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On Binomial Sums and Alternating Binomial Sums of Generalized Fibonacci Numbers with Falling Factorials

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Abstract
In this paper, we consider and obtain binomial sums and alternating binomial sums including falling factorial of the summation indice. For example, For integers n and m such that $0 \leq m < n$,

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = \frac{n^{\underline{m}}}{(p^2+4)^m} \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} V_{2(m-i)}^{n-m} V_{2(m+n)(m-i)} - (-1)^m 2^{n-m} \binom{2m}{m} \right),$$

and for positive odd integer m,

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} V_k^{2m} = n^{\underline{m}} \left(\sum_{i=0}^{m-1} (-1)^{n(i+1)} \binom{2m}{i} V_{m-i}^{n-m} V_{(m+n)(m-i)} + \binom{2m}{m} 2^{n-m} \right).$$

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1. Introduction

For $n \geq 2$, define second order linear sequences $\{U_n\}$ and $\{V_n\}$ by

$$U_n = pU_{n-1} + U_{n-2} \text{ and } V_n = pV_{n-1} + V_{n-2},$$

with $U_0 = 0, U_1 = 1$ and $V_0 = 2, V_1 = p$, respectively. When $p = 1$, $U_n = F_n$ (nth Fibonacci number) and $V_n = L_n$ (nth Lucas number). The Binet formulae are

$$U_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \text{ and } V_n = \alpha^n + \beta^n,$$

where $\alpha, \beta = \left(p \mp \sqrt{\Delta}\right)/2$ such that $\Delta = p^2 + 4$.

$x^{\underline{m}}$ stands for the falling factorial defined by

$$x^{\underline{m}} = x(x-1)...(x-m+1).$$

Wiemann and Cooper [11] gave certain conjectures for the Melham sum

$$\sum_{k=0}^{n} F_{2k+\delta}^{2m+\varepsilon},$$

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where m is a positive integer and $\varepsilon, \delta \in \{0,1\}$. Ozeki [9] considered Melham's sum and gave an explicit expansion for it as a polynomial in F_{2n+1} and Prodinger [10] obtained a formula for the sum

$$\sum_{k=0}^{n} F_{2k+\delta}^{2m+\varepsilon},$$

where m, ε and δ are as before. Kılıç et. al [2] considered alternating Melham's sums for Fibonacci and Lucas numbers of the form

$$\sum_{k=1}^{n} (-1)^k F_{2k+\delta}^{2m+\varepsilon} \text{ and } \sum_{k=1}^{n} (-1)^k L_{2k+\delta}^{2m+\varepsilon},$$

where m, ε and δ are as before.

Kılıç and Taşdemir [7] derived various binomial-double-sums involving the Fibonacci numbers as well as their alternating analogues and they [8] gave and computed various sum families of binomial sums namely binomial-sums including double sums and one binomial coefficient of the forms and their alternating analogues

$$\sum_{0\le i,j\le n} \binom{i}{j} U_{ri+tj}, \quad \sum_{0\le i,j\le n} \binom{i}{j} (-1)^i V_{ri+tj},$$

where r and t are odd integers.

Khan and Kwong [6] obtained the sums

$$\sum_{i=0}^{n} \binom{n}{i} i^m U_i \text{ and } \sum_{i=0}^{n} \binom{n}{i} (-1)^i i^m U_i,$$

for some nonnegative integer m.

Kılıç et. al [3] computed the weighted binomial sums including the powers of the summation index:

$$\sum_{i=0}^{n} \binom{n}{i} i^m U_{ti}^{2m+\varepsilon} \text{ and } \sum_{i=0}^{n} \binom{n}{i} i^m (-1)^{n+i} V_{ti}^{2m+\varepsilon} \ ,$$

where positive integers t, m and $\varepsilon \in \{0,1\}$.

Kılıç et. al [4] introduced and computed new kinds of binomial sums including falling factorial of the summation indice. For example,

$$\sum_{i=0}^{n} \binom{n}{i} (-1)^i i^{\underline{m}} U_{kti} V_{k(n-ti)} = (-1)^{kn(t+1)+m} n^{\underline{m}} U_{kt}^{n-m} \Delta^{(n-m)/2} \times \begin{cases} U_{k(tn+tm-n)} & \text{if } n \equiv m \pmod 2, \\ -\Delta^{1/2} V_{k(tn+tm-n)} & \text{if } n \equiv m+1 \pmod 2, \end{cases}$$

where k and t are any integers, and m is a nonnegative integer. They [5] gave closed formulæ for weighted and alternating weighted binomial sums with the generalized Fibonacci and Lucas numbers including both falling factorials and powers of indices. Furthermore they derived closed formulæ for weighted binomial sums including odd powers of the generalized Fibonacci and Lucas numbers.

