\partial p}{\partial \bar{x}} + \frac{\partial P_{xy}}{\partial \bar{y}} + \frac{\mu_e H_0}{4 \pi \varrho} \frac{\partial H_x}{\partial \bar{y}},$$

(2) $$-\frac{\partial H_x}{\partial \bar{y}} = 4 \pi \sigma \left[E_z + \mu_e H_0 U_x - \mu_e v_0 H_x\right],$$

(3) $$P_{xy} = \pm P_0 + \mu \frac{\partial U_x}{\partial \bar{y}}.$$

We now transform these equations to the non-dimensional form with the relations

(4)
$$u_n = u_0 u, \qquad v_0 = m u_0, \qquad H_x = H_0 H, \qquad \bar{y} = L y, \qquad P_{xy} = \varrho u_0^2 \tau,$$

$$P_0 = \varrho u_0^2 \tau_0, \qquad R = \frac{\varrho u_0 L}{\mu}, \qquad R_m = 4 \pi \sigma \mu_e u_0 L, \qquad M = \mu_e H_0 L \sqrt{\frac{\sigma}{\mu}},$$

$$P = -\frac{L}{\varrho u_0^2} \frac{\partial P}{\partial \bar{x}} \qquad \text{and} \qquad E_z = \mu_e U_0 H_0 E.$$

In the above R is the REYNOLD's number, R_m the magnetic REYNOLD's number, M the HARTMANN number, τ the non-dimensional shearing stress and $2L$ is the normal distance between the two plates. With these relations the equations in non-dimensional form are:

(5) $$m \frac{du}{dy} = P + \frac{d\tau}{dy} + \frac{M^2}{RR_m} \frac{dH}{dy},$$

(6) $$u = mH - \frac{1}{R_m} \frac{dH}{dy} - E,$$

(7) $$\tau = \pm \tau_0 + \frac{1}{R} \frac{du}{dy}.$$

Integrating equation (5), we get

(8) $$\tau = mu - \frac{M^2}{RR_m} H - Py + \text{Const.}$$

Eliminating u and τ between equations (6), (7) and (8) and solving we obtain,

(9) $$H = A e^{\alpha y} + B e^{\beta y} + \frac{PRR_m}{m^2 RR_m - M^2} y + c,$$

where A, B and C are constants and

(10) $$\alpha, \beta = \frac{m}{2}(R + R_m) \pm \sqrt{\frac{m^2}{4}(R - R_m)^2 + M^2},$$

Substituting H from equation (9) in equation (6) we obtain

(11)
$$u = A\left(m - \frac{\alpha}{R_m}\right) e^{\alpha y} + B\left(m - \frac{\beta}{R_m}\right) e^{\beta y} + \frac{m PR R_m}{m^2 R R_m - M^2} y - \frac{PR}{m^2 R R_m - M^2} + mc - E.$$

3. Boundary Conditions. We take the plates to be non - conducting and the lower plate to be moving with the constant velocity $u_0 U_0'$ and the plug to be moving with the velocity $u_0 U_0$. It is assumed that there is no external magnetic field parallel to the plates. Therefore the continuity of the magnetic field gives that the tangential component of the field must vanish at both plates. Let the plug be extending from $y = -y_2$ to $y = y_1$ and the value of H at the lower and upper surface of the plug be H_2 and H_1 respectively. Again in the upper region, u is decreasing with increase of y, that is, du/dy is negative. Therefore to make τ numerically greater than τ_0, we take the sign of τ_0 at $y = y_1$ to be negative. Then the sign of τ_0 at the lower region is positive. Thus the boundary conditions are

(12)

$$\text{at} \quad y = -1, \quad u = U_0' \quad H = 0; \quad \text{at } y = -y_2, \quad u = U_0, \quad \tau = \tau_0, \quad H = H_2,$$

$$\text{at} \quad y = 1, \quad u = 0, \quad H = 0; \quad \text{at } y = y_1, \quad u = U_0, \quad \tau = -\tau_0, \quad H = H_1.$$

4. Solution for the plug region. In the plug region the equations of motion are:

(13) $$P + \frac{d\tau}{dy} + \frac{M^2}{RR_m} \frac{dH}{dy} = 0,$$

(14) $$U_0 = mH - \frac{1}{R_m} \frac{dH}{dy} - E.$$

Solving these we get

(15) $$\tau = -Py - \frac{M^2}{RR_m} H + \text{Const.},$$

(16) $$H = D\, e^{mR_m y} + \frac{E + U_0}{m},$$

where D is an arbitrary constant.

