FRAUD DETECTING WITH BENFORD'S LAW: AN ALTERNATIVE APPROACH WITH BDS AND CRITIC VALUES[*]

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ABSTRACT

Despite has many critical as much as defenders, Benford's Law has been used many different ways in accounting. The purpose of the study is to create an alternative standard and critical values that can be used to assess financial data compliance with Benford's law for using as an audit procedure. To this end, the allowable deviation limits were determined on an average basis by analysing the quarterly (20 periods total) balance sheet and income statements of the companies traded in BIST for the years 2013-2017. Analysis done in Excel software by performing 1. Digit, 2. Digit and First 2 Digit tests of Benford Analysis. As a result, a new measuring standard BDS (Benford Digit Score) and conformity limits have been created that evaluated is more effective for audit aims.

Keywords: Benford's Law, Fraud Detection, Fraudulent Financial Statement

JEL Classification: M42, M41, M40

BENFORD YASASI İLE HİLE DENETİMİ: BDS VE KRİTİK DEĞERLER İLE ALTERNATİF BİR YAKLAŞIM

ÖZ

Birçok savunucusu kadar eleştireni de olmasına rağmen Benford Yasası çok farklı şekillerde muhasebede kullanılagelmiştir. Bu çalışmanın amacı Benford yasasının finansal tabloların denetiminde kullanılmasında alternatif bir standart ve yasa ile uyumun değerlendirilmesinde yeni kritik değerler oluşturmaktır. Bu amaçla BİST'te işlem gören şirketlerin 2013-2017 yılları arası çeyrek dönemlik finansal tabloları (toplam 20 dönem) veri

seti olarak kullanılmış, Benford analizi testlerinden 1. Basamak Testi, 2. Basamak Testi ve ilk 2 Basamak testleri uygulanmıştır. Sonuç olarak Benford yasasına uyumun değerlendirilmesinde daha anlamlı sonuçlar veren yeni kritik değerler üretilmiş ve BDS (Benford Basamak Skoru) alarak adlandırılan yeni bir ölçüm standardı hesaplanmıştır.

Anahtar Kelimeler: Benford Yasası, Hile Denetimi, Hileli Finansal Raporlama

JEL Sınıflandırması: M42, M41, M40

1. INTRODUCTION

Most of the frauds in the world of economics are based on changing the numbers. Detecting the changed numbers in this case also means to reveal the fraud. Benford's law states that the probability of finding numbers in a naturally occurring number of digits is not equal. According to the Benford's law, the probability of the first digit being 1 is almost 6 times higher than the probability of being 9 (Nigrini 2011, 1). For example, when a country's city population or a company's expenditure bills are dealt with, the numbers that make up this data show that those starting with 1 and those starting with 9 do not have the same quantity and that these quantities will decrease as they go from 1 to 9. This is the Benford's law, also known as the "first digit phenomenon" or the "first digit law". This law has a wide range of uses ranging from fraud detection to computer disk space allocation (Fewster 2012, 26).

As the use of software designed for specific and general audit purposes becomes widespread, audit coverage expands. Nonetheless, according to the ACFE 2016 report, it was found that firms lost 5% of their annual income due to fraud and the total loss caused by the scam was $ 6.3 billion. Turkey is a similar situation; GİB, according to the 2017 annual report in 2017, in Turkey taxpayers examined, which constitutes 1.9% of total taxpayers, the tax loss determined per taxpayer 126 thousand and base average was declared 10% missing in all types of taxes in 2017.

Various methods are used to gain an idea of the correctness of financial statements. These are generally based on the analysis of various financial ratios. In spite of this, Benford's law, which is based on the frequency of the figures in the number of steps, is another way of finding practice in fraud control, even though there is no financial infrastructure. However, for both methods, critical values generally accepted as being present, Turkey does not have the critical customized values to be applied to financial statements.

There are a number of problems that weaken the effectiveness of auditing. First of all, most of the financial data is kept in the business and there are obstacles to access. Controlling financial data is a time-consuming and expensive process. Declared public and publicly available financial data are limited and the effectiveness of methods that will give definite results for supervised use is weak. Financial

