Araştırma Makalesi

INVESTIGATION OF FLEXURAL AND ELASTIC BUCKLING BEHAVIOR OF CELLULAR BEAMS

Research Article

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DAİRESEL BOŞLUKLU KİRİŞLERDE EĞİLME VE YANAL BURULMALI BURKULMA DAVRANIŞININ İNCELENMESİ

Alıntı / Cite

Akgönen, A.İ., Güneş, B., Nassani, D.E., (2020). Investigation of Flexural and Elastic Buckling Behavior of Cellular Beams, Journal of Engineering Sciences and Design, 8(3), 869-882.

1. Introduction

Castellated beams with circular holes are named as cellular beams (AISC-31, 2017). Cellular beams are produced by cutting steel profiles using a computer-controlled torch. Cuts are made in a circular pattern. Separated two

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halves are welded back together from their webs to form a cellular beam. Castellated beams with various hole patterns are widely used in steel structures. Hole geometries are generally preferred as circular and hexagonal shapes. In recent years, usage of perforated beams with sinusoidal openings is also increasingly being used in steel framing systems. The cutting process of a beam with a hexagonal opening is performed in one pass, while the cellular beam is cut in two passes. Therefore, the cutting process for a cellular beam takes a longer time and the cellular beam manufacturing process produces more waste material than the beam with a hexagonal hole.

Cellular beams are widely applied in the roof and floor systems of structures that have wide openings and where the floor height should be limited (Parking structures, etc. Fig. 1) due to mechanical systems (ventilation, air conditioning, heating, etc.). They are also preferred in restoration works owing to its modern appearance.

Figure 1. Parking structure (Arcelormittal, 2020a)

The usage of the castellated beam provides a serious advantage for structure height. Perforated beam web decreases required structure height. A structure consists of 7 stories with ordinary beams can be built as an 8 story building with perforated beams. Thus, fewer stories decrease construction costs and construction time. Early completion of construction moves service time to earlier time. Beams that have a high moment of inertia decrease the need for the required column and foundation element quantity (Arcelormittal, 2020b). Lighter construction with fewer elements will be subjected to less earthquake load.

On the other hand, the stability problem (buckling) takes a very important place for steel perforated beam design. Parallel to advancing technology, material strength is increasing and required element cross-section size is decreasing. Furthermore, the effort to reduce construction weight to minimize construction material cost canalize design engineers to use thin-walled steel elements. Therefore, more attention has to be paid for the buckling problem of steel perforated beam. Elastic lateral-torsional buckling (LTB) can also be critical during the erection of steel structures. Perforated beams are generally preferred at large spans because of their increased carrying capacity. During erection, long perforated beams are subjected to its own dead load and worker live load before the installation of lateral beam supports. At this stage, the lateral movement of the compression flange is not prevented. This case most likely cause elastic lateral-torsional buckling (Bradley, 2003).

Numerical studies, parametric studies, and elastic buckling problems have attracted the attention of many researchers. (Akgöz and Civalek, 2017; Avcar, 2014a; Avcar, 2014b; Civalek and Kiracıoglu, 2010). Extensive numerical, analytical, and experimental studies were found in the literature highlighting the flexural and elastic buckling behavior of cellular beams. Tsavdaridis and D'Mello (2011) conducted an experimental and analytical study on the behavior of perforated steel beams with closely spaced web openings. The finite element model (FEM) was established to investigate the effect of web opening spacing/web opening depth of web-posts and web opening depth/web thickness. An empirical formula was proposed to predict the ultimate vertical shear capacity of the web-post. Sweedan (2011) numerically investigated the lateral stability of the cellular beam. A 3D FEM with 2D shell elements was established. Effects of beam geometry and slenderness were studied on simply supported cellular beam which was subjected to various load types. Ellobody (2012) investigated nonlinear analysis of cellular steel beams under combined buckling modes. A 3D FEM with 2D shell elements was established which considers material nonlinearity, residual stress, and initial geometric imperfection of cellular beam. A parametric analysis also performed to study the effect of change in steel strength, cross-section geometry, and beam length on buckling behavior and capacity of cellular beam. Erdal and Saka (2013) ultimate load carrying capacities and finite element analysis of optimally designed steel cellular beams under loading conditions. Specimens were

