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Iterative Perturbation Technique for Solving a Special Magnetohydrodynamics Problem

Sinan Deniz[1*]

Manisa Celal Bayar University, Department of Mathematics, Manisa/TURKEY
*sinan.deniz@cbu.edu.tr

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Abstract

In this study, we use iterative perturbation technique for struggling MHD Jeffery-Hamel flow problem for some special values of Re and Ha numbers. This problem aroused from the classical work by Navier and Stokes and their equations. We exploit Maxwell's electromagnetism governing equations via reducing them to nonlinear differential equations to reform the main problem. After simplifying the well-known equation, we get a basic problem and we can readily investigate the emerged problem. In order to check the power of the technique, we prove that the results are well agreed with the numerical solutions. The present graphics prove that perturbation iteration technique has high accuracy for different α, Ha and Re numbers.

Keywords: Fluid mechanics, Jeffery-Hamel flows, perturbation iteration method.

1. Introduction

In many fields of engineering, it is very crucial to analyze an incompressible viscous fluid. These fluids generally have been occurred in nonparallel walls. Additionally, liquids through convergent-divergent channels are one of the most applicable cases in fluid mechanics. All these afore mentioned models have been criticized in civil, electronic and ocean engineering problems [1]. Besides all of that, one needs to model all these types of equations to handle the mathematical cases and correspondingly the real phenomena of the physical situations. The mathematical analysis of that problem was also extensively studied by two successful researchers Jeffery and Hamel (J-H) in 1916 [2,3]. On the other hand, the term of magnetohydrodynamic (MHD) was first used in 1970 [4]. Additionally, the aforementioned J-H flows are an exact similarity solution of the Navier and Stokes equations. Especially, these equations appear in the special case of two-dimensional (2-D) flow through a channel with inclined plane walls. These walls have been considered as meeting at a vertex with a source. Or it can be sink at the vertex [5]. Many researchers have studied to get approximate solutions to this flow problem. Most of them encounters with the highly difficult nonlinear terms and nonhomogeneous power terms. In order to get well-enough solutions, they use numerical techniques [1-4].

It is well known that there are many linear and nonlinear differential equations which are used in the study of several fields for example engineering, chemistry, physics, etc. The solutions of these equations can provide more information about the described process. However, because of the complexity of the nonlinear differential equations such as Jeffery-Hamel flows and other fluid problems, it is complicated to get the exact solutions. Therefore, a broad class of semi -analytical and analytical techniques have been proposed to solve these types of equations such as variational iteration method (VIM) [6], Adomian decomposition method (ADM) [7]. Besides these semi-analytical methods there are some semi-numerical techniques such as homotopy analysis method (HAM) [8], optimal homotopy asymptotic method [9]. Pandır has used generalized F expansion method for solving to Sine-Gordon equation [10]. Sezer et al. have implemented many kinds of collocation methods in their papers [11-12]. Inan has implemented exponential finite difference technique to nonlinear equations [13]. Lie symmetry is applied to handle ordinary differential equations [14]. In addition to these techniques, the well-known perturbation method has been recently used to construct the perturbation iteration method. This new effective technique has been used to solve some strongly nonlinear systems and yields better results than many other methods in literature [15-19]. Besides all these, stability analysis of these methods is very crucial concept to understand the qualitative idea behind the methods. Therefore, many papers are devoted analyzing uniform continuity, convergence and stability analysis for all these types of techniques [20].

Geometrical analysis for many real word phenomena is also very important such as in fluid dynamics. For instance, null quaternionic rectifying curves and null quaternionic similar curves in special Minkowski space has been discussed by Kahraman in 2018 [21]. He also studied null quaternionic slant helices in Minkowski Spaces in 2019 [22]. In 2004, Hopkins et. al have investigated the effects of arterial geometry on aneurysm growth as three-dimensional computational fluid dynamics study [23]. Besides all that, approximate solutions for MHD squeezing fluid flow has been obtained by Akgül [24]. Reproducing kernel Hilbert space method based on reproducing kernel functions for investigating boundary layer flow of a Powell–Eyring non-Newtonian fluid has been also used by same author [25]. Solitary wave solutions of time–space nonlinear fractional Schrödinger's equation has been analyzed via two analytical approaches in [26]. Akgül has also published a paper about fluid equations including reproducing kernel Hilbert space method to solve MHD Jeffery-Hamel flows problem in nonparallel walls [26, 27].