information contains information that is unreasonable and infects information users. Developing techniques that can test cheap, fast, and large data can increase audit effectiveness. Benford analysis offers a useful method for this. This study has set out to solve some of the problems encountered in the use of Benford's Law in financial statements audit. The critical values used to investigate whether the frequency values obtained by the analysis of the data set give a signal of fraud may not be up to date and the suitability for the financial statements should be discussed. In addition, it is not clear which frequency values the comparison is based on. (For example, 1. digit, 2. digit or first 2 digit frequencies?). The purpose of this study is to establish a specific and current set of critical values for use determination of financial statements health. Also, in study we examined an alternative measure standard as a different variation of MAD which commonly used in Benford analysis. The study contributes to the literature in several ways. Primarily, this paper is first study that suggest calculated critical values for measuring Benford's law conformity. Also based on a large set of financial statements and presenting a current tabulation of critical values. Suggested critic values table is authentic and updatable, which drafted from financial statements from Turkey. Secondly this paper suggesting an alternative measurement standard to the MAD value, named BDS (Benford Digit Score), in Benford's law compliance measure. Results shows that BDS is a more useful and effective supplement of MAD. These two innovations offer a new method of fraud control with Benford analysis. This new method provides a useful tool for all users of information, especially managers and accounting professionals.

The structure of the paper is as follows; Section 1 background and motivation of study. Section 2 review related previous studies. Section 3 outlines the conceptual basis for Benford`s law theory. Section 4 explains the research method and rationale behind the study, whilst Section 5 summarizes the results obtained. This is followed by Section 6, which discusses the results and suggests further research.

2. LITERATURE REVIEW

The application of Benford's law to accounting data was first made by Carslaw (1988). In Carslaw's study, the frequency of second digit (especially zero) appearances in the earnings numbers of New Zealand companies was investigated in accordance with Benford's law. Between 1981 and 1985, financial statements of 220 companies traded in the New Zealand stock market were investigated. The compatibility of the step frequencies calculated within the Benford's law with the expected frequencies was evaluated by z-statistics and chi-square tests. In particular, reported gains have a much higher expected 0 frequency and a lower expected 9 frequency. It has been stated that this abnormality may be evidence of an alternative income-healing behavior that firms may be trying to reach. According to the author, such targets are based on the existence of cognitive reference points.

Since then Benford analysis has found a wide range of applications in accounting with a wide variety of methods. The analysis can be applied to any kind of accounting data that has a number of records over a certain size and is expected to occur naturally. Generally; accounting statements, financial statements, tax declarations and macroeconomic data.

Nigrini (1994) conducted a case study on embezzlement by a security chief working in a large housing estate. The first two-digit combinations of fraudulent checks gave a hint to the fraudulent process of deviation from Benford's law. In Hill's (1998) analysis of a 1995 tax bill that is known to be fraudulent in New York, it was determined that the numbers did not follow Benford's law. Nigrini (1995) applied Benford's analysis of US President Clinton's tax payments, 13-year tax payments have been observed to follow the Benford distribution. Tam Cho and Gaines (2012) tested years of US election campaign spending by Benford analysis. In 2011, an analyst of China-based company implemented the Benford analysis, First and Second digit tests using the data from the last 10 quarters of the income table, and found that 5 and 8 figures were higher than expected and 4 figures were lower than expected. This situation has come to the conclusion that the 4 figures of Chinese culture originated from the fact that the ominous 5 and 8 figures were regarded as auspicious, and they may have been played with figures (Özdemir 2014, 25) The summary information of similar studies is given in the Table 1.

Table 1. Benford's Law Applications in the field of Accounting

Author	Year	Variables	Digit Test	Conformity Tets
Carslaw	1988	Net Profit, Ordinary Profit	1.and 2.	Z-statistics
Thomas	1989	Profit-loss, quarterly profits, earnings per share	1.and 2.	Z-statistics
Niskanen and Keloharju	2000	Profit	1.and 2.	Z-statistics
Skousen all	2004	Profit	1. 2. 3. and 4.	Z-statistics
Jordon and Clark	2011	Profit	2.	Z-statistics
Hsieh Hsieh and Lin	2013	Quarterly profits	2.	Z-statistics
Van Ceneghem	2002, 2016	Profit, Financial Statements	2.	Chi-square, z-stat, MAD test
Kinnunen and Koskela	2003	Net profit and loss	2.	Z-statistics
Das and Zhang	2003	Earnings per share	2.	Z-statistics