optimally designed using harmony search method following BS5950 (2000). Experimental work and finite element analysis (FEA) were conducted with NPI profiles and S355 steel quality. El-Sawy et al., (2014) studied moment gradient factor of cellular steel beams under inelastic flexure. A 3D FEM with 2D shell elements was established. Simply supported cellular beam under various load types was considered. Parametric analysis is conducted to evaluate the effect of various geometrical properties on the inelastic stability of cellular steel beams. Pachpor et al., (2015) studied the behavior of cellular beam using design methods according to BS5950, considering the strength of the tee section and web-post element. A cellular beam was designed following BS5950 (2000) and the theoretical calculation result was compared with FEA for verification. Tsavdaridis and Galiatsatos (2015) conducted a parametric finite element analysis investigating closely spaced cellular beams with double concentric transverse stiffeners. Parameters such as (S/D) ratio, stiffener, and web thickness were investigated. Kwani and Wijaya (2017) studied LTB of castellated beams analyzed using the collapse analysis. FEM was analyzed by ADINA 8.9 (2013). At the end of the study, a correction factor for the AISC formula was offered to determine the critical moment of cellular beams. There are other studies in the literature where different types of steel elements are examined by FEA method (Pachideh et al., 2019; Yin et al., 2019; Jiang et al., 2019; Taheri et al., 2019; Roy et al., 2018; Roy and Lim, 2019).

As it is seen in the literature studies, most of the experimental studies on elastic buckling and flexural behavior of cellular beams were conducted with American wide flange steel profiles, American steel quality. Besides, 3D finite element studies presented in literature were conducted by using 2D shell elements. In this study, it was aimed to carry out more sensitive FEA with 3D solid elements for cellular beams to study flexural and elastic buckling behavior of beam by using European steel quality and European steel shapes. In addition, the validity of the theoretical calculation of critical LTB load for a cellular beam with both gross section and net section was studied by using European steel quality and European steel shapes.

2. Material and Method

2.1. Verification Study

Erdal and Saka (2013) conducted twelve experimental studies using NPI240, NPI260, and NPI280 steel profiles. S355 steel quality was preferred for all cellular beams. Specimens were supported as a simply supported beam with an approximately 3-meter span. The vertical force was applied at the mid-point of the beam. In this study, experimental study results conducted with the NPI240 profile is used for verification of FEA. Table 1 shows the dimensional properties of NPI240 based cellular beam and Table 2 ultimate load and failure mode of specimens reported by Erdal and Saka (2013).

Table 2. Ultimate load capacities and failure modes of NPI_240 sections (Erdal and Saka, 2013)

Test 1 and Test 2 graphics show that specimens laterally buckled around 200 kN or (Mcr=140 kNm) before reaching ultimate bending capacity due to lack of lateral support. Arrows show the laterally buckled points of the specimens as illustrated in Fig. 2 and Fig. 3 As it is seen, LTB occurred in the elastic region. U_F_Def shows upper flange vertical deflection data, L_F_Def shows lower flange deflection data and Lat_Def shows lateral movement measured using LVDT.

Figure 2. Load deflection graphics for NPI 240 (Test1) (Erdal and Saka, 2013)

Figure 3. Load deflection graphics for NPI 240 (Test2) (Erdal and Saka, 2013)

2.2. Lateral Torsional Buckling (LTB) Behavior of Cellular Beams

LTB generates two kinds of stresses which are Saint Venant and Warping stresses. While Saint Venant stress occurs due to the rotation of section about a longitudinal axis, warping stress occurs due to the torsion of the section about a vertical axis (Timoshenko and Gere, 1961; Martini and Mohammad, 2011).

$$
M_T = M_{SV} + M_W \tag{1}
$$

$$
M_{sv} = GJ \frac{d\varphi}{dz}
$$
 (2)

$$
M_{w} = -EC_{w} \frac{d^{3} \phi}{dz^{3}}
$$
 (3)

$$
M_T = M_{SV} + M_W = GJ \frac{d\phi}{dz} - EC_w \frac{d^3\phi}{dz^3}
$$
 (4)

Solution of Equation of Equilibrium for laterally buckled case is given in the following equation.

$$
EC_{w}\frac{d^{4}\phi}{dz^{4}} - GJ\frac{d^{2}\phi}{dz^{2}} - \frac{M_{0}^{2}}{EI_{y}}\phi = 0
$$
\n(5)

After the solution of differential equation considering boundary conditions, Critical Moment can be obtained as follows.

$$
M_{o\ cr} = \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}
$$
 (6)

L is the laterally un-braced length for beam compression flange. E is the modulus of elasticity. I^y is the moment of inertia about the y-axis. G is the shear modulus, J is the polar moment of inertia and C_w is the warping constant.

$$
M_n = C_b M_{ocr}
$$
 (7)

$$
C_{\rm b} = \frac{M_{\rm n}}{M_{\rm ocr}}\tag{8}
$$

Previous calculation is valid for a beam which is subjected to fixed bending moment along its length. This is the most critical load case for LTB and may occur if the beam ends are subjected to the same amount of moment with opposite directions. However, beams may be exposed to various load types and moments during service life. Variation of moment along a beam length is considered by moment gradient factor (C_b) . C_b factor differs for some national steel design codes which are presented in Table 3 for mid-span concentrated load as follows.

