Sosyal Bilimler Dergisi 2010, (2), 116-121

MEASURING THE EFFICIENCY OF PORTFOLIOS WITH DATA ENVELOPMENT ANALYSIS

Dr. Leyla YÜCEL [1]

ABSTRACT

This paper presents an innovative approach about measuring the efficiency of portfolios with Data Envelopment Analysis (DEA). Portfolios are accepted as production units which are assigned to provide return to their investors at the end of the investment period, t=1. Unlike the common sense regarding funds as portfolios and measuring their performances, in this paper the portfolios which are developed from the rough were handled. The efficiency of portfolios were measured with Data Envelopment Analysis by using the set of inputs and outputs which were derived from Markowitz's Modern Portfolio Theory. Finally, the effects of efficiency concept on realized returns of portfolios at t=1 were examined so as to demystify if realized returns rise or not.

Key Words: Data envelopment analysis, Efficiency of portfolios, DEA-CCR model, Excel-Solver Module, Frontier Analyst.

ÖZET

Bu çalışma, portföylerin etkinliklerinin Veri Zarflama Analizi (VZA) ile ölçülmesine dair yeni bir yaklaşım sunmaktadır. Portföyler, yatırım dönemi sonunda (t=1'de) yatırımcısına getiri sağlamakla görevli birer üretim birimi olarak kabul edilmeketedirler. Yatırım fonlarını portföy olarak ele alarak bunların performanslarını ölçmeye dayanan geleneksel çalışmalardan farklı olarak, gerçek verilerle, sıfırdan oluşturulmuş olan portföyler bu çalışmaya konu olmuştur. Portföylerin Veri Zarflama Analizi ile ölçülmesinde kullanılan girdiler ve çıktılar, Markowitz'in Modern Portföy Teorisi'nden faydalanılarak elde edilmişlerdir. Çalışmanın sonunda, etkinlik kavramının, portföylerin dönem sonu getirileri üzerinde herhangi bir artış sağlayıp sağlamadığı incelenmiştir.

Anahtar Kelimeler: Veri Zarflama analizi, Portföylerin etkinliği, VZA-CCR Model, Excel-Solver Eklentisi, Frontier Analyst yazılımı.

[1] İstanbul Üniversitesi, İktisat Fakültesi, Ekonometri Bölümü.

INTRODUCTION

Data Envelopment Analysis measures the efficiency of production units which produce similar outputs by using similar inputs. DEA is a nonparametric extreme point technique (Nakanishi and Falcocchio, 2004:186). Because, production function is not required and production units (also called decision making units, shortly dmu's) are compared with the most productive ones, while parametric methods such as regression analysis accepts average dmu's as productive units.

DEA is based on Farrells' Total Factor Productivity approach which is the ratio of weighted outputs to wieghted inputs. Actually, measuring the efficiency of production units by comparing them with a benchmark technology was Farrell's idea. Although, Farrell laid the foundation of DEA in 1957 on his article named "The Measurement of Productive Efficiency" (Farrell, 1957); Charnes, Cooper and Rhodes made DEA known in 1978 with their article named "Measuring the Efficiency of Decision Making Units" (Charnes et al., 1978). This article took more than 700 references between 1978-1999 in the area of social sciences (Forsund and Sarafoglu, 2002). DEA became very popular because it makes understanding of the measurements easy; analyses many inputs and outputs at the same time, calculates optimal weights to find most efficient production unit, finds what makes some production units non-efficient and provides the solution to make them efficient by acting like other efficient production units.

In this study, a group of portfolio which are produced from the rough with real data were handled and their efficiency scores were measured with data envelopment analysis. The input-output set of this problem was extracted from the modern portfolio theory. Markowitz defines efficient portfolio by two ways; the portfolio which has the most realised return at a definite level of risk or the portfolio which has the minimum level of risk at a definite level of realised return is called efficient portfolio (Markowitz, 1952: 82). The ratios of assets in portfolios are determined according to their risk and return levels. Risky portfolios probably contains risky assets. When the risks and returns of the portfolios are plotted on risk and return diagram, efficient portfolios take place on the efficient frontier. At this juncture, the efficiency concept of modern portfolio theory and data envelopment analysis concurs. Because in data envelopment analysis an efficient production unit has to produce the most output by using definite amount of input or produce a definite amount of output by using minimum input.