Jhonson	2009	Earnings per share	1.	Z-statistics
Jordon all	2009	Sales	2.	Z-statistics
Nigrini	1994, 2000	Payrolls	First 2	Z-statistics; MAD test
Nigrini and Mittermaier	1997	Invoices	1. 2. And First 2.	z-statistics
Möller	2014	Financial Statements	2.	z-stat, Chi-square
Nigrini	2015	Financial Statements	First 2	MAD test
Jhonson and Weggenman	2013	Financial Statements	1.	Z-statistics, MAD test
Archambault	2011	Financial Statements	1.	Chi-square
Amiram all	2015	Financial Statements	1.	FSD score
Uzuner	2014	Financial Statements	1.	Chi-square
Hanselman	2012	Financial Statements	1.	Chi-square, Z-statistics, MAD test
Quick and Wolz	2005	Financial Statements	First 2	Chi-square, Z-statistics
Tilden and Janes	2012	Financial Statements	1.	Z-statistics
Nigrini	1992, 1996	Income Tax Returns	1. 2. and First 2.	Chi-square, Z-statistics MAD test
Chiristian and Gupta	1993	Income Tax Returns	First 2	Z-statistics
Rauch	2011	Macroeconomic Data	1.	Chi-square
Krakar and Zgela	2009	Swift messages	1. 2. and First 2	Chi-square
Durtschi	2004	Purchase checks, insurance claims	1.	NA
Tam Cho all	2007	Election Campaign financing	1.	Euclidean distance
Geyer and Drechsler	2014	Long Term Debt	1.	Chi-square,Z-statistics

Dorfleiner and Klein	2008	Stock price	2.	Chi-square
Akkaş	2007	Stocks accounts	1., 2. and First 2.	Chi-square
Çakır	2004	Stock closing	1., 2.and First 2.	MAD test
Gönen and Rasgen	2014	Stocks trading volume	1., 2.and First 2.	MAD test
Çubukçu	2009	Payment Checks	First 2	Chi-square
Boztepe	2013	Budget revenues and expenditures	1.	NA
Yanık and Samancı	2013	General administrative expenses	1., 2., First 2 and First 3	Chi-square
Demir	2014	Marketing sales distribution expenses	1., 2.and First 2.	Chi-square

3.THEORETICAL FRAMEWORK

3.1. Benford's Law

The emergence of Benford's law is based on a two-page article published in the American Journal of Mathematics in 1881 on the frequency of numbers on the number of digits by American astronomer and mathematician Simon Newcomb's. Accordingly, the frequency of the first digit in the first step decreases from 1 to 9. In step 3, the probabilities are very close to each other, and from the fourth step onwards the difference becomes unclear (Newcomb 1881, 40). Since the calculator was not invented during the periods in which this claim was put forward, logarithmic rulers were used. Newcomb has come up with the idea that the first pages of logbook books are always dirtier and worn than the next page.

Newcomb's model has been almost forgotten for 57 years despite all the excitement and functionality until famous physicist Frank Benford's made similar observations. Benford showed on the table the frequency of each digit in the number of digits taking the average of the distribution results obtained from 20,229 different data sets in the 1938, edition of The Law of Anamorphous Number in the American Philosophical Society(Benford 1937, 553).Table 1 shows Benford's observations and results.

As seen in Table 2, the first digits in data sets which collected from a variety of fields show almost the same distribution. From 20 different data sets, on average, 30.6% of the total 20,229 data starts with 1. The ratio of those who start with 2 is 18.5% on average, and this ratio is decreasing as the number grows. Benford (1938) formulated these conclusions in a distribution hypothesis to be called the

"Benford's Law", a universal law regulating the digits of numbers. Benford's law is strong enough to raise suspicions about the authenticity of data sets that do not comply with this distinction (Benford 1957, 551). This rule is also regarded as a universal nature law because it maintains its validity when scale and number base change (Fewster 2012, 27).

