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Fixed Point Theorems in b-Rectangular Metric Spaces

Vildan Öztürk[1]

[1] *Department of Mathematics, Faculty of Polatli Science and Arts, University of Ankara Haci Bayram Veli, Ankara, Turkey*

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Abstract

The concept of b-rectangular metric space is introduced as a generalization of b-metric space and rectangular (generalized) metric space. In this paper, we introduce generalized almost contraction for two mappings and prove common fixed point theorems in b-rectangular metric spaces.

1. Introduction and Preliminaries

Banach contraction principle is one of the earlier and main results in fixed point theory. Banach contraction principle was proved in complete metric spaces. In last years, many generalizations of the concept of metric spaces are defined and some fixed point theorems was proved in these spaces. In particular, $b-$metric spaces was introduced by Bakhtin [2] and Czerwik [6] as a generalization of metric spaces. They proved Banach contraction principle in b-metric spaces. Since then, some authors proved fixed point theorems in b-metric spaces [11], [13], [18], [19], [20], [21]. Another generalization of metric spaces is generalized metric spaces (g.m.s.) or rectangular metric spaces (r.m.s.). Branciari [5] introduced the concept of generalized metric space by replacing the triangle inequality by a more general inequality - by the rectangular inequality. Thereafter, many authors initiated and studied many existing fixed point theorems in such spaces [7], [8], [12], [15]. Also, the concept of $b-$rectangular metric space is introduced as a generalization of $b-$metric space and rectangular (generalized) metric space by Geoge et al. [11]. Also see [9], [10], [14] , [16], [17], [22].

Definition 1.1. *[2], [6] Let X be a nonempty set and $s \geq 1$ be a given real number. A function $d : X \times X \to [0,\infty)$ is a b-metric on X if, for all $x,y,z \in X$, the following conditions hold:*

(b1) $d(x,y) = 0$ *if and only if* $x = y$,
(b2) $d(x,y) = d(y,x)$,
(b3) $d(x,z) \leq s[d(x,y)+d(y,z)]$ *(b-triangular inequality).*

In this case, the pair (X,d) is called a b-metric space.

Definition 1.2. *[5] Let X be a nonempty set, and let $d : X \times X \to [0,\infty)$ be a mapping such that for all $x,y \in X$ and all distinct points $u,v \in X$, each distinct from x and y :*

(r1) $d(x,y) = 0$ *if and only if* $x = y$,
(r2) $d(x,y) = d(y,x)$,
(r3) $d(x,y) \leq d(x,u)+d(u,v)+d(v,y)$ *(rectangular inequality).*

Then (X,d) is called rectangular or generalized metric space.

Definition 1.3. *[11] Let X be a nonempty set, $s \geq 1$ be a given real number and let $d : X \times X \to [0,\infty)$ be a mapping such that for all $x,y \in X$ and distinct points $u,v \in X$, each distinct from x and y :*

(br1) $d(x,y) = 0$ *if and only if* $x = y$,
(br2) $d(x,y) = d(y,x)$,
(br3) $d(x,y) \leq s[d(x,u)+d(u,v)+d(v,y)]$ *(b-rectangular inequality).*

Email address and ORCID Number: **vildan.ozturk@hbv.edu.tr, https://orcid.org/0000-0001-5825-2030 (V. Öztürk)**

Then (X,d) is called a b-rectangular metric space or a $b-$generalized metric space (b-g.m.s.).

Note that every metric space is a rectangular metric space (g.m.s) and every rectangular metric space is a rectangular $b-$metric space (with coefficient $s=1$). However the converse is not necessarily true. Also, every metric space is a b-metric space and every b-metric space is a $b-$rectangular metric space (not necessarily with the same coefficient) [11].