2. Mathematical Formulation of Jeffery-Hamel Flow

In this part of the paper, we analyze the analytical plan with the help of fluid flow equation problems. They have been reviewed by many scientists in the literature [1-5]. We take the immovable and fixed two-dimensional (2-D) flow. Of course, these flows have been existed with incompressible conducting viscous fluid. These fluids arise from a source or sink. We also suppose that these phenomena occur at the intersection between two rigid plane walls. The angle between these walls are taken as 2α. The rigid walls are taken to be divergent if $\alpha > 0$. Reversely, convergent if $\alpha < 0$. We now imagine that the velocity is only along the radial direction and depends on r and θ so that $\mathbf{v} = (u(r;\theta); 0)$. Using continuity and the Navier-Stokes equations in polar coordinates,

$$\frac{\rho}{r}\frac{\partial}{\partial r}(ru(r,\theta)) = 0, \quad \textbf{(2.1)}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r} = -\frac{1}{p}\frac{\partial P}{\partial r} + \nu\left[\begin{array}{l}\nabla^2 u(r,\theta)\\ -\dfrac{u(r,\theta)}{r^2}\end{array}\right] - \frac{\sigma B_0^2}{\rho r^2}u(r,\theta)$$

$$= -\frac{1}{p}\frac{\partial P}{\partial r} + \nu\left[\begin{array}{l}\dfrac{\partial^2 u(r,\theta)}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u(r,\theta)}{\partial r}\\ +\dfrac{1}{r^2}\dfrac{\partial^2 u(r,\theta)}{\partial \theta^2} - \dfrac{u(r,\theta)}{r^2}\end{array}\right] - \frac{\sigma B_0^2}{\rho r^2}u(r,\theta), \quad \textbf{(2.2)}$$

$$0 = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \frac{2\nu}{r^2}\frac{\partial u(r,\theta)}{\partial \theta}. \quad \textbf{(2.3)}$$

Here P denotes the fluid pressure. The constant B_0 represents the electromagnetic induction and σ symbols the conductivity of the fluid. As in many papers, ρ denotes the fluid density. Finally, ν shows the coefficient of kinematic viscosity. The continuity Eq. (1) implies that

$$u(\theta, r) = \frac{f(\theta)}{r} \quad \textbf{(2.4)}$$

For simplification, we can use the following dimensionless parameters,

$$D(x) = \frac{f(\theta)}{f_{max}} \text{ where } x = \frac{\theta}{\alpha} \quad \textbf{(2.5)}$$

and eliminating P between Eqs. (2.2) and (2.3), we obtain an ordinary differential equation for the normalized function profile $D(x)$:

$$D'''(x) + 2\alpha^2\frac{f_{max}}{\nu}D(x)D'(x) + \left(4 - \sqrt{\sigma\frac{B_0^2}{\rho\nu}}\right)\alpha^2 D'(x) = 0 \quad \textbf{(2.6)}$$

or equivalently

$$D''' + 2\alpha ReDD' + (4 - Ha)\alpha^2 D' = 0 \quad \textbf{(2.7)}$$

Since one has a symmetric geometry here, we can take the boundary conditions as follows

$$D(0) = 1,\ D'(0) = 0,\ D(1) = 0. \quad \textbf{(2.8)}$$

3. A Short Description of the Perturbation Iteration Method

In this section, we give a short description of the perturbation iteration technique. For much more information, we refer to the papers [16-18]. This technique was firstly introduced by Pakdemirli et al. and applied to many types of nonlinear problems [28-30].

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Many different kinds of ordinary and partial differential equations are solved by using perturbation iteration techniques and even fractional differential equations are also considered modified forms of this method [31-38].