Table 2. Benford's Digit Observations

Data	1. Digit									N
	1	2	3	4	5	6	7	8	9	
Rivers, Area	31,0	16,4	10,7	11,3	7,2	8,6	5,5	4,2	5,1	335
Population	33,9	20,4	14,2	8,1	7,2	6,2	4,1	3,7	2,2	3259
Constants	41,3	14,4	4,8	8,6	10,6	5,8	1,0	2,9	10,6	104
Newspapers	30,0	18,0	12,0	10,0	8,0	6,0	6,0	5,0	5,0	100
Spec.Heat	24,0	18,4	16,2	14,6	10,6	4,1	3,2	4,8	4,1	1389
Pressure	29,6	18,3	12,8	9,8	8,3	6,4	5,7	4,4	4,7	703
H.P. Lost	30,0	18,4	11,9	10,8	8,1	7,0	5,1	5,1	3,6	690
Mol. Wgt.	26,7	25,2	15,4	10,8	6,7	5,1	4,1	2,8	3,2	1800
Drainage	27,1	23,9	13,8	12,6	8,2	5,0	5,0	2,5	1,9	159
Atomic Wgt.	47,2	18,7	5,5	4,4	6,6	4,4	3,3	4,4	5,5	91
n-1 √n,…	25,7	20,3	9,7	6,8	6,6	6,8	7,2	8,0	8,9	5000
Design	26,8	14,8	14,3	7,5	8,3	8,4	7,0	7,3	5,6	560
Digest	33,4	18,5	12,4	7,5	7,1	6,5	5,5	4,9	4,2	308
Cost Data	32,4	18,8	10,1	10,1	9,8	5,5	4,7	5,5	3,1	741
X-Ray Volts	27,9	17,5	14,4	9,0	8,1	7,4	5,1	5,8	4,8	707
Am.League	32,7	17,6	12,6	9,8	7,4	6,4	4,9	5,6	3,0	1458
Black Body	31,0	17,3	14,1	8,7	6,6	7,0	5,2	4,7	5,4	1165
Adresses	28,9	19,2	12,6	8,8	8,5	6,4	5,6	5,0	5,0	342
n1, n2…n!	25,3	16,0	12,0	10,0	8,5	8,8	6,8	7,1	5,5	900
Death Rate	27,0	18,6	15,7	9,4	6,7	6,5	7,2	4,8	4,1	418
Average	30,6	18,5	12,4	9,4	8,0	6,4	5,1	4,9	4,7	Total
Probable Error	±0,8	±0,4	±0,4	±0,3	±0,2	±0,2	±0,2	±0,2	±0,3	20229

Source: (Benford 1938, 553)

In the study Benford (1882) experiment was repeated with various data sets and Benford's law tested. Table 3 shows the results of the first digit test of a total 408.000 data consisting of 21 data sets collected by the author at year 2018. The results are very close to the results of Benford (1881).

Table 3. Updated Digit Observations

Data	1. Digit									N
	1	2	3	4	5	6	7	8	9	
Election Results	33,6	19,2	12,7	9,5	6,9	5,6	4,6	4,4	3,4	12003
BIST Num.Transct	31,6	18,2	12,2	9,3	8,6	6,3	5,3	4,6	4,0	178601
BIST Vol.Transct	29,3	17,7	12,3	1,3	7,4	6,2	5,3	4,7	4,2	189585
Foreign Trade, TR	42,3	17,8	10,5	6,6	5,0	3,5	3,7	5,3	1,0	456
Population, Towns, TR	29,9	19,2	13,9	9,5	6,6	7,4	4,6	6,1	2,8	970
Cost Statement	31,6	17,9	14,2	9,5	7,3	5,4	5,1	4,7	4,2	7380
Area, TR	33,6	14,7	9,1	10,4	6,7	6,3	6,5	7,9	4,7	1058
Newspapers	43,2	7,6	20,8	6,9	18,2	13,0	4,0	8,0	8,0	9742
Population, Countries	29,9	14,0	13,3	10,6	9,1	8,0	5,7	3,8	4,9	264
Debt Statement, EU	28,3	20,1	13,9	9,8	7,7	6,4	4,9	4,7	4,2	1565
Population, Cities, EU	32,9	18,4	12,8	8,9	8,5	5,8	5,2	3,9	3,5	1640
Soccer Audience	28,6	18,8	8,2	10,6	9,1	8,8	9,1	4,3	2,4	329
GDP, OECD	30,9	17,2	11,9	9,0	8,0	6,5	5,4	3,5	4,2	2542
CO2 Emission,	24,2	18,2	9,5	12,5	9,1	8,0	4,9	4,9	3,8	264
Forest, Area	26,1	15,2	13,6	10,6	6,4	8,3	2,7	5,3	7,2	232
Labour' Countries	25,4	18,2	11,0	10,2	6,4	8,0	2,7	3,0	3,0	264
GDP Per Capita	22,3	11,7	13,3	12,5	7,6	5,3	8,7	4,2	3,8	264
Refugee, Countries	26,6	14,8	8,0	9,1	6,5	6,1	6,1	4,2	1,9	263
Area, Countries	28,0	20,8	11,7	10,2	7,6	5,7	6,1	4,5	4,9	263
Fish Species	27,3	21,2	14,0	9,1	10,6	4,9	3,8	3,0	3,0	264
Goodwill Resources	25,9	14,8	9,5	7,2	6,1	4,2	1,9	4,9	2,7	203
Average	30,1	16,9	12,2	9,2	8,1	6,6	5,1	4,8	3,9	Total
Probable Error	-0,8	-0,4	-0,4	-0,3	-0,2	-0,2	-0,2	-0,2	-0,3	408152