Using the boundary conditions we get

(17) $$\tau = -\tau_0 + P(y_1 - y) + \frac{P(y_1 + y_2) - 2\tau_0}{e^{-mR_m y_1} - e^{-mR_m y_1}} (e^{mR_m y_1} - e^{mR_m y}),$$

(18) $$H = \frac{RR_m}{M^2} \frac{P(y_1 + y_2) - 2\tau_0}{e^{-mR_m y_2} - e^{mR_m y_2}} e^{mR_m y} + \frac{E + U_0}{m},$$

and

(19) $$H_1 = \frac{RR_m}{m^2} \frac{P(y_1 + y_2) - 2\tau_0}{e^{-mR_m y_2} - e^{mR_m y_2}} e^{mR_m y_1} + \frac{E + U_0}{m},$$

(20) $$H_2 = \frac{RR_m}{m^2} \frac{P(y_1 + y_2) - 2\tau_0}{e^{-mR_m y_2} - e^{mR_m y_2}} e^{-mR_m y_2} + \frac{E + U_0}{m}.$$

5. Solution for the upper region. Using thd values of H_1 and H_2 from (19) and (20) and the other boundary conditions, we obtain the expressions for the variables, for upper region as

(21)
$$u = \frac{m\,P\,RR_m}{m^2\,RR_m - M^2}(y-1) + \frac{RR\,R_m}{(\beta-\alpha)(m^2\,RR_m - M^2)^2}\Big[\alpha^2\left(m - \frac{\beta}{R_m}\right)(e^{\beta y} - e^{\beta})\,e^{\beta - y_1}$$
$$- \beta^2\left(m - \frac{\alpha}{R_m}\right)e^{-\alpha y_1}(e^{\alpha y} - e^{\alpha}) - \frac{m\,(m^2\,RR_m - M^2)}{e^{-mR_m y_2} - e^{mR_m y_2}}(y_1 + y_2 - 2\tau_0/P)\,e^{-mR\,y_1}$$
$$\{\alpha\,e^{\alpha y_1}(e^{\beta y} - e^{\beta}) - \beta\,e^{\beta y_1}(e^{\alpha y} - e^{\alpha})\},$$

(22)
$$H = \frac{P^2\,RR_m}{m^2\,RR_m - M^2}(y-1) + \frac{P\,RR_m}{(\beta-\alpha)(m^2\,RR_m - M^2)^2}\Big[\alpha^2\,e^{-\beta y_1}(e^{\beta y} - e^{\beta}) - \beta^2\,e^{\alpha y_1}$$
$$(e^{\alpha y} - e^{\alpha}) - \frac{mR_m^2\,(m^2\,RR_m - M^2)}{M^2\,(e^{-mR_m y_2} - e^{-mR_m y_1})}(y_1 + y_2 - 2\tau_0/P)\Big\{\alpha\left(m - \frac{\alpha}{R_m}\right)e^{\alpha y_1}$$
$$(e^{\beta y} - e^{\beta}) - \beta\left(m - \frac{\beta}{R_m}\right)e^{\beta y_1}(e^{\alpha y} - e^{\alpha})\Big\}\Big],$$