Table 3. Moment gradient factor given by national design codes, C*^b* (Martini and Mohammad, 2011)

Loading Type	AISC360-16 (2016)	EC3 (2005)	AS4100 (1998)	
Mid-Span	1.32	1.37	1,35	
Concentrated Load				

Cross-section of beam elements that are subjected to bending moment can be classified as compact, non-compact, and slender according to AISC360-16 (2016) and TSDC-2016 (Turkish Steel Design Code, 2016). Geometric slenderness limitations are given in Table 4.

Element		\mathcal{A}_{p}	Лr	
Flange	D_f $2t_f$	Е 0,38 г.	Е 1,00	
Web	t_w	Е 3,76 L.	Е 5,70	

Table 4. Geometric Slenderness limitations (AISC360-16 (2016) and TSDC-2016 (2016))

After the Northridge Earthquake (1994), many experimental studies conducted in the USA to analyze damage occurred at steel structures. AISC 360-16 took its final form in the light of these experiences. American steel profiles (W profiles) and American steel quality (A992, A572 Gr. 50) were used in experimental studies. Turkish Steel Design Code (TSDC-2016) that came into force in 2016 seriously affected by AISC360-16. However, steel quality and steel shapes used in Turkey are quite different from the ones used in the USA. For this reason, a calculation conducted based on AISC360-16 with steel shape and steel quality used in Turkey. An experimental study conducted by Erdal and Saka (2013) with S355 steel quality and NPI240 steel profile was used for verification of the AISC360-16 design theory to determine LTB point. The effective radius of gyration (its) was calculated by using the following equation for gross and net section solution. Where bf is flange width, tf is the flange thickness, tw is the web thickness and h is the height of the beam web.

$$
i_{\text{ts}} = \frac{b_{\text{f}}}{\sqrt{12(1 + \frac{ht_w}{6b_{\text{f}}t_{\text{f}}})}}
$$
(9)

Equation 9 considers the total cross-section area of beam flange and one-sixth of beam web height for calculation of an effective radius of gyration. LTB behavior of the prismatic beam is investigated based on preceding information given above according to AISC 360-16 and Turkish steel design code (TSDC-2016). These design codes state that the LTB problem for perforated beam can be examined using the gross section area of the beam. In this study, theoretical study results using gross-section and net section were compared with an experimental study results performed by Erdal and Saka (2013). Table 5 shows a comparison of critical LTB load with the net section and gross section. The verification study shows that LTB occurs around 200 kN (Mcr=140 kNm) for cellular beam obtained from the NPI240 root profile. This result proves that LTB load calculation with gross section and net section overestimates the critical LTB capacity. For this reason, LTB load calculation with the net section and a more conservative safety factor (less than 0.9) is recommended by authors for design purposes.

Table 5. Hand Calculations with Gross and Net Section

2.3. Finite Element Model

2.3.1. Material Model and Material Properties

According to coupon tests given at the verification study, modulus of elasticity, yield strength, and ultimate rupture strength of steel were taken as 190.000,00 MPa, 390 MPa, and 495 MPa, respectively. Bilinear isotropic hardening material model (Fig. 4) was used in FEA. Tangent modulus in the plastic region was taken 1.000,00 MPa to prevent iteration problems (Akgönen et al., 2015). SOLID187 tetrahedral element with 10 nodes was selected which is well suited for irregular meshes (Sharcnet, 2017). SOLID187 element also has plasticity, large deflection, and large strain capabilities. Von Mises yield criteria (Beer et al.,2014), which are suitable for ductile materials, used to analyze stresses in the beam. When Von Mises stress (σ_{vm}) value (equation 10) exceeds the yield capacity of material (σ_y>σ_{vm}), the yield of material occurs. σ_x , σ_y, σ_z are normal stresses along x,y,z axes, respectively.

$$
\sigma_{\text{vm}} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_y)^2}
$$
(10)

Figure 4. Bilinear elasto-plastic material model

2.3.2. Boundary Conditions

The FEM was established as a simply supported beam model. Remote displacement command was used to apply boundary conditions. The following constraints were used for FEA. Table 6 shows the boundary conditions applied to the cellular beam and Fig. 5 shows X, Y, and Z directions.