The Benford's law, based on the principle that people cannot produce numbers by chance, is an example of Hill's (1998) experiment. In the course of the theory of probability, one group is asked to write 200 rounds of the results of the coin tour, and the other group is asked to write the estimated results of the 200 rounds. Although the same face-to-face situation often occurs six times in succession, in practice, this scenario has never been seen in the prediction group results (Hill 1998, 362). In another experiment, 742 students were asked to create random 6-digit numbers, and the numbers were found to be less compatible or incompatible with Benford's law (Nigrini and Mittermaier 1997, 56). This test also repeated by us. In accounting final examination, students asked for write randomly 6-digit numbers on paper. As a result of the Benford analysis of the data set collected from 343 paper, it appears that the data set is incompatible with the Benford's law. If people are asked to generate random numbers, their response will indeed vary significantly from random sequences (Hill 1988, 967). When people think they are producing a random number, they often reflect on their own experiences and the numbers in their experiences.

3.2. Mathematical Basis of Benford's Law

Under Benford's law lies a simple mathematical logic. Let's consider the market value of a company. The company with a market value of 1 million £ needs to double its value in order to reach a value of "2" in the first digit, in other words the value must increase by 100%. This operator, which is supposed to grow at an annual rate of 10%, should be 7.3 years to reach a value of 2 million £, which is the first digit 2. This time a 50% growth is needed to reach a value of 3 next. For a market value starting at "4", the company should grow only 33 percent. After this, the time to reach TL 5 million is 5 years. 5 million will reach 6 million in 1.9 years, and this figure will continue to decrease as the numbers increase. In this example, the first name of the company's market value will remain 1 for 7.3 years and 9 for only 1 year. As we can see, while small numbers remain unchanged for longer, large numbers change in a shorter time. This is the basic reason why small numbers are more likely to be in first steps (Nigrini 1999a, 80).

For this reason, it is farther away from the first digit spread of many financial data, from a purchase order to the stock market, with the number 1 being 2 and the number 8 being 9. Thus, in the first digit of observed events, small values are greater than large values (Durtschi 2004, 21). A similar example can be observed on staff salaries. A salary increases from 1,000 £ to 2,000 £, which is longer than 8,000 £ to 9000 £. At the same time, the number of low-paid workers is higher than the number of high-paid workers, and the number of small companies is higher than that of large companies. This may be an explanation for the greater observation of small numbers in nature.

Benford's law is a general law concerning naturally occurring numbers that maintain their validity under different circumstances. Pinkham (1961) stated that if there is a law governing digital

distributions, it is a premise that this law is constant in terms of scale. This means that if all the numbers in a data set appropriate to the Benford's law are multiplied by a non-zero constant, the new set will follow the Benford law (Nigrini 2011, 30). If you apply this law to the monetary system, the consequences of the data being denominated in Dollar, Euro, Pound, Peso, Yen or Lira do not change, so there is no need to deal with the exchange rates (Geyer and Willliamson 2004, 232).

Hill (1995) has similarly shown that Benford's law is independent of the number base. Accordingly, the number base of a data set following the Benford's law in the 10 base of numbers is raised or lowered again to the law.

3.3. General Formula

The approximate values of the expected frequencies from Benford's observations can be calculated by the logarithmic formulation. The probability of having a significant non-zero number in the first digit of the number is calculated as follows (Hill 1998, 358):

$$P\,(D1 = d1)\log10(1 + 1/d1);\ d1 = \{1,2,3\ldots9\} \tag{1}$$

For example the probability that the first digit of a number is 6:

P(D1=6) =log(1+1/6)=0,0669 = %6,69

Calculation for probability 1 and 2 in the first step:

P (D1:1) =log102=0,03010 = %30,1

P(D1=2)=log103/2=0,1760 = %17,6

Figure 1 shows that the expected frequencies calculated by Benford's observations and the logarithmic formula are very close to each other. Likewise, the probability of a digit being in the second digit of the number can be calculated by the following formula:

$$P\,(D2 = d2) = \sum_{d1=1}^{9} \log10(1 + 1/\,d1\,d2)\,;\ d2 = \{1,2,3\ldots0\} \tag{2}$$