Definition 1.4. *[11] (X,d) be a $b-$rectangular metric space and $\{x_n\}$ be a sequence in X and $x \in X$. Then*
***(i)** The sequence $\{x_n\}$ is said to be convergent in (X,d) and converges to x, if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) < \varepsilon$ for all $n > n_0$ and this fact is represented by*

$$\lim_{n\to\infty} x_n = x \text{ or } x_n \to x \text{ as } n \to \infty.$$

***(ii)** The sequence $\{x_n\}$ is said to be $b-$rectangular-Cauchy in (X,d) if for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_{n+p}) < \varepsilon$ for all $n > n_0, p > 0$ or equivalently,*

$$\lim_{n\to\infty} d(x_{n+p}, x_n) = 0 \text{ for all } p > 0.$$

***(iii)** (X,d) is said to be complete if every $b-$rectangular$-$Cauchy sequence in (X,d) converges to an element of X.*

Note that, limit of a sequence in a rectangular $b-$metric space is not necessarily unique.

Example 1.5. *[11] Let $X = A \cup B$, where $A = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ and B is the set of all positive integers. Define $d : XxX \to [0,\infty)$ such that $d(x,y) = d(y,x)$ for all $x,y \in X$ and*

$$d(x,y) = \begin{cases} 0 \text{ if } x = y \\ 2\alpha \text{ if } x,y \in A \\ \frac{\alpha}{2n} \text{ if } x \in A \text{ and } y \in \{2,3\} \\ \alpha \text{ otherwise} \end{cases}$$

where $\alpha > 0$ is a constant. Then (X,d) is a b-rectangular metric space with coefficient $s = 2 > 1$. However we have the following:
***1.** (X,d) is not a rectangular metric space, as $d\left(\frac{1}{2},\frac{1}{3}\right) = 2\alpha > \frac{17}{12} = d\left(\frac{1}{2},4\right) + d(4,3) + d\left(3,\frac{1}{3}\right)$ and hence not a metric space.*
***2.** There does not exist $s > 1$ satisfying $d(x,y) \le s[d(x,z) + d(z,y)]$ for all $x,y,z \in X$, and so (X,d) is not a $b-$metric space.*
***3.** $B_{\frac{\alpha}{2}}\left(\frac{1}{2}\right) = \left\{2,3,\frac{1}{2}\right\}$ and there does not exist any open ball with centre 2 and contained in $B_{\frac{\alpha}{2}}\left(\frac{1}{2}\right)$. So $B_{\frac{\alpha}{2}}\left(\frac{1}{2}\right)$ is not an open set.*
***4.** The sequence $\left\{\frac{1}{n}\right\}$ converges to 2 and 3 in b-rectangular metric space and so limit is not unique. Also $d\left(\frac{1}{n},\frac{1}{n+p}\right) = 2\alpha \nrightarrow 0$ as $n \to \infty$, therefore $\left\{\frac{1}{n}\right\}$ is not a b-rectangular$-$Cauchy sequence in b-rectangular metric space.*
***5.** There does not exist any $r_1, r_2 > 0$ such that $B_{r_1}(2) \cap B_{r_2}(3) = \varnothing$ and (X,d) is not Hausdoff.*

In this paper, we prove some fixed point theorems for mappings satisfying almost contractive condition in b-rectangular metric spaces. Berinde [3] defined the notion of a weak contraction mapping which is more general than a contraction mapping. Afterward, many authors have studied this problem and obtained significant results [1], [4], [21], [23].

2. Fixed point theorems

Theorem 2.1. *Let (X,d) be a complete b-rectangular metric space with $s > 1$, and let $f,g : X \to X$ be two self maps satisfying*

$$d(fx,gy) \le \delta M(x,y) + LN(x,y) \tag{2.1}$$

for all $x,y \in X$, where $\delta \in \left[0,\frac{1}{s}\right)$ and $L \ge 0$ and

$$M(x,y) = \max\{d(x,y), d(x,fx), d(y,gy)\}$$

$$N(x,y) = \min\{d(x,fx), d(y,gy), d(x,gy), d(y,fx)\}.$$

Then f and g have a unique common fixed point.