This article is arrenged as follows: Section 2 is about the methodology. Section 3 is the application part. The last section is the conclusion part.

II. METHODOLOGY

Efficiency can be defined as generating maximum output by using minimum input with an optimal production cycle. In this study output for the system is; "realized return" at the end of investment period t=1. Determining the output is relative easy to determine the inputs. Because there are a lot of feasible factors affecting the security prices. We can never entirely list all of these factors, but we can guess by examining the final quotations of the securities. However this way of finding feasible factors, generally gives the same result. Generally there is a big factor which explains more than %85 of total change, and a lot of small factors having %1-%2 explanation. Trzcinka tried to find the feasible factors by using covariance matrix of stocks' final quotations in 1986. He found only one important factor and 4 unimportant factors. Brown also find only one factor for NYSE in 1989. In 1993, Connor and Korajczyk found one or two factors for their research about NYSE (Özçam, 1997:30-31).

II.A. DETERMINING THE INPUTS AND THE OUTPUTS OF THE SYSTEM

An efficient portfolio must generate maximum realised return with minimum inputs compared to other portfolios. Within this objective, the inputs of this study are determined as follows;

Input1: The standart deviation of the portfolio (this is a kind of risk that arise from gathering assets in a portfolio and can be controlled and completely eliminated by diversification)

Standart deviation of the portfolio;

$$s_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j Cov(ij)} \quad \text{Eq. 1.}$$

Input2: The difference between expected return and realized return at the end of investment period, t=1.

Expected return;

$$\mathbf{E_1(R_p) = \sum_{i=1}^{N} w_i E(R_i)} \quad \text{Eq. 2.}$$

Realised return;

$$\mathbf{R_1(R_p) = \sum_{i=1}^{N} w_i * R_i} \quad \text{Eq. 3.}$$

The output of the system is the realised return at t=1. Briefly, the input-output set for measuring the efficiency of an investment portfolio problem can be seen below;

Output1: Realized return at the end of investment period, t=1, "$R_1(R_P)$"

Input1: Standart deviation of portfolio at the end of investment period, t=1, "σ_p"

Input2: $E_1(R_p) - R_1(R_p)$

Input1, σ_p, is the standart deviation of portfolio. It is calculated by using covariance matrix of returns. If a portfolio is well diversified, its standart deviation decreases. Input2 is the difference between expected return and realized return of the portfolio at the end of investment term, (t=1). This positive difference have to be decreased to the level which will be offered by DEA as a result of efficiency measurement process.

II.B. DETERMINING THE APPROPRIATE DATA ENVELOPMENT ANALYSIS MODEL FOR MEASURING PORTFOLIO EFFICIENCY

At the preliminary stage, the orientatiton of the DEA model have to be determined. DEA models can be input oriented, output oriented or non oriented models. An input oriented model aims to decrease inputs without intermeddling the outputs. An output oriented model aims to increase outputs without intervening the inputs. A non oriented model such as Additive Model, interferes both inputs and outputs, namely, it aims to decrease the inputs and increase the outputs of the system simultaneously.

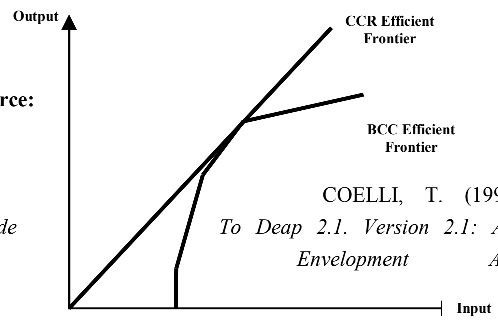
In this study, our model is required to be "input oriented", because the only output of the system, which is "realized return at t=1" occurs surprisingly all the time. Because the level of it can not be guaranteed. But it is always probable to interfere the inputs. For example, Input1 (standart deviation of portfolio) can be decreased with a better diversification. Input3 is substantially a numerical value, there is no issue in minimising it.