Figure 5. Axis used for boundary conditions

Table 6. Boundary conditions										
Part	Translation			Rotation						
	X	v	z	$r_{\rm x}$	r_{v}	$r_{\rm z}$				
Left side				free						
Right side			free	free						

Table 6. Boundary conditions

2.3.3. Investigation of Eigen-Value Problem by FEA

The rigidity matrix consists of elastic rigidity matrix $[K_E]$ and geometric rigidity matrix $[K_G]$. These matrixes are established automatically by ANSYS (2019). Buckling occurs when the total rigidity matrix equals to zero. This case is named as an Eigenvalue problem. The rigidity of bodies varies due to load type whether it is subjected to tension load or compression load. This variation is shown with $[K_{\sigma}]$ named as geometric rigidity matrix or initial stiffness matrix. [K_σ] is a symmetric matrix that takes into account the increase or decrease of stiffness due to load type. The relationship between force and stiffness matrix for linear elastic finite element analysis is shown in equation 11. Where *F* is the force matrix applied to structure, *u* is the displacement matrix of nodes and *λ* is the eigenvalue. Buckling occurs when the rigidity matrix equals to zero (Martini and Mohammad, 2011).

$$
\{F\} = [[K_E] + [K_{\sigma}] \lambda] \{u\}
$$
 (11)

Equation 12 (Martini and Mohammad, 2011) shows the total rigidity matrix of the structural element. 1 kN external vertical load was applied at mid-point of cellular beam to initiate elastic buckling analysis. Required lowest eigenvalue factor to solve rigidity matrix gives buckling load and related eigenvector gives buckling mode shape.

$$
[[KE] + [Ko] \lambda] = 0 \tag{12}
$$

2.3.4. Parametric Study

Four different types (Fig. 6) and seven different parameters were investigated in the parametric study. Firstly, (M1) was established (Fig. 6a) for verification of FEM which was obtained by the NPI240 root profile (Erdal and Saka (2013)) with 345 mm hole spacing and 251 mm hole diameter. Secondly, the hole diameter was varied for a fixed 345 mm hole spacing for M2-M7 models. Thirdly, hole spacing was varied for a fixed 251 mm hole diameter for M8-M11 models. Fourthly, vertical rigidity plates were placed (Fig. 6b) to verified model (M1) along web post centers for various thicknesses (M12-M15). Fifthly, vertical rigidity plate length was varied for vertical stiffener with 5 mm thickness (M16-M19). Sixthly, horizontal rigidity plates (Fig. 6c) were placed to verified model (M1) at 130mm above and below the center point of holes for various horizontal stiffener thicknesses (M20-M23). Seventhly, horizontal rigidity plate length was varied for vertical stiffener with 10 mm thickness (M23-M26). Lastly, a circular rigidity plate was added to the verified model (M1) for various circular rigidity plate thickness (M27-M30) (Fig. 6d). The geometrical properties of studied models can be seen in Table 7.

s : spacing between holes.
Do : hole diameter.

Do : hole diameter.
bvrp : width of vertical : width of vertical rigidity plate.

tvrp : thickness of vertical rigidity plate.

lvrp : length of vertical rigidity plate.

: length of vertical rigidity plate.

bhrp : width of horizontal rigidity plate.

thrp : thickness of horizontal rigidity plate.

lhrp : length of horizontal rigidity plate.

bcrp : with of circular rigidity plate.

tcrp : thickness of circular rigidity plate

Figure 6. Studied cellular beam types

3. Results and Discussion

Finite element analysis was conducted to investigate the flexural and lateral-torsional buckling behavior of the cellular beam (Fig. 7). An experimental study from literature was chosen for verification of FEM. Parametric studies were performed with a verified numerical model to investigate the effect of some geometrical properties of the cellular beam such as hole diameter, hole spacing, and various types of rigidity plates on buckling and bending capacity of beams.

Figure 7. Laterally buckled shape of cellular beam

Fig. 8a shows the load-deflection curve of the experimental study for buckling-prevented specimens (Test3 and Test4) that reported by Erdal and Saka (2013) and Fig. 8b shows the FEA result. As it is seen, the FEA result suits well with experimental study results.