2. Some Results

In this section, firstly, we will continue our work with the following lemma:

Lemma 2.1. *[1]Let n and m be integers such that $0 \le m < n$. For $a \ne -1$,*

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} a^k = a^m n^{\underline{m}} (1+a)^{n-m} \ . \tag{2.1}$$

Now, we shall present one of our main result.

Theorem 2.1. *For integers n and m such that $0 \leq m < n$, we have*

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = \frac{n^{\underline{m}}}{\Delta^m} \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} V_{2(m-i)}^{n-m} V_{2(m+n)(m-i)} - (-1)^m \, 2^{n-m} \binom{2m}{m} \right),$$

and

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} U_{2k}^{2m+1} = n^{\underline{m}} \Delta^{(n-3m)/2} \times \begin{cases} \sum\limits_{i=0}^{m} (-1)^i \binom{2m+1}{i} U_{2m-2i+1}^{n-m} U_{(2m-2i+1)(m+n)} & \textit{if } n \equiv m \pmod 2, \\ \Delta^{-1/2} \sum\limits_{i=0}^{m} (-1)^i \binom{2m+1}{i} U_{2m-2i+1}^{n-m} V_{(2m-2i+1)(m+n)} & \textit{if } n \equiv m+1 \pmod 2. \end{cases}$$

Proof. Consider that

$$\begin{aligned} &\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = \frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \left(\alpha^{2k} - \beta^{2k}\right)^{2m} \\ = &\frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \sum_{i=0}^{2m} (-1)^i \binom{2m}{i} \alpha^{2ki} \beta^{2k(2m-i)} \\ = &\frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} \left(\alpha^{2ki} \beta^{2k(2m-i)} + \alpha^{2k(2m-i)} \beta^{2ki} \right) - (-1)^m \binom{2m}{m} (\alpha\beta)^{2km} \right), \end{aligned}$$

and by $\alpha\beta = -1$ and Lemma 2.1, the above expression equals

$$\begin{aligned} = &\frac{1}{(\alpha-\beta)^{2m}} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \left(\alpha^{2k(2m-2i)} + \beta^{2k(2m-2i)} \right) \\ &- (-1)^m \frac{1}{(\alpha-\beta)^{2m}} \binom{2m}{m} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} (\alpha\beta)^{2km} \\ = &\frac{1}{(\alpha-\beta)^{2m}} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \left(\alpha^{4k(m-i)} + \beta^{4k(m-i)} \right) \\ &- (-1)^m \frac{1}{(\alpha-\beta)^{2m}} \binom{2m}{m} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} \\ = &\frac{n^{\underline{m}}}{\Delta^m} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \left(\alpha^{4m(m-i)} \left(1 + \alpha^{4(m-i)}\right)^{n-m} + \beta^{4m(m-i)} \left(1 + \beta^{4(m-i)}\right)^{n-m} \right) \\ &- (-1)^m \binom{2m}{m} n^{\underline{m}} \frac{2^{n-m}}{\Delta^m}. \end{aligned}$$

Since $\alpha^2 = \beta^{-2}$ and $\beta^2 = \alpha^{-2}$, we write

$$\begin{aligned}
&\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = \frac{n^{\underline{m}}}{\Delta^m} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \\
&\times \left(\alpha^{4m(m-i)} \left(1 + \frac{\alpha^{2(m-i)}}{\beta^{2(m-i)}}\right)^{n-m} + \beta^{4m(m-i)} \left(1 + \frac{\beta^{2(m-i)}}{\alpha^{2(m-i)}}\right)^{n-m} \right) - (-1)^m \frac{n^{\underline{m}}}{\Delta^m} 2^{n-m} \binom{2m}{m} \\
= \; & \frac{n^{\underline{m}}}{\Delta^m} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} V_{2(m-i)}^{n-m} \left(\alpha^{4m(m-i)} \beta^{2(m-i)(m-n)} + \alpha^{2(m-i)(m-n)} \beta^{4m(m-i)} \right) \\
& - (-1)^m \binom{2m}{m} n^{\underline{m}} \frac{2^{n-m}}{\Delta^m} \\
= \; & \frac{n^{\underline{m}}}{\Delta^m} \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} V_{2(m-i)}^{n-m} \left(\beta^{-2(m+n)(m-i)} + \alpha^{-2(m+n)(m-i)} \right) - (-1)^m 2^{n-m} \binom{2m}{m} \right) \\
= \; & \frac{n^{\underline{m}}}{\Delta^m} \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} V_{2(m-i)}^{n-m} V_{2(m+n)(m-i)} - (-1)^m 2^{n-m} \binom{2m}{m} \right),
\end{aligned}$$

as claimed. The proof of the binomial sum of the odd powers can be similarly done. □