Consider the following third order nonlinear differential equation:

$$F\left(D''', D', D, \varepsilon\right) = 0 \quad \textbf{(3.1)}$$

where $D = D(x)$. Here ε is the perturbation parameter. To obtain perturbation iteration algorithms (PIA), we will use only one correction term from classical perturbation expanding as

$$D_{n+1} = D_n + \varepsilon\left(D_c\right)_n \quad \textbf{(3.2)}$$

where $n \in \mathrm{N} \cup \{0\}$ and $\left(D_c\right)_n$ is the n th correction term of the iteration algorithm. Upon substitution of (3.2) into (3.1) then expanding it in a Taylor series with *n*th derivatives yields the PIA $-n$'s. With only first derivatives, we have PIA-1 as

$$\begin{aligned} &F + F_D (D_c)_n \varepsilon + F_{D'} (D_{c'})_n \varepsilon \\ &+ F_{D'''} (D_{c'''})_n \varepsilon + F_\varepsilon \varepsilon = 0 \end{aligned} \quad \textbf{(3.3)}$$

where subscripts of *F* symbolize partial differentiation. Note here that all derivatives and functions are computed at $\varepsilon = 0$. We can now start to iterate. First of all, we require a trial function which is called D_0. This function can be chosen judiciously according to the given described conditions. After this step, $\left(D_c\right)_0$ is evaluated from the algorithms (3.3) with the help of D_0 and recommended condition(s). After this step, the first approximate PIM solution D_1 is obtained by using $\left(D_c\right)_0$ and so on.

4. Pim Solutions for Jeffery-Hamel Flow

Let us now apply OPIM to Jeffery-Hamel flow. Artificial perturbation parameter is inserted to Eq. (2.7) as follows:

$$\begin{aligned} &F\left(D''', D', D, \varepsilon\right) = D''' + 2\varepsilon\alpha ReDD' \\ &+\varepsilon(4-Ha)\alpha^2 D' = 0. \end{aligned} \quad \textbf{(4.1)}$$

Using the Eqs. (3.2) and (3.3) and setting $\varepsilon = 1$ we get the following perturbation iteration algorithm:

$$\begin{aligned} &(D_c)_{n'''} = \\ &-\left(D_{n'''} + 2\alpha Re D_n D_{n'} + (4-Ha)\alpha^2 D_{n'}\right). \end{aligned} \quad \textbf{(4.2)}$$

As a starting function we can use

$$D_0 = 1 - x^2 \quad \textbf{(4.3)}$$

which satisfies the boundary conditions (2.8). Substituting D_0 into Eq. (4.2) gives a first-order problem:

$$(D_c)_0''' = 2\alpha^2(4-\mathrm{Ha})x + 4\alpha \mathrm{Re} x\left(1-x^2\right). \quad \textbf{(4.4)}$$

By solving (4.4), first correction term is obtained as:

$$(D_c)_0 = \frac{1}{60}\begin{pmatrix} -5\alpha^2 \mathrm{Ha} x^4 + 5\alpha^2 \mathrm{Ha} x^2 \\ -2\alpha \mathrm{Re} x^6 + \\ 10\alpha \mathrm{Re} x^4 - 8\alpha \mathrm{Re} x^2 \\ +20\alpha^2 x^4 - 20\alpha^2 x^2 \end{pmatrix}. \quad \textbf{(4.5)}$$