The second step is determining the type of DEA model. As it is well known, there are mainly four types of DEA models. They are; CCR Model, BCC Model, Additive Model and Multiplicative Model. In this paper, input oriented CCR Model was selected. The first reason why CCR model has preferred is;

- CCR model measures *general efficiency* and this is more suitable for an investment portfolio than *pure technical efficiency* and *scale efficiency* of BCC model. Eventually, portfolio is a financial investment instrument and total efficiency is more suitable than technical or scale efficiencies. Technical and scale efficiencies are convenient for measuring the efficiencies of factories, hospitals, restaurants, etc.

- Second reason is that, CCR model carries out the most accurate and sensitive efficiency measurement. As it is well known, the efficient frontier of CCR, is the leftmost one. This can be seen in Figure.1(Coelli, 1996:20)

Figure.1. The Efficient Frontiers Of CCR and BCC Models



Source: COELLI, T. (1996), *A Guide To Deap 2.1. Version 2.1: A Data Envelopment Analysis*

Computer Program, CEPA Working Paper 96/08, Internet Address; http://www.owlnet.rice. edu/~econ380/DEAP.PDF, Access Time: 2010.12.10.

- Moreover a CCR efficient decision making unit is efficient according to all other DEA models. But inverse is not true all the time. For example, a BCC or Additional efficient dmu, namely a dmu which is efficient according to Additional DEA Model, is generally not CCR efficient. Consequently, in this study, input oriented CCR model is preffered.

II.C. DETERMINING THE NUMBER OF PRODUCTION UNITS

The recommended number of dmus for measuring efficiency with DEA may vary depending on different point of views. In a system with “m” inputs and “k” outputs; there must be at least “m+k+1” dmus in DEA process (Bakırcı, 2006:168). Furthermore, according to some others, the number of dmus have to be more than “2x(m+k)”, “3x(m+k)” or “mxk”. But, if the number of dmus ascends without any rule, the explanation of the study may become irrelevant. Because DEA finds a very little subset of dmu’s as efficient. For instance let’s assume that there are two sets which have “10” and “100” dmu’s respectively. As DEA is an extreme point technique, it tends to find very few dmus efficient. So, while interpreting the results of the research, especially about the larger set with “100” dmu’s, it might be concluded, there are “3” efficient units and the remainder 97 of them are inefficient. In such a case, these “3” efficient units will be the benchmark dmu’s for the others. On the other hand, if we handle the smaller set which has “10” dmu’s, these “3” efficient units will be the benchmark for the remainder “7” dmu’s. It is relatively easy to imitate the efficient units for these “7” dmus rather “97” dmu’s. So, when big set of dmu’s are in question, it is recommended to divide the big set to subsets. Finally, compare the efficient dmu’s of each subset in the same set which is developed from those subsets.

By the way, determining the number of inputs and outputs is also important in DEA. Because the size of input-output set states the number of dmu’s. Here, the related inputs and related outputs may be grouped with factor analysis in order to decrease the size of input-output set, correspondingly the number of dmu’s required for the analysis.

III. APPLICATION

The data were obtained from Istanbul Stock Exchange, between January 2005 – December 2009. Daily closing prices were turned in to monthly data by calculating the average of the two mid days, such as 15^{th} and 16^{th} days for 30 days month. Portfolios contain gold bullion, American Dollar and stocks with different ratios according as their expected returns at the beginning of investment period, t=0.