For example, if we take $m = 1$,

$$\sum_{k=0}^{n} k \binom{n}{k} U_{2k}^2 = \frac{n}{\Delta} \left((\Delta - 2)^{n-1} V_{2(n+1)} - 2^n \right),$$

and

$$\sum_{k=0}^{n} \binom{n}{k} k U_{2k}^3 = n \Delta^{(n-3)/2} \begin{cases} (\Delta - 3)^{n-1} U_{3(n+1)} - 3U_{n+1} & \text{if } n \text{ is odd}, \\ \Delta^{-1/2} \left((\Delta - 3)^{n-1} V_{3(n+1)} - 3V_{n+1} \right) & \text{if } n \text{ is even}. \end{cases}$$

Theorem 2.2. *Let n and m be integers such that $0 < m < n$. For even number m,*

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} V_k^{2m} = n^{\underline{m}} \left(\sum_{i=0}^{m-1} (-1)^{ni} \binom{2m}{i} V_{m-i}^{n-m} V_{(m+n)(m-i)} + (-1)^m \binom{2m}{m} 2^{n-m} \right),$$

and for odd number m,

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} V_k^{2m} = n^{\underline{m}} \Delta^{(n-m)/2} \begin{cases} \Delta^{1/2} \sum\limits_{i=0}^{m-1} (-1)^{ni} \binom{2m}{i} U_{m-i}^{n-m} U_{(m+n)(m-i)} & \text{if } n \text{ is even}, \\ \sum\limits_{i=0}^{m-1} (-1)^{ni} \binom{2m}{i} U_{m-i}^{n-m} V_{(m+n)(m-i)} & \text{if } n \text{ is odd}. \end{cases}$$

Theorem 2.3. *Let n and m be integers such that $0 \le m < n$. We have*

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} V_{2k}^{2m+1} = n^{\underline{m}} \Delta^{(n-m)/2} \begin{cases} \sum\limits_{i=0}^{m} \binom{2m+1}{i} U_{2m-2i+1}^{n-m} V_{(2m-2i+1)(m+n)} & \text{if } n \equiv m \pmod{2}, \\ \Delta^{1/2} \sum\limits_{i=0}^{m} \binom{2m+1}{i} U_{2m-2i+1}^{n-m} U_{(2m-2i+1)(m+n)} & \text{if } n \equiv m+1 \pmod{2}, \end{cases}$$

and

$$\sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} V_{2k}^{2m} = n^{\underline{m}} \left(\sum_{i=0}^{m-1} \binom{2m}{i} V_{2(m-i)}^{n-m} V_{2(m+n)(m-i)} + \binom{2m}{m} 2^{n-m} \right).$$

Now, we will give alternating binomial sums of the powers of generalized Fibonacci and Lucas numbers with falling factorials.

Theorem 2.4. *Let n and m be integers such that $0 \leq m < n$. We have*

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = (-1)^m n^{\underline{m}} \Delta^{(n-3m)/2}$$
$$\times \begin{cases} \sum\limits_{i=0}^{m} (-1)^i \binom{2m}{i} U_{2(m-i)}^{n-m} V_{2(m+n)(m-i)} & \text{if } n \equiv m \pmod 2, \\ \Delta^{1/2} \sum\limits_{i=0}^{m} (-1)^{i+1} \binom{2m}{i} U_{2(m-i)}^{n-m} U_{2(m+n)(m-i)} & \text{if } n \equiv m+1 \pmod 2, \end{cases}$$

and

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} U_{2k}^{2m+1} = \frac{(-1)^n}{\Delta^m} n^{\underline{m}} \sum_{i=0}^{m} (-1)^i \binom{2m+1}{i} V_{2m-2i+1}^{n-m} U_{(2m+1-2i)(m+n)}.$$

Proof. From Binet formula of $\{U_n\}$, we have

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = \frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}}$$
$$\times \left(\sum_{i=0}^{m} (-1)^i \binom{2m}{i} \left(\alpha^{2ki}\beta^{2k(2m-i)} + \alpha^{2k(2m-i)}\beta^{2ki} \right) - (-1)^m \binom{2m}{m} (\alpha\beta)^{2km} \right),$$

and applying some elementary operations, equals

$$= \frac{1}{(\alpha-\beta)^{2m}} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} \left(\alpha^{4k(m-i)} + \beta^{4k(m-i)} \right)$$
$$- (-1)^m \binom{2m}{m} \frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} \binom{n}{k} k^{\underline{m}} (-1)^k (\alpha\beta)^{2km}.$$