Thus, the first approximate solution is

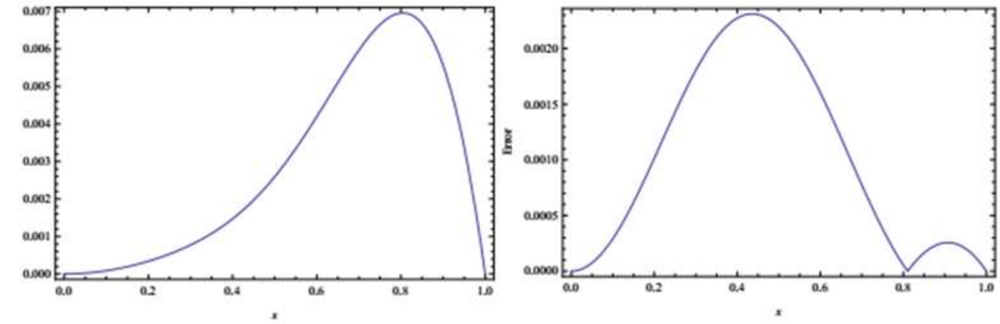
$$D_1 = 1 - x^2 + \frac{1}{60}\begin{pmatrix} -5\alpha^2 \mathrm{Ha} x^4 + 5\alpha^2 \mathrm{Ha} x^2 \\ -2\alpha \mathrm{Re} x^6 + 10\alpha \mathrm{Re} x^4 \\ -8\alpha \mathrm{Re} x^2 + 20\alpha^2 x^4 \\ -20\alpha^2 x^2 \end{pmatrix}. \quad \textbf{(4.6)}$$

Proceeding as mentioned in previous section, one can get second order solution and so on.

Higher order iterations can be reached in a similar manner. Manifestly, the more iterations, the more approximate solution becomes more sophisticated, which wants powerful computer programs. We use Mathematica 9.0 to handle the complex computations throughout this study.

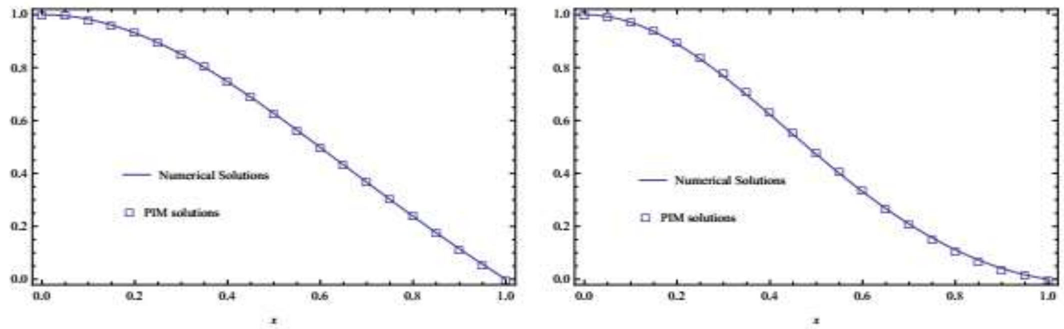
In [5], optimal homotopy asymptotic method is used to get the second order approximate solution for $\alpha = \frac{\pi}{36}, Ha = 0$ and $Re = 50$.

In the following plots and tables, a comparison of the OHAM, PIM and the numerical results is shown. It is also clear from figure 4.1 and tables that PIM solutions are better than those of OHAM solutions. Figures 4.2 - 4.3 show the magnetic field effect on the velocity profiles for convergent and divergent channels for some fixed Reynolds, Hartmann numbers with angels α's.



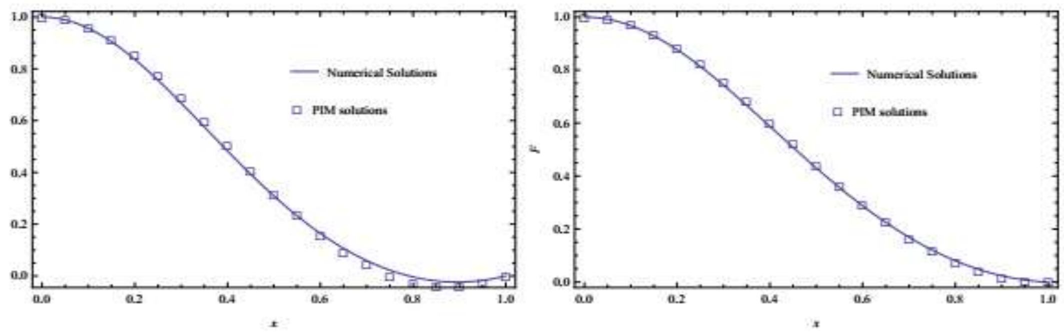
(a) Errors for OHAM (b) Errors for PIM

Figure 4.1: Comparison of the absolute erros obtained by second order OHAM and PIM approximate solutions