(23)
$$U_0 = \frac{m\,PR\,R_m}{m^2\,RR_m - M^2}(y_1 - 1) + \frac{P\,RR_m}{(\beta-\alpha)(m^2\,RR_m - m^2)}\Big[\alpha_2\left(m - \frac{\beta}{R_m}\right)(1 - e^{\beta(1-y_1)})$$
$$- \beta^2\left(m - \frac{\alpha}{R_m}\right)(1 - e^{\alpha(1-y_1)}) - \frac{m\,(m^2\,RR_m - M^2)\,e^{-mRy_1}}{e^{-mR_m y_2} - e^{mR_m y_1}}(y_1 + y_2 - 2\tau_0/P)$$
$$\{\alpha\,(e^{(\alpha+\beta)\,y_1} - e^{\beta + \alpha y_1}) - \beta\,(e^{(\alpha+\beta)\,y_1}) - e^{\alpha + \beta y_1})\}\Big].$$

(24)
$$H_1 = \frac{P\,RR_m}{m^2\,RR_m - M^2}(y_1 - 1) + \frac{P\,RR_m}{(\beta-\alpha)(m^2\,RR_m - m^2)^2}\Big[\alpha^2\,(1 - e^{\beta\,((1-y_1))})$$
$$- \beta^2\,(1 - e^{\alpha(1-y_1)}) + \frac{m\,(m^2\,RR_m - M^2)\,R_m^2\,e^{-mRy_1}}{M^2\,(e^{-mR_m y_2} - e^{mR_m y_1})}(y_1 + y_2 - 2\tau_0/P)$$
$$\Big\{\alpha\left(m - \frac{\alpha}{R_m}\right)e^{\alpha y_1}(e^{\beta y_1} - e^{\beta}) - \beta\left(m - \frac{\beta}{R_m}\right)e^{\beta y_2}(e^{\alpha y_1} - e^{\alpha})\Big\}\Big],$$

(25)

$$E=\frac{PR}{(\alpha-\beta)\,(m^2 RR_m-M^2)}\Big[\beta-\alpha-\beta\, e^{\alpha(1-y_1)}+\alpha\, e^{\beta(1-y_1)}-\frac{mR_m^2\,(m^2 RR_m-M^2)}{M^2\,(e^{-mR_m y_2}-e^{mR_m y_1})}$$

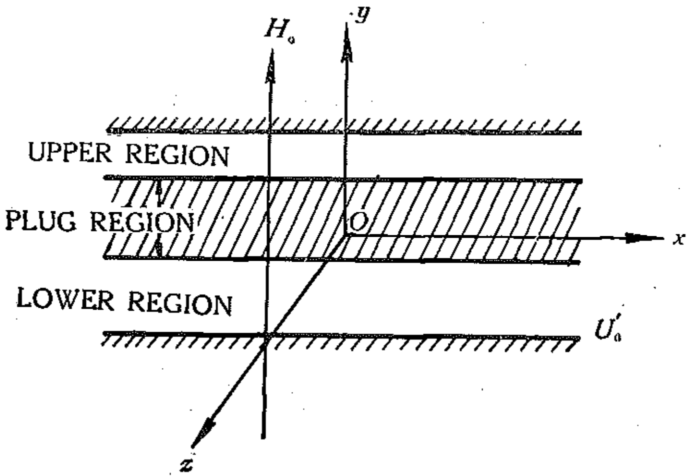
$$e^{-mR\, y_1}\Big\{\Big(m-\frac{\beta}{R_m}\Big)e^{\alpha+\beta y_1}-\Big(m-\frac{\alpha}{R_m}\Big)e^{\beta+\alpha y_1}\Big\}\Big],$$

(26)

$$\tau=-\tau_0+\frac{PR_m}{(\beta-\alpha)\,(m^2 RR_m-M^2)}\Big[\alpha\Big(m-\frac{\beta}{R_m}\Big)e^{\beta(y-y_1)}-\beta\Big(m-\frac{\alpha}{R_m}\Big)e^{\alpha(y-y_1)}$$

$$+\frac{m\,(m^2 RR_m-M^2)\,e^{-mR\,y_1}}{e^{-mR_m y_2}-e^{mR_m y_1}}\,(y_1+y_2-2\,\tau_0/P)\,(e^{\alpha y_1+\beta y}-e^{\beta y_1+\alpha y})\Big]+\frac{mPR_m}{m^2 RR_m-M^2}.$$