As there are two inputs and only one output, 5 portfolios were handled in connection with the rule about the number of production units which says there must be at least “m+k+1” dmus in DEA process. Portfolios were produced with Excel- Solver module in light of Markowitz’s efficient portfolio definition. Table 1 presents the compositions of the assets in the portfolios:

Table.1. The Composition of Securities in Portfolios

PORTFOLIOS	GOLD BULLION	US $	AKCNS	AKSA	ALARKO	ALKIM	ANHYT	ASELSAN	$\sum_{i=1}^{k} w_i$
P1	0.20	0.64	0	0.12	0	0	0.01	0.03	1
P2	0	0.20	0	0.10	0	0.40	0	0.30	1
P3	0	0.20	0	0.10	0	0.60	0	0.10	1
P4	0.30	0	0	0	0	0.10	0	0.60	1
P5	0	0.10	0	0	0	0	0	0.90	1

After producing the portfolios, their efficiencies will be measured with DEA.

DEA, divides the portfolios in to two subsets such as “efficient portfolios” and “inefficient portfolios”. Then, the inefficient portfolios will be tried to be worked up into the efficient portfolios, when they become efficient it will be seeked whether the realised return increases.

The steps are like this;

1^{st} Step : calculating the inputs and outputs of portfolios at t=1,

2nd Step: measuring their efficiencies with DEA CCR model,

3rd Step: applying required minimisation to related inputs which are designated by DEA results.

4th Step : reproducing the portolios in the light of DEA's offers (with the new expected returns, the new standart deviations),

Last Step: checking the level of the realised returns of portfolios after applying DEA's offers.

The inputs and the outputs of these portfolios are calculated at t=1 can be seen in Table.2:

Table.2. The Set of Inputs and Outputs

Portfolios	I1 σ_p	I2 $E_1(R_p) - R_1(R_p)$	O1 $R_1(R_p)$
P1	0.0654	0.0455-0.0014=0.0441	0.0014
P2	0.0801	0.0366-0.0306=0.006	0.0306
P3	0.1125	0.0357-0.0439= -0,0082	0.0439
P4	0.0531	0.0409-0.0163=0.0246	0.0163
P5	0.0733	0.0529-0.0007=0.0522	0.0007

Below, Table.3 denotes the efficiency scores of the portfolios. These are the results of DEA obtained by using Frontier Analyst Software:

Table.3. Efficiency Scores of The Portfolios

Portfolios	Efficiency Scores	DEA's Offers *(DEA's Offers are generated by Frontier Analyst Software for inefficient dmus in order to make them efficient units.)*
P1	0.055	Decrease I1 %94, I2 %100.
P2	0.979	Decrease I1 %2, I2 %195.
P3	1	No offer for this efficient dmu.
P4	0.787	Decrease I1 %21, I2 %112.
P5	0.024	Decrease I1 %97, I2 %100.

Frontier Analyst Software calculated only P3 portfolio as efficient. Hence, it has no offers for P3. But it says, P1 must decrease its I1 with the ratio of %94, I2 %100. P2 must decrease its I1 with the ratio of %2, I2 %195. P4 must decrease its I1 with the ratio of %21, I2 %112. P5 must decrease its I1 with the ratio of %97, I2 %100 in order to become efficient units. The recommended new inputs after applying required minimisation for P1, P2, P4 and P5 can be seen below in Table.4:

Table.4. The Recommended New Amounts of Inputs For Inefficient Portfolios

Portfolios	I1 σ_p	I2 $E_1(R_p) - R_1(R_p)$
P1	0.0039	0
P2	0.0784	-0.0057
P4	0.0419	-0.0029
P5	0.0021	0

The new expected return of P2 is calculated below:

$E_1(R_p) - R_1(R_p) = -0.0057$

$E_1(R_p)=R_1(R_p)-0.0057=0.0306-0.0057 = 0.0249$

Eq. 4.

The others can also be calculated by this way.

After applying the DEA's offers (producing the portfolios with their new standart deviations and expected values again), the new efficiency scores and amended realized returns can be seen below in Table.5:

Table.5. Realized Returns of Portfolios After Applying DEA's Offers

Portfolios	Efficiency Scores	Realized Return After Applying DEA's Offers
P1	0.920	0.0149
P2	0.999	0.0711
P3	1	------
P4	0.997	0.0531
P5	0.854	0.0149

As can be seen from Table.5., the efficiency of P1 increased from %5.5 (see Table.3) to %92 and its realized return increased from %0.14 (see Table.2) to %1.49. The efficiency of P2 increased %97.9 to %99.9

and its realized return increased from %3.06 to %7.11. The efficiency of P4 increased from %78.7 to %99.7 and its realised return increased %1.63 to %5.31. The efficiency of P5 increased from %2.4 to %85.4 and its realized return increased from %0.007 to %1.49.