(a) $Re = 50$ (b) $Re = 100$

Figure 4.2: Comparison of the numerical results against the second order PIM approximation for the velocity profile using $\alpha = \frac{\pi}{36}$ and $Ha = 0$ for different Reynolds numbers



(a) $\alpha = \frac{\pi}{18}$ (b) $\alpha = \frac{\pi}{24}$

Figure 4.3: Comparison of the numerical results against the second order PIM approximation for the velocity profile using $Re = 75$ and $Ha = 0$ for different α

Table 1. Absolute errors for sixth order PIM solutions for different Reynolds and Hartmann numbers.

x	Re= 175, Ha=5	Re=200, Ha=5	Re=50, Ha= 10	Re=50, Ha=20
0.1	3.124×10^{-6}	4.632×10^{-7}	7.111×10^{-5}	2.044×10^{-5}
0.2	8.052×10^{-6}	1.057×10^{-7}	8.041×10^{-5}	6.034×10^{-5}
0.3	8.711×10^{-7}	1.086×10^{-7}	2.222×10^{-6}	4.706×10^{-6}
0.4	5.011×10^{-8}	2.997×10^{-7}	6.411×10^{-6}	7.055×10^{-6}
0.5	2.058×10^{-6}	5.088×10^{-7}	6.520×10^{-6}	3.524×10^{-6}
0.6	2.779×10^{-7}	2.410×10^{-6}	6.030×10^{-5}	4.001×10^{-5}
0.7	3.087×10^{-6}	5.085×10^{-6}	1.005×10^{-6}	3.041×10^{-5}
0.8	3.045×10^{-7}	6.047×10^{-6}	8.006×10^{-5}	7.770×10^{-5}
0.9	4.056×10^{-6}	8.047×10^{-6}	9.056×10^{-5}	9.055×10^{-5}

Table 2. Absolute errors for sixth order OHAM solutions for different Reynolds and Hartmann numbers.

x	Re= 175, Ha=5	Re=200, Ha=5	Re=50, Ha= 10	Re=50, Ha=20
0.1	5.004×10^{-5}	1.018×10^{-4}	5.032×10^{-4}	2.005×10^{-4}
0.2	2.067×10^{-4}	9.044×10^{-6}	4.502×10^{-4}	5.660×10^{-4}
0.3	8.046×10^{-6}	4.222×10^{-6}	5.177×10^{-5}	9.995×10^{-6}
0.4	1.110×10^{-6}	5.002×10^{-6}	9.025×10^{-5}	9.755×10^{-6}
0.5	8.096×10^{-6}	9.023×10^{-6}	5.023×10^{-5}	1.067×10^{-5}
0.6	1.009×10^{-6}	2.055×10^{-5}	5.023×10^{-6}	5.905×10^{-4}
0.7	9.066×10^{-5}	7.502×10^{-6}	6.024×10^{-3}	1.010×10^{-3}
0.8	4.068×10^{-6}	5.065×10^{-6}	2.014×10^{-4}	6.012×10^{-4}
0.9	9.098×10^{-7}	2.023×10^{-6}	5.014×10^{-4}	7.463×10^{-4}

5. Conclusion and Results

In this study, we implement perturbation iteration method to find the approximate solutions of nonlinear differential equation governing Jeffery-Hamel flow. Our results clearly demonstrate that PIM can solve nonlinear problems with successive rapidly convergent approximations without any restrictive assumptions or transformations causing changes in the physical definition of the considered problem. One of the fundamental advantages of this method is to be applicable directly to the nonlinear terms. Also, resulting equations can also be solved by using simple analytical methods. We also make a comparison with the OHAM solution and see that PIM is more useful because it reduces the size of calculations and also its iterations are direct and straightforward. The figures and tables also reveal that new solutions agree very well with the numerical solutions obtained from Mathematica 9.0. Finally, we can say that PIM can be safely used to handle fluid mechanics and other problems in engineering.

Ethics

There are no ethical issues after the publication of this manuscript.

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