Proof. Let x_0 be an arbitrary point in X. Define the sequence $\{x_n\}$ in X as $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for $n \ge 1$. Suppose that there is some $n \ge 1$ such that $x_n = x_{n+1}$. If $n = 2k$, then $x_{2k} = x_{2k+1}$ and from (2.1),

$$d(x_{2k+1}, x_{2k+2}) = d(fx_{2k}, gx_{2k+1}) \le \delta M(x_{2k}, x_{2k+1}) + LN(x_{2k}, x_{2k+1})$$

where

$$\begin{aligned} M(x_{2k}, x_{2k+1}) &= \max\{d(x_{2k}, x_{2k+1}), d(x_{2k}, fx_{2k}), d(x_{2k+1}, gx_{2k+1})\} \\ &= \max\{d(x_{2k}, x_{2k+1}), d(x_{2k}, x_{2k+1}), d(x_{2k+1}, x_{2k+2})\} \\ &= \max\{0, 0, d(x_{2k+1}, x_{2k+2})\} \end{aligned}$$

and

$$\begin{aligned} N(x_{2k}, x_{2k+1}) &= \min\{d(x_{2k}, fx_{2k}), d(x_{2k+1}, gx_{2k+1}), d(x_{2k}, gx_{2k+1}), d(x_{2k+1}, fx_{2k})\} \\ &= \min\{d(x_{2k}, x_{2k+1}), d(x_{2k+1}, x_{2k+2}), d(x_{2k}, x_{2k+2}), d(x_{2k+1}, x_{2k+1})\} \\ &= 0 \end{aligned}$$

Thus we have

$$d\left(x_{2k+1},x_{2k+2}\right)\leq \delta d\left(x_{2k+1},x_{2k+2}\right)$$

which is a contradiction with $\delta\in\left[0,\frac{1}{s}\right)$. Therefore $x_{2k+1}=x_{2k+2}$. Hence we have $x_{2k}=x_{2k+1}=x_{2k+2}$. It means that $x_{2k}=fx_{2k}=gx_{2k}$,i.e. x_{2k} is a common fixed point of f and g.
If $n=2k+1$, then using same arguments, it can be shown that x_{2k+1} is a common fixed point of f and g.
Now suppose $x_n\neq x_{n+1}$ for all $n\geq 1$.

$$d\left(x_{2n+1},x_{2n+2}\right)=d\left(fx_{2n},gx_{2n+1}\right)\leq \delta M\left(x_{2n},x_{2n+1}\right)+LN\left(x_{2n},x_{2n+1}\right) \tag{2.2}$$

where

$$\begin{aligned}M\left(x_{2n},x_{2n+1}\right)&=\max\left\{d\left(x_{2n},x_{2n+1}\right),d\left(x_{2n},fx_{2n}\right),d\left(x_{2n+1},gx_{2n+1}\right)\right\}\\&=\max\left\{d\left(x_{2n},x_{2n+1}\right),d\left(x_{2n},x_{2n+1}\right),d\left(x_{2n+1},x_{2n+2}\right)\right\}\\&=\max\left\{d\left(x_{2n},x_{2n+1}\right),d\left(x_{2n+1},x_{2n+2}\right)\right\}\end{aligned}$$

and

$$\begin{aligned}N\left(x_{2n},x_{2n+1}\right)&=\min\left\{d\left(x_{2n},fx_{2n}\right),d\left(x_{2n+1},gx_{2n+1}\right),d\left(x_{2n},gx_{2n+1}\right),d\left(x_{2n+1},fx_{2n}\right)\right\}\\&=\min\left\{d\left(x_{2n},x_{2n+1}\right),d\left(x_{2n+1},x_{2n+2}\right),d\left(x_{2n},x_{2n+2}\right),0\right\}\\&=0.\end{aligned}$$

If $M\left(x_{2n},x_{2n+1}\right)=d\left(x_{2n+1},x_{2n+2}\right)$, then by (2.2)

$$d\left(x_{2n+1},x_{2n+2}\right)\leq \delta d\left(x_{2n+1},x_{2n+2}\right)$$

which is a contradiction. Thus $M\left(x_{2n},x_{2n+1}\right)=d\left(x_{2n},x_{2n+1}\right)$ and from (2.2)

$$d\left(x_{2n+1},x_{2n+2}\right)\leq \delta d\left(x_{2n},x_{2n+1}\right).$$

Similarly it can be proved that

$$d\left(x_{2n+3},x_{2n+2}\right)\leq \delta d\left(x_{2n+2},x_{2n+1}\right).$$

So

$$d\left(x_{n+1},x_n\right)\leq \delta d\left(x_n,x_{n-1}\right)\leq \delta^n d\left(x_1,x_0\right).$$