Lets have a look to the final composition of securities in portfolios after applying DEA's offers. This can be seen below in Table.6.

Table.6. The Final Composition of Securities In Portfolios After Applying DEA's Offers

Portfolios	Gold Bullion	US $	AKCNS	AKSA	ALARKO	ALKIM	ANHYT	ASELSAN	$\sum_{i=1}^{k} W_i$
P1	0.37	0.28	0	0	0.02	0.05	0	0.28	1
P2	0.25	0	0	0	0	0.39	0	0.36	1
P3	0	0.20	0	0.10	0	0.60	0	0.10	1
P4	0.47	0	0	0	0	0.08	0	0.45	1
P5	0.37	0.28	0	0	0.02	0.05	0	0.28	1

After applying DEA's offers to the portfolios, it can be seen from Table.6 that, the securities are more equally distributed. For example P5 was composed of %90 Stock 5 and %10 US dolar before DEA (see Table 1). But after applying DEA's offers, it became well balanced on account of assests, also its realised return arised from %0.007 to %1.49, which means 21.28 times more than the first case. Similar comments can be done for the other inefficient portfolios.

CONCLUSION

This new approach can be described as ***"diversifying the securities in order to increase the realised return at t=1 in the light of DEA's offers, by applying required reduction to the inputs"***. Thereby, it can be seen that increasing the efficiency score of an investment portfolio brings muchmore realised return at the end of investment period t=1. The findings of Section 3 shows this. Surely it is not an absolute way to enrich an investor on its own. Some times DEA's offers may not be satisfactory, especially when inadequate diversified portfolios are in case. Because such portfolios are not sufficient to neutralise possible stock-market declines. As far as its efficiency is increased, presumably uncontrollable realised returns will be generated at the end of investment period by reason of its deficient structure which includes few investment such as only two stocks or only one stock and US $, etc. So, being efficient is not enough for an investment portfolio on its own. At first a good investment portfolio has to be well diversified, then it must be efficient. It can be suggested that, efficiency is complementary to well diversification of investment portfolios and efficiency brings extra realised return.

REFERENCES

BAKIRCI, Fehim. (2006), **Üretimde Etkinlik ve Verimlilik Ölçümü Veri Zarflama Analizi Teori ve Uygulama**, Ankara: Atlas Yayınları.

CHARNES, Abraham, COOPER, W. W., & RHODES, E. (1978), "Measuring The Efficiency of Decision Making Units". **European Journal of Operations Research**, Vol:2, pp: 429-444.

COELLI, Tim. (1996), **A Guide To Deap 2.1. Version 2.1: A Data Envelopment Analysis Computer Program**, CEPA Working Paper 96/08, Internet Address; http://www.owlnet.rice.edu/~econ380/DEAP.PDF, Access Time: 2010.12.10.

FARRELL, M. J. (1957), "The Measurement of Productive Efficiency", **Journal of The Royal Statistical Society,** Series A, Vol:120, pp: 253-290.

FØRSUND, F. R. & SARAFOGLOU, N. (2002), "On The Origins of Data Envelopment Analysis", **Journal of Productivity Analysis**, p: 23-40.

MARKOWITZ, Harry. (1952), "Portfolio Selection", **The Journal of Finance**, Vol:7, Issue 1, pp: 77-91.

NAKANISHI, Y. J. & FALCOCCHIO, J. C. (2004), "Performance Assesment of Intelligent Transportation Systems Using Data Envelopment Analysis", **Research in Transportation Economics**, Vol:8, pp: 181-197.

ÖZÇAM, Mustafa. (1997), **Varlık Fiyatlama Modelleri Aracılığıyla Dinamik Portföy Yönetimi**, Ankara: SPK Yayınları, No:104.