By Lemma 2.1 and $\alpha\beta = -1$, we have

$$\begin{aligned}
&= \frac{1}{(\alpha-\beta)^{2m}} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} \left(\alpha^{4k(m-i)} + \beta^{4k(m-i)} \right) \\
&\quad - (-1)^m \binom{2m}{m} \frac{1}{(\alpha-\beta)^{2m}} \sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} \\
&= \frac{n^{\underline{m}}}{\Delta^m} \sum_{i=0}^{m} (-1)^{i+m} \binom{2m}{i} \left(\alpha^{4m(m-i)} \left(1-\alpha^{4(m-i)}\right)^{n-m} + \beta^{4m(m-i)} \left(1-\beta^{4(m-i)}\right)^{n-m} \right) \\
&= (-1)^m \frac{n^{\underline{m}}}{\Delta^m} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} \left(\alpha^{4m(m-i)} \left(1-\frac{\alpha^{2(m-i)}}{\beta^{2(m-i)}}\right)^{n-m} + \beta^{4m(m-i)} \left(1-\frac{\beta^{2(m-i)}}{\alpha^{2(m-i)}}\right)^{n-m} \right) \\
&= (-1)^m \frac{n^{\underline{m}}}{\Delta^m} (\alpha-\beta)^{n-m} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} U_{2(m-i)}^{n-m} \left((-1)^{n-m} \alpha^{2(m+n)(m-i)} + \beta^{2(m+n)(m-i)} \right).
\end{aligned}$$

For $n \equiv m \pmod 2$, we get

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^{\underline{m}} U_{2k}^{2m} = (-1)^m \Delta^{(n-3m)/2} n^{\underline{m}} \sum_{i=0}^{m} (-1)^i \binom{2m}{i} U_{2(m-i)}^{n-m} V_{2(m+n)(m-i)}.$$

For $n \equiv m+1 \pmod 2$, the desired result is similarly obtained. We can prove the other claim by using same approach. □

For positive integer $n > 1$ and $m = 1$,

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k U_{2k}^2 = \begin{cases} -n\Delta^{(n-3)/2} U_2^{n-1} V_{2(n+1)} & \text{if } n \text{ is odd}, \\ n\Delta^{(n-2)/2} U_2^{n-1} U_{2(n+1)} & \text{if } n \text{ is even}, \end{cases}$$

and for $m=2$,

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}k\left(k-1\right)U_{2k}^{5}$$
$$=(-1)^{n}p^{n-2}\frac{n\left(n-1\right)}{\Delta^{2}}\left(\left(p^{4}+5p^{2}+5\right)^{n-2}U_{5(n+2)}-5\left(p^{2}+3\right)^{n-2}U_{3(n+2)}+10U_{(n+2)}\right).$$

Theorem 2.5. *Let n and m be integers such that $0<m<n$. For even number m,*

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}k^{\underline{m}}V_{k}^{2m}=n^{\underline{m}}\Delta^{(n-m)/2}\begin{cases}\sum\limits_{i=0}^{m-1}\binom{2m}{i}U_{m-i}^{n-m}V_{(m+n)(m-i)} & \text{if } n \text{ is even},\\ \Delta^{1/2}\sum\limits_{i=0}^{m-1}(-1)^{i+1}\binom{2m}{i}U_{m-i}^{n-m}U_{(m+n)(m-i)} & \text{if } n \text{ is odd},\end{cases}$$

and for odd number m,

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}k^{\underline{m}}V_{k}^{2m}=n^{\underline{m}}\left(\sum_{i=0}^{m-1}(-1)^{n(i+1)}\binom{2m}{i}V_{m-i}^{n-m}V_{(m+n)(m-i)}+\binom{2m}{m}2^{n-m}\right).$$

Theorem 2.6. *Let n and m be integers such that $0<m<n$. We have*

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}k^{\underline{m}}V_{2k}^{2m+1}=(-1)^{n}n^{\underline{m}}\sum_{i=0}^{m}\binom{2m+1}{i}V_{2m-2i+1}^{n-m}V_{(2m-2i+1)(m+n)},$$

and

$$\sum_{k=0}^{n}(-1)^{k}\binom{n}{k}k^{\underline{m}}V_{2k}^{2m}=(-1)^{m}n^{\underline{m}}\Delta^{(n-m)/2}\begin{cases}\sum\limits_{i=0}^{m-1}\binom{2m}{i}U_{2(m-i)}^{n-m}V_{2(m+n)(m-i)} & \text{if } n\equiv m \pmod 2,\\ \Delta^{1/2}\sum\limits_{i=0}^{m-1}\binom{2m}{i}U_{2(m-i)}^{n-m}U_{2(m+n)(m-i)} & \text{if } n\equiv m+1 \pmod 2.\end{cases}$$

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