for all $n\geq 1$. Similarly, we can show $d\left(x_{n+2},x_n\right)\leq \delta^n d\left(x_2,x_0\right)$.
We can show that $\{x_n\}$ is a $b-$rectangular$-$Cauchy sequence. Using b-rectangular inequality and $x_n\neq x_{n+1}$ for all $n\geq 1$ and $d_n=d\left(x_n,x_{n+1}\right)$, $d_n^*=d\left(x_n,x_{n+2}\right)$

$$\begin{aligned}d\left(x_n,x_{n+2m+1}\right)&\leq s\left[d\left(x_n,x_{n+1}\right)+d\left(x_{n+1},x_{n+2}\right)+d\left(x_{n+2},x_{n+2m+1}\right)\right]\\&\leq s\left[d_n+d_{n+1}\right]+s^2\left[d_{n+2}+d_{n+3}\right]+s^3\left[d_{n+4}+d_{n+5}\right]\\&+...+s^{m+1}d_{n+2m}\\&\leq s\left[\delta^n d_0+\delta^{n+1}d_0\right]+s^2\left[\delta^{n+2}d_0+\delta^{n+3}d_0\right]\\&+s^3\left[\delta^{n+4}d_0+\delta^{n+5}d_0\right]+...+s^m\delta^{n+2m}d_0\\&\leq s\delta^n\left[1+s\delta^2+s^2\delta^4+...+s^m\delta^{2m}\right]d_0+s\delta^{n+1}\left[1+s\delta^2+s^2\delta^4+...+s^m\delta^{2m}\right]d_0\\&=\frac{1+\delta}{1-s\delta^2}s\delta^n d_0\quad (s\delta^2<1).\end{aligned}$$

Hence,

$$d\left(x_n,x_{n+2m+1}\right)\leq \frac{1+\delta}{1-s\delta^2}s\delta^n d_0. \tag{2.3}$$

Similarly, we can show

$$d\left(x_n,x_{n+2m}\right)\leq \frac{1+\delta}{1-s\delta^2}s\delta^n d_0+\delta^{n-2}d_0^*. \tag{2.4}$$

Thus form (2.3) and (2.4), we obtain that

$$\lim_{n\to\infty}d\left(x_n,x_{n+p}\right)=0,$$

for all $p=1,2,3,....$ Hence $\{x_n\}$ is a b-rectangular-Cauchy sequence in (X,d). By completeness of (X,d), there exists $r\in X$ such that $x_n=fx_{n-1}\to r$ as $n\to\infty$.

Now we prove that $fr=r$. By $b-$rectangular inequality ,

$$\begin{aligned}\frac{1}{s}d(fr,r) &\leq d(fr,gx_n)+d(gx_n,x_n)+d(x_n,r)\\ &\leq \delta M(r,x_n)+LN(r,x_n)+d(x_{n+1},x_n)+d(x_n,r)\end{aligned}$$

where

$$M(r,x_n)=\max\{d(r,x_n),d(r,fr),d(x_n,gx_n)\}\to d(r,fr),$$

as $n\to\infty$ and

$$N(r,x_n)=\min\{d(r,fr),d(x_n,gx_n),d(r,gx_n),d(x_n,fr)\}\to 0,$$

as $n\to\infty$.
Hence, taking the limit as $n\to\infty$, we obtain

$$\frac{1}{s}d(fr,r)\leq \delta d(r,fr)+L.0+0+0$$

that is $fr=r$. Hence r is a fixed point of f.
Now we show $gr=r$. Suppose $r\neq gr$, by (2.1)

$$d(r,gr)=d(fr,gr)\leq \delta M(r,r)+LN(r,r)$$

where

$$\begin{aligned}M(r,r)&=\max\{d(r,r),d(r,fr),d(r,gr)\}\\ &=\max\{0,0,d(r,gr)\}\\ &=d(r,gr)\end{aligned}$$

and

$$\begin{aligned}N(r,r)&=\min\{d(r,fr),d(r,gr),d(r,gr),d(r,fr)\}\\ &=0.\end{aligned}$$