P: Probability

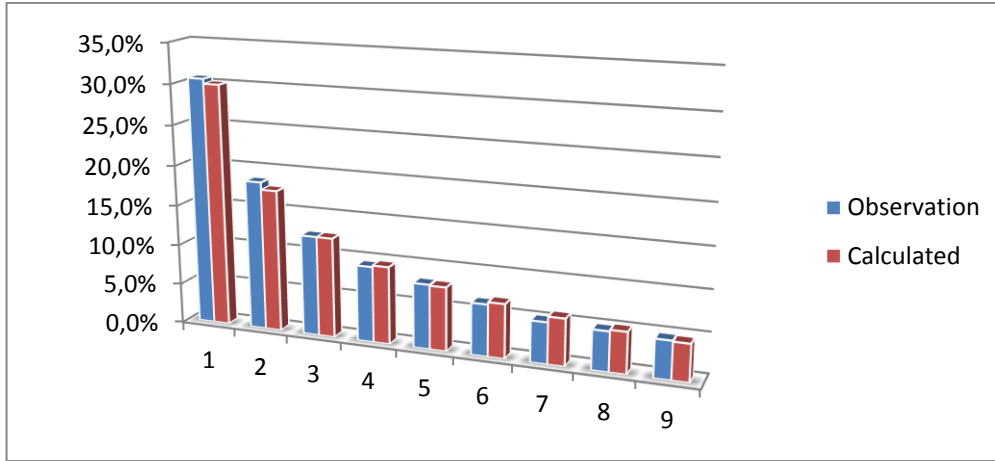D1:First digit

D2:Second Digit

D3:Third Digit

Figure 1. Frequency of the Numbers in the First Digit

Source: (Raimi 1969, 109)

3.4. Benford Analysis

There are five important tests for the use of the Benford's law. These; The first step test, the second step test, the first two step test, the first three step test and the last two step test. The first and second digit tests are high level conformity tests in the selection of data. The first two Digits and the First Three Digits tests can be used to select audit targets. The last two digits test is a strong test for detecting the derived digits, it can be used to determine the rolls. The poor compatibility of data sets with Benford's law may be a signal of an anomaly related to the data. Therefore, if a researcher with 4 datasets in hand has 1 set of incompatibilities while being compatible with 3 sets of Benford, the strategy should be to focus on incongruities because cheating is high risk (Nigrini 2012, 74). Data sets to be tested for compliance with the Benford's law should meet the following requirements (Quick 2005, 1290).

- The dataset should define the size of similar occurrences; The data must express the same kind events. The example is all city-based or all-year sales.

- The lower or upper limit of the values in the dataset should not exist. The maximum and minimum limits disrupt the distribution.

- The values in the data set should not be assigned numbers. It is one of the main conditions of the law that the numbers are formed randomly from the natural way (Akbaş 2007, 196).

The Benford's law can best be applied to large data sets. As the size of the dataset grows, the efficiency of the analysis also increases. (Drake and Nigrini 2000, 132) Research has shown that the level of compliance with the Benford's Law is higher for multi-digit numbers at four quarters (Cakir 2004, 77).

3.5. Benford Conformity Tests

The results of Benford's law tests need to be subjected to an evaluation of significance with statistical tests. For this, there are several methods used in the literature. Chi-square, Z-test and Kolmogorov Simirnov tests can be applied to test the results. According to Nigrini, these tests, which take into account the data size, can give incorrect results, especially in large data sets, due to the excessive power problem. A deviation that should be considered normal due to the size of the data set can be perceived as incompatibility in these tests. According to Nigrini (2011), therefore, the absolute average deviation (MAD) test, which does not account for the number of data, is preferred over other statistical tests. When testing compatibility with Benford's law, the deviation between the expected result and the expected value (Average Absolute Deviation - MAD) is calculated by subtracting the expected values from the actual values and dividing the result by the number of digits. The number of digits takes 9 for the first step test and 90 for the first two step tests. Assume that the distribution of the first step 1 in a data set is 0.320 (expected ratio is 0.301). Deviation will come out at 0.019. The absolute deviation is the positive value of the deviation (minus sign is ignored if the actual value is lower than expected). For example, if the first digit 1 ratio is 0.290, the absolute deviation is 0.011 (0.290-0.301). Absolute deviations are collected for nine steps and then divided into 9 for MAD (90 for the first two digits) (Drake and Nigrini 2000, 133).