6. Solution for the lower region. Similarly the solution for lower region is,



(27)

$$u=\frac{P\,RR_m}{(\alpha-\beta)\,(m^2 RR_m-M^2)^2}\Big[\beta^2\Big(m-\frac{\alpha}{R_m}\Big)e^{\alpha y_2}\,(e^{\alpha y}-e^{-\alpha})-\alpha^2\Big(m-\frac{\beta}{R_m}\Big)e^{\beta y_2}$$

$$\times\,(e^{\beta y}-e^{-\beta})+\frac{m\,(m^2 RR_m-M^2)\,e^{mR\,y_2}}{e^{-mR_m y_2}-e^{mR_m y_1}}\,(y_1+y_2-2\,\tau_0/P)\,\{\alpha\, e^{-\alpha y_2}\,(e^{\beta y}-e^{-\beta})$$

$$-\beta\, e^{\beta y_2}\,(e^{\alpha y}-e^{-\alpha})\}\Big]+\frac{mP\,RR_m}{m^2 RR_m-M^2}\,(y+1)+U_0'.$$

(28)

$$H=\frac{P\,RR_m}{(\alpha-\beta)\,(m^2 RR_m-M^2)^2}\Big[\beta^2\, e^{\alpha y_2}\,(e^{\alpha y}-e^{-\alpha})-\alpha^2\, e^{\beta y_2}\,(e^{\beta y}-e^{-\beta})$$

$$+\frac{mR_m^2(m^2RR_m-M^2)\,e^{mRy_2}}{(e^{-mR_my_2}-e^{mR_my_1})\,M^2}(y_1+y_2-2\tau_0/P)\left\{\alpha\left(m-\frac{\alpha}{R_m}\right)e^{-\alpha y_2}(e^{\beta y}-e^{-\beta})\right.$$

$$\left.\left.-\beta\left(m-\frac{\beta}{R_m}\right)e^{-\beta y_2}(e^{\alpha y}-e^{-\alpha})\right\}\right]+\frac{PRR_m}{m^2RR_m-M^2}(y+1).$$

(29)

$$U_0=\frac{PRR_m}{(\alpha-\beta)(m^2RR_m-M^2)^2}\left[\alpha^2\left(m-\frac{\beta}{R_m}\right)(1-e^{-\beta(1-y_2)})-\beta^2\left(m-\frac{\alpha}{R_m}\right)\right.$$

$$\times(1-e^{-\alpha(1-y_2)})-\frac{m(m^2RR_m-M^2)\,e^{mRy_2}}{e^{-mR_my_2}-e^{mR_my_1}}(y_1+y_2-2\tau_0/P)\{\alpha\,e^{-\alpha y_2}(e^{-\beta y_2}-e^{-\beta})$$

$$\left.-\beta\,e^{-\beta y_2}(e^{-\alpha y_2}-e^{-\alpha})\}\right]+\frac{mPRR_m}{m^2RR_m-M^2}(1-y_2),$$

(30)

$$H_2=\frac{PRR_m}{(\beta-\alpha)(m^2RR_m-M^2)^2}\left[\alpha^2(1-e^{-\beta(1-y_2)})-\beta^2(1-e^{-\alpha(1-y_2)})\right.$$

$$-\frac{m(m^2RR_m-M^2)R_m^2e^{mRy_2}}{M^2(e^{-mR_my_2}-e^{mR_my_1})}\left\{\alpha\left(m-\frac{\alpha}{R_m}\right)e^{-\alpha y_2}(e^{-\beta y_2}-e^{-\beta})-\beta\left(m-\frac{\beta}{R_m}\right)\right.$$

$$\left.\times e^{-\beta y_2}(e^{-\alpha y_2}-e^{-\alpha})\right]+\frac{PRR_m(1-y_2)}{m^2RR_m-M^2},$$