Fig. 9a shows the load-deflection behavior of cellular beam for various hole sizes. Center to center hole spacing was 345mm. Elastic rigidity and ultimate load capacity decreases due to hole size. Fig. 9b shows the ultimate load capacity-hole size relation for cellular beam. Maximum flexural capacity decreased almost linearly between D0- D200 models. A sudden capacity decrease was observed for D251 and D300. Ultimate load capacity decreased by 70% due to the increasing hole size compared to the no hole model. Fig. 9c shows vertical rigidity-hole size relation for cellular beam. Vertical rigidity was seriously affected by hole size for 150 mm or larger hole diameters. Vertical rigidity decreased 74% for D300 compared to D0. It was assumed that LTB was prevented to reach the ultimate flexural capacity of the cellular beam for Fig. 9a,b,c. Fig. 9d shows the ultimate elastic LTB load-hole diameter relation. Center to center hole spacing was 345mm. A parabolic decrease was observed due to the hole diameter increase. Elastic LTB capacity was not affected as much as the ultimate flexural capacity of the cellular beam. However, the 200 mm hole diameter can also be accepted as a critical value for LTB. LTB capacity for 300 mm hole diameter decreased 22,7% compared to the no hole model.

Figure 8. Comparison of Experimental study (Erdal and Saka, 2013) and FEA

Figure 9. Graphics of parametric studies (1/2)

Fig. 10a shows the effect of vertical rigidity plate thickness on elastic LTB capacity. The length is 329,4 mm for all vertical rigidity plates. Almost a linear increase was observed due to the increase of vertical rigidity plate thickness. Vertical rigidity plate with 12 mm thickness (M15) increased the LTB capacity 40,6 % compared to the M1 model with no rigidity plate. Fig. 10b shows the effect of vertical rigidity plate length on elastic LTB capacity. Thickness is 5 mm for all rigidity plates. Vertical rigidity plate with 329,4 mm length increased LTB capacity up to 20,9% compared to the M1 model with no rigidity plate. Fig. 10c shows that horizontal rigidity plate thickness has a slight effect on LTB capacity. Rigidity plate with 15 mm thickness and 100 mm length increased LTB capacity around 2%. Fig. 10d shows the effect of horizontal rigidity plate length on elastic LTB capacity. Rigidity plate with 10 mm thickness and 250 mm length increased LTB capacity around 5,8% compared to the M1 model with no rigidity plate. Fig. 10e shows the effect of hole spacing on elastic LTB capacity. It was seen that elastic LTB capacity increased due to increased hole spacing. However, unexpected capacity decrease was observed for 500 mm hole spacing. Hole spacing affected the quantity of hole and the number of holes was also given in parenthesis. Fig. 10f shows the effect of circular rigidity plate thickness on elastic LTB capacity. Rigidity plate was used for all perimeter of hole (Fig. 6d). Circular rigidity plate with 12 mm thickness (M30) increased LTB capacity around 18,8% compared to the M1 model with no rigidity plate.

Figure 10. Graphics of parametric studies (2/2)

 (e)

 (f)

4. Conclusions

3D finite element analysis was performed to determine the flexural and elastic lateral-torsional buckling behavior of a cellular beam. Parametric studies were performed with a verified numerical model to investigate the effect of some geometrical properties of the cellular beam such as hole diameter, hole spacing, and various types of rigidity plates on buckling and flexural capacity of beams. The major conclusions of this study are as follows.

- Usage of gross-section area is recommended to estimate the LTB capacity of cellular beam following AISC360-16 (2016) and TSDC-2016 (2016). Theoretical calculation of critical LTB load with gross-section area gave a larger result than the critical LTB load found at experimental work. However, critical LTB load obtained by using the net-section area with safety factor perfectly matched to experimental study result. More conservative safety factor and net cross-section area usage is recommended for safe cellular beam design.
- Most of the experimental studies on LTB were conducted with American wide flange steel profiles in literature. Theoretical calculations following AISC360-16 and TSDC-2016 showed good agreement with experimental and finite element study results conducted with European steel profiles and European steel quality. Effect of various hole geometries and various types of rigidity plates were studied on flexural and elastic LTB capacity of the cellular beam. Numerical studies showed that rigidity plates contribute significantly to LTB capacity depending on their type.
- Most of 3D finite element models were established using two-dimensional shell elements in the literature. In this study, a 3-D finite element model was established using three-dimensional SOLID187 tetrahedral elements with 10 nodes that provide higher sensitivity. Finite element analysis can satisfactorily predict both linear and non-linear flexural character of cellular beam with acceptable accuracy in terms of rigidity and ultimate beam flexural capacity. The finite element model showed satisfactory agreement to predict elastic buckling capacity and elastic buckling mode of the cellular beam.

Notation

The following symbols are used in this paper:

Conflict of Interest

No conflict of interest was declared by the authors.

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