Jhonson and Wegenman (2013) used a different method to calculate MAD values. In this method, the differences between the realized and expected ratios are subtracted from the average of these differences and calculated by dividing K to K.J&W MAD $= \frac{\sum_{i=1}^{K}|(AP-EP)-av(AP-EP)|}{K}$

4. METHODOLOGY

4.1. Research Design

The data set was subjected to the 1st Digit test, 2nd Digit test and fisrt-2 digit test under the Benford analysis. From the result, BDS values calculate for each observation. According to BDS critic values table companies classified as conformity and nonconformity companies. BDS is different version of MAD and calculate by taking average of digit test MAD values.

4.2. Data Set

The study attempted to analyze a large set of data, covering a wide range of time series for the construction of the critical value tables to be used in assessing Benford's law compliance with financial statement data. Balance sheets and income tables covering 347 companies (20-quarter) period between 2013 and 2017, which are traded at BIST in 2017, were used as data set. The data set consists of a total of 34700 financial statements. The data is collected from FINNET database and ww.kap.gov.tr.

4.3. Creating the Critical Values Table

Acceptable deviation limits must be defined in order to be able to evaluate Benford's law compliance with a data set. In the literature, chi-square and z-statistics are the most commonly used methods of measuring the goodness of fit. However, these methods are criticized because they point out minor discrepancies even in large data sets (excessive power problem). Instead, the MAD test is another method commonly used in the literature. MAD is calculated as below:

$$= \frac{\sum_{i=1}^{K} |AP - EP|}{K} \tag{3}$$

Here, the AP represents the percentage realized, the EP expected percentage, and the K variable number (equal to 90 for the first two digits). The number of records is not used in the calculation of N, MAD. In the determination of the level of harmony of the results of the MAD test, the critical value table created by Nigrini (2011) is used.

The aim of the study is to create a current and unique critical values from the data set of BIST companies financial statements. For this purpose, 1st, 2nd, and First 2 digit tests were applied to the whole data set for 1 year and 5 years separately, and the MAD values of each digit test were calculated. In our study, instead of evaluating the MAD value of each digit test separately, a single critical value was determined by taking the average of the MAD value of the three (1st, 2nd, and First 2 digit tests) digit tests. In the literature, one by one critical values are used for each digit frequency. In practice, however, it is not clear how to comment the conformity when different digit test results are not consistent. For this reason, a single value was generated by taking the average of the MAD values calculated for the three different digit values calculated in this study, and this value was called "Benford Digit Score (BDS)".

BDS = Average (1. Digit MAD, 2. Digit MAD, First 2 Digit MAD) (4)

In order to test whether BDS is effective in measuring compliance with the Benford's law, four randomized financial statements of a randomly selected company traded in BIST are manipulated to increase the net profit of the quarter in 2017, and the original financial statements and the manipulated financial statements test results are compared. Manipulation process, 4 item of balance sheet and income table were made by changing. In the income table in this framework, the "Sales" item is increased and the "Operating Expenses" is reduced. In Balance sheet, "Short-Term Receivables" and "Debt and Expense Provisions" were increased. The Benford analysis results of the manipulated and original financial statements are presented in Table 4.

Table 4. BDS Activity Test

DATA SET	1.DIGIT	2.DIGIT	FIRST 2 DIGIT	BDS
Manipulated	0,0158	0,0188	0,0051	0,0132
Original	0,0162	0,0177	0,0049	0,0129

The first Digit MAD value of the manipulated data set is lower than that of the original data sets by 0.0158, according to the results of Table 4. According to this result, the manipulated data set seems more compatible with Benford's law. According to the results of the second digit and the first two digits, the MAD values of the original data set were lower and gave the correct signal. BDS results are parallel to the results of the second and first two digits. The test was repeated in the direction of reducing the profit, the experiment was tried on the data of 3 randomly chosen companies and similar results were obtained. Therefore, it can be said that the BDS value is effective in measuring the Benford's law compliance, but it would be useful to repeat the test with more data.

5. RESULTS AND DISCUSSION

In this section quarterly financial statements of the companies have been subjected to Benford analysis separately for the last 5 years (2013-2017) and annually (2017). The quarterly detailed balance sheet and income tables of the companies that were traded at BIST, for the period of 2013-2017 constitute the first data set and the quarterly balance sheet and income tables of 2017 are the second. The data sets created by sub-submission of 20 quarterly financial statements of each company were subjected to Benford analysis, 1st Digit, 2nd Digit and First 2 digit tests. The size of the data set differs because the number of items that contain value is different in financial statements of each company.