(31)

$$E=\frac{PR}{(\alpha-\beta)(m^2RR_m-M^2)}\left[\beta-\alpha+\alpha\,e^{\beta(y_2-1)}-\beta\,e^{\alpha(y_2-1)}-\frac{mR_m^2(m^2RR_m-M^2)}{M^2(e^{-mR_my_2}-e^{mR_my_1})}\right.$$

$$\left.\times e^{mRy_2}(y_1+y_2-2\tau_0/P)\left\{\left(m-\frac{\beta}{R_m}\right)e^{-\alpha-\beta y_2}-\left(m-\frac{\alpha}{R_m}\right)e^{-\beta-\alpha y_2}\right\}\right],$$

(32)

$$\tau=\tau_0+\frac{PR_m}{(\beta-\alpha)(m^2RR_m-M^2)}\left[\alpha\left(m-\frac{\beta}{R_m}\right)e^{\beta(y+y_2)}-\beta\left(m-\frac{\alpha}{R_m}\right)e^{\alpha(y+y_2)}\right.$$

$$\left.-\frac{m(m^2RR_m-M^2)\,e^{mRy_2}}{e^{-mR_my_2}-e^{mR_my_1}}(y_1+y_2-2\tau_0/P)(e^{\beta y-\alpha y_2}-e^{\alpha y-\beta y_2})+m(\beta-\alpha)\right].$$

Equating the values of E from (25) and (31) we get

(33)

$$\tau_0=P\left[1+\tfrac{1}{2}(y_1+y_2)-\frac{M^2(e^{-mR_my_2}-e^{mR_my_1})}{2mR_m(m^2RR_m-M^2)}\{\alpha(e^{\beta(1-y_1)}-e^{\beta(y_2-1)})\right.$$

$$-\beta(e^{\alpha(1-y_1)}-e^{\alpha(y_2-1)})\}\{(mR_m-\beta)(e^{\alpha+(\beta-mR)y_1}-e^{-\alpha-(\beta-mR)y_2})-(mR_m-\alpha)$$

$$\left.\times(e^{\beta+y_1(\alpha-mR)}-e^{-\beta-(\alpha-mR)y_2}\}^{-1}\right].$$

Again equating (23) and (29) we shall get an other equation. This equation along with (33) shall determine the values of y_1 and y_2. The total flux is given by

(34)

$$Q = \int u dy = \frac{P R R_m}{(\beta-\alpha)(m^2 R R_m - M^2)^3} \Big[\alpha^3 \left(m - \frac{\beta}{R_m} \right) \{ e^{\beta(1-y_1)} (1-\beta) - (1+\beta) e^{\beta(y_2-1)}$$

$$+ \beta (y_1+y_2)\} - \beta_3 \left(m - \frac{\alpha}{R_m} \right) \{(1-\alpha) e^{\alpha(1-y_1)} - (1+\alpha) e^{-\alpha(1-y_2)} + \alpha (y_1+y_2)\}$$

$$+ \frac{m (m^2 R R_m - M^2)}{e^{-mR_m y_2} - e^{mR_m y_1}} (y_1 + y_2 - 2\tau_0/P) \Big\{ \frac{\beta}{\alpha} e^{(\beta - mR) y_1} (e^{\alpha} - e^{\alpha y_1} - \alpha e^{\alpha})$$

$$- \frac{\beta}{\alpha} e^{-(\beta - mR) y_2} (e^{-\alpha} - e^{-\alpha y_2} + \alpha e^{-\alpha}) - \frac{\alpha}{\beta} e^{(\alpha - mR) y_1} (e^{\beta} - e^{\beta y_1} - \beta e^{\beta}) +$$

$$+ \frac{\alpha}{\beta} e^{(mR-\alpha) y_2} (e^{-\beta} - e^{-\beta y_2} + \beta e^{-\beta}) + (\alpha + \beta)(y_2 e^{-mR_m y_2} + y_1 e^{mR y_1}) \Big]$$

$$- \frac{mP R R_m}{2 (m^2 R R_m - M^2)} (y_1 + y_2)^2 + U_0' (1 - y_2).$$

7. Limiting cases. **7a.** *Hydrodynamic case with suction and injection.*

When $M \to 0$, we have $\alpha \to mR$, $\beta \to mR_m (=0)$. Substituting these values and taking the limits we have for the upper region