By (2.1),

$$d(r,gr)\leq \delta d(r,gr)$$

which is a contradiction. Thus $gr=r$.
Now we show that uniqueness, Suppose r and t are different common fixed points of f and g. By (2.1),

$$d(r,t)=d(fr,gt)\leq \delta M(r,t)+LN(r,t) \tag{2.5}$$

where

$$\begin{aligned}M(r,t)&=\max\{d(r,t),d(r,fr),d(t,gt)\}\\ &=d(r,t)\end{aligned}$$

and

$$\begin{aligned}N(r,t)&=\min\{d(r,fr),d(t,gt),d(r,gt),d(t,fr)\}\\ &=0.\end{aligned}$$

Thus from (2.5),

$$d(r,t)\leq \delta d(r,t)$$

So $d(r,t)=0$, i.e. $r=t$. □

Example 2.2. *Let $X=A\cup B$, where $A=\left\{\frac{1}{n}:n\in\{2,3,4,5\}\right\}$ and $B=[1,2]$. Define $d:X\times X\to[0,\infty)$ such that $d(x,y)=d(y,x)$ for all $x,y\in X$ and*

$$\begin{aligned}&d\left(\frac{1}{2},\frac{1}{3}\right)=d\left(\frac{1}{4},\frac{1}{5}\right)=0.03;d\left(\frac{1}{2},\frac{1}{5}\right)=d\left(\frac{1}{3},\frac{1}{4}\right)=0.02;\\ &d\left(\frac{1}{2},\frac{1}{4}\right)=d\left(\frac{1}{5},\frac{1}{3}\right)=0.06;\ d(x,y)=|x-y|^2 \text{ otherwise.}\end{aligned}$$

Then (X,d) is a complete b-rectangular metric space with coefficient $s=3>1$. But (X,d) is neither a metric space nor a a rectangular metric space. Let $f,g:X\to X$ be defined as

$$f(x)=\begin{cases}\frac{1}{4}, & x\in A\\ \frac{1}{5}, & x\in B\end{cases},\quad g(x)=\begin{cases}\frac{1}{4}, & x\in A\\ \frac{1}{6}, & x\in B\end{cases}$$

Then f and g satisfy all conditions of Theorem 2.1 with $\delta=\frac{1}{4}$ and $L\geq 0$ and $x=\frac{1}{4}$ is a unique fixed point of f and g.

Corollary 2.3. *Let (X,d) be a b-rectangular metric space with $s>1$, and let $f,g:X\to X$ be self maps satisfying*

$$d(fx,gy)\leq \delta d(x,y)+L\min\{d(x,fx),d(y,gy),d(x,gy),d(y,fx)\}$$

for all $x,y\in X$, where $\delta\in\left[0,\frac{1}{s}\right)$ and $L\geq 0$. Then f and g have a unique fixed point.

Corollary 2.4. *Let (X,d) be a complete $b-$rectangular metric space with $s>1$, and let $f:X\to X$ be a self map satisfying*

$$d(fx,fy)\leq \delta M(x,y)+LN(x,y)$$

for all $x,y\in X$, where $\delta\in\left[0,\frac{1}{s}\right)$ and $L\geq 0$ and

$$M(x,y)=\max\{d(x,y),d(x,fx),d(y,fy)\}$$

$$N(x,y)=\min\{d(x,fx),d(y,fy),d(x,fy),d(y,fx)\}$$

then f has a unique fixed point.

Corollary 2.5. *Let (X,d) be a $b-$rectangular metric space with $s>1$, and let $f:X\to X$ be a self map satisfying*

$$d(fx,fy)\leq \delta d(x,y)+L\min\{d(x,fx),d(y,fy),d(x,fy),d(y,fx)\}$$

for all $x,y\in X$, where $\delta\in\left[0,\frac{1}{s}\right)$ and $L\geq 0$. Then f has a unique fixed point.

3. Conclusion

The development of the field of fixed point theory depends on the generalization of the Banach Contraction principle on complete metric spaces. This generalization or extension comes up by either introducing new types of contractions or by working on a more general structured space such as b-rectangular metric spaces. In this article, we have proven some fixed point theorems for almost contraction on b-rectangular metric spaces and hence our results generalize many existing results in the literature.

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