Table 5. Averages of MAD Values for Companies

Company	Data	Term	1.Digit	2.Digit	First 2 Digit	BDS
347	1.694	Last 5 years	0,0142	0,0110	0,0033	0,0095
358	350	Last 1 years	0,0218	0,0192	0,0060	0,0157

Table 5 shows the average MAD values obtained at the end of the Benford analysis on a company basis. Since the MAD value is affected by the data size, the result is always lower in large data sets. This is a factor that must be taken into account when establishing the critical values to be used in Benford's Law compliance assessment. For this reason, the critical value table generated is calculated separately

for short-term and long-term data sets. Nigrini (2000;2011) suggests a 4-level harmonization table, while still forming the MAD critical values used in the literature.

Table 6. Nigrini (2011) MAD Critical Values Table

1.Digit	2.Digit	First 2 Digit	Result
0,000 – 0,006	0,000 – 0,008	0,0000 – 0,0012	Close Comformity
0,006 – 0,012	0,008 – 0,010	0,0012 – 0,0018	Acceptable Comformity
0,012 – 0,015	0,010 – 0,012	0,0018 – 0,0022	Marginal Acceptable Comformity
>0,015	>0,012	>0, 0022	Nonconformity

Source: (Nigrini 2011, 160)

Nigrini (2011) does not provide information on how the thresholds are calculated in his work on the critical values table. The question posed by e-mail in this regard is that he has organized critical values according to his personal experience. This removes the possibility of testing or updating the table.

A new tabulation of critical values based on the financial statements of BIST companies is proposed, which may be an alternative to the critical values table in the literature.

Table 7. BDS Critical Values Table

BDS Value	Result
0,000 - 0,0095	Comformity
0,0095 - 0,0157	Acceptable Comformity
>0,157	Nonconformity

Source: (Ozevin 2018, 116)

BDS value is average of 1st Digit test, 2nd digit test and first 2 digit test of MAD values. The aim here is to make it easier to reach a more precise judgment by means of a one-stage adaptation test. Sensitivity analysis was not required because the minimum and maximum BDS values were not exceeded.

In the study Benford's analysis of the data set applied, according to the table of alternative BDS critical values in Table 7, the following results were obtained in the compatibility test; In the 2013-2017 period data set, 195 financial statements of 347 companies were in compliance with Benford's law and 152 companies were out of compliance. In the analysis carried out on the year 2017 data set, the financial statements of 199 companies from 358 companies were in compliance with Benford's law and 152 companies were incompatible.

The inconsistency of the dataset with Benford's law may be fraudulent, or it may also be due to non-fraudulent causes. Non-compliance with Benford's law should only be perceived as a red flag, and if there are no other reasonable reasons for non-compliance, in-depth auditing should be exercised.

6. CONCLUSION

The Benford's law is widely used in auditing and the studies note that the method is working effectively. The basic rule is that the incompatible data of Benford's law will be regarded as red flags giving a fraud signal. However, the criterion for determining this inconsistency must be controversial. In this study, an alternative goodness-of-fit test and a table of critical values are proposed that can be used to assess Benford's law compliance. The table of critical values used in the literature was first created by Drake and Nigrini (2000) and then updated by Nigrini (2011). However, the general validity and update of the critical values table can be criticized. For this reason, it is evaluated that it will be more effective in evaluating compliance with the Benford's law of the financial statement data and the critical value table derived from a large data set.

In the study, a different method was applied to the use of the MAD value to calculate the deviation between expected distributions and distributions according to the Benford's law. Here, instead of calculating a separate deviation values for each step test, it is recommended to take into account a single deviation value of the batch for all step tests applied. It has been observed that the results of the tests made are more effective in comparing the harmonization of the value of the deviation results of all step tests called BDS.

A tabulation of the critical values derived from the financial statements of the BIST companies from 34.700 financial statements, was established in the evaluation of compliance. In addition, the conformity limits were simplified from the literature and a three-stage (Comformity, Accaptable Comformity, Nonconformity) table of critical values was established. This is because the average of the difference between the expected values and the actual values will decrease as the number of data increases. The proposed critical values are evaluated to be more effective because the table is more up-to-date and specific. In the study, two innovations have been developed for fraud detection with Benford's law. First of them is presenting new, authentique, updatable and wide data based critical values table. Second is alternative measurement standard named BDS as alternative of MAD. These two innovations offer a new method of fraud control with Benford analysis. This new method provides a useful tool for all users of information, especially managers and accounting professionals.

For supporting the effectiveness of new method would be tested with some definitive fraudulent statements, however, is hard to reach. An alternative way would be testing different groups of companies

or financial statements according to fraud expectations. In addition, the method can also be used to test different financial statements of the same company for fraud detection.

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