(35) $$u = \frac{P}{m} \left[-1 + y - \frac{1}{mR} \{ e^{mR(y-y_1)} - e^{mR(1-y_1)} \} \right],$$

(36) $$U_0 = \frac{P}{m} \left[y_1 - 1 - \frac{1}{mR} \{ 1 - e^{mR(1-y_1)} \} \right],$$

(37) $$\tau = -\tau_0 + \frac{P}{mR} \left[1 - e^{mR(y-y_1)} \right].$$

For the lower region, we have

(38) $$u = u_0' + \frac{P}{m} \left[y + 1 - \frac{1}{mR} \{ e^{mR(y+y_2)} - e^{-mR(1-y_2)} \} \right],$$

(39) $$U_0 = U_0' + \frac{P}{m} \left[1 - y_2 - \frac{1}{mR} \{ 1 - e^{-mR(1-y_2)} \} \right],$$

(40) $$\tau = \tau_0 + \frac{P}{mR} \left[1 - e^{mR(y+y_2)} \right].$$

7b. *Limiting hydromagnetic case without suction or infection.* When $m \to 0$, $U_0' = 0$ we have $\alpha \to M$, $\beta \to -M$, and $y_1 = y_2 = y_0$. Substituting these values we get

$$u = \frac{PR}{m} \frac{\operatorname{Cosh} m(1-y_0) - \operatorname{Cosh} m(y-y_0)}{\operatorname{Sinh} m(1-y_0) + My_0}, \tag{41}$$

$$U_0 = \frac{PR}{m} \frac{\operatorname{Cosh} m(1-y_0) - 1}{\operatorname{Sinh} m(1-y_0) + my_0}, \tag{42}$$

$$\tau = P \frac{\operatorname{Sinh} m(y_0-y) - my_0}{\operatorname{Sinh} m(1-y_0) + my_0}, \tag{43}$$

$$\tau_0 = P \frac{MP\, y_0}{\operatorname{Sinh} m(1-y_0) + my_0}. \tag{44}$$

Instead of equations (41) to (44), Turgut SARPKAYA[1] has obtained the following expressions:

$$u = \frac{PR}{m} \frac{\operatorname{Cosh} m(1-y_0) - \operatorname{Cosh} M(y-y_0)}{(\operatorname{Sinh} M - \operatorname{Sinh} My_0) \operatorname{Cosh} My_0 + y_0 M}, \tag{45}$$

$$U_0 = \frac{PR}{m} \frac{\operatorname{Cosh} M(1-y_0) - 1}{(\operatorname{Sinh} M - \operatorname{Sinh} My_0) \operatorname{Cosh} My_0 + y_0 M}, \tag{46}$$

$$\tau = P \frac{\operatorname{Sinh} My_0 + \operatorname{Sinh} m(y-y_0)}{(\operatorname{Sinh} m - \operatorname{Sinh} my_0) \operatorname{Cosh} my_0 + my_0}, \tag{47}$$

$$\tau_0 = P \frac{\operatorname{Sinh} my_0}{(\operatorname{Sinh} m - \operatorname{Sinh} my_0) \operatorname{Cosh} my_0 + my_0}. \tag{48}$$

The difference occurs from the error in computing the integral

$$\int (E - \mu_e U_x M_0)\, dy.$$

It also appears that he has assumed that the expression for τ in the liquid region also holds for the plug region and thus derives the expression for τ_9 by substituting the condition $\tau = 0$ at $y = 0$, which in fact does not hold in the plug region.

7c. *Hydromagnetic flow of Newtonian fluid with suction and injection.*

If $\tau \to 0$, we have the Newtonian fluid. Substituting this we have

(49)

$$y_1 = -y_2, \quad m(m^2 RR_m - m^2)(\beta - \alpha) + \alpha^2 e^{-\beta y_1} \left(m - \frac{\beta}{R_m}\right) \operatorname{Sinh} \beta$$

$$+ \beta^2 e^{-\alpha y_1} \left(m - \frac{\alpha}{R_m}\right) \operatorname{Sinh} \alpha = 0,$$

(50)

$$u = \frac{P\,RR_m}{m^2\,RR_m - M^2}\left[\beta\left(m - \frac{\alpha}{R_m}\right)\frac{\text{Cosh}\,\alpha - e^{\alpha y}}{(\alpha-\beta)\,\text{Sinh}\,\alpha} - \alpha\left(m - \frac{\beta}{R_m}\right)\frac{\text{Cosh}\,\beta - e^{\beta y}}{(\alpha-\beta)\,\text{Sinh}\,\beta} + my\right]$$

$$+ \frac{U_0'}{2}\left\{1 + (mR_m - \beta)\,\frac{\text{Cosh}\,\beta - e^{\beta y}}{(\alpha-\beta)\,\text{Sinh}\,\beta} - (mR_m - \alpha)\,\frac{\text{Cosh}\,\alpha - e^{\alpha y}}{(\alpha-\beta)\,\text{Sinh}\,\alpha}\right\},$$

(51)

$$H = \frac{P\,RR_m}{m^2\,RR_m - M^2}\left\{\alpha\,\frac{\text{Cosh}\,\beta - e^{\beta y}}{(\alpha-\beta)\,\text{Sinh}\,\beta} - \beta\,\frac{\text{Cosh}\,\alpha - e^{\alpha y}}{(\alpha-\beta)\,\text{Sinh}\,\alpha} + y\right\}$$

$$+ \frac{U_0\,R_m}{2\,(\alpha-\beta)}\left\{\frac{\text{Cosh}\,\beta - e^{\beta y}}{\text{Sinh}\,\beta} - \frac{\text{Cosh}\,\alpha - e^{\alpha y}}{\text{Sinh}\,\alpha}\right\}$$

(52)

$$E = -\frac{U_0}{2}\left[\frac{1}{\alpha-\beta}\,(\alpha\,\text{Coth}\,\alpha - \beta\,\text{Coth}\,\beta)\right] + \frac{PR}{M^2 - m^2\,RR_m}\left[1 + \frac{\alpha\beta}{\alpha-\beta}(\text{Coth}\,\alpha - \text{Coth}\,\beta)\right].$$

7d. HARTMANN's *flow* :

When $m \to 0$ and $\tau \to 0$ we have the **HARTMANN** flow. For this case we have the solution :

(53) $$u = \frac{U_0'}{2}\cdot\frac{\text{Sinh}\,M - \text{Sinh}\,My}{\text{Sinh}\,M} + \frac{P\,(\text{Cosh}\,M - \text{Cosh}\,My)}{M\,\text{Sinh}\,M},$$

(54) $$H = \frac{PRR_m}{m^2}\,\frac{\text{Sinh}\,My - y\,\text{Sinh}\,M}{\text{Sinh}\,M} + \frac{U_0'R_m}{2M}\,\frac{\text{Cosh}\,My - \text{Cosh}\,M}{\text{Sinh}\,M},$$

(55) $$E = -\frac{U_0'}{2} + \frac{PR}{M^2}(1 - M\,\text{Coth}\,M).$$

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ÖZET

Bu yazıda iki paralel düzlem levha arasında bulunan iletken bir BINGHAM plâstik akışkanının akımı, sabit emme ve püskürtme, hareket doğrultusuna dik bir manyetik alan ve levhalardan bir tanesinin hızı sabit bir hareket ilo kaydırılması halinde incelenmiştir. Emme ve püskürtmenin tıkaç kalınlığı üzerinde bir tesiri olmadığı, fakat tıkaçı, emmenin vuku bulduğu levhaya doğru kaydırdığı gösterilmiştir.