Universal Journal of Mathematics and Applications, 3 (1) (2020) 33-37
Research paper

UJMA

Universal Journal of Mathematics and Applications

Journal Homepage: www.dergipark.gov.tr/ujma
ISSN 2619-9653
DOI: https://doi.org/10.32323/ujma.615166

Parameter Reduction Method for Pythagorean Fuzzy Soft Sets

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Article Info

Keywords: *Parameter reduction, Pythagorean fuzzy soft set.*
2010 AMS: *03E72, 68T37*
Received: *4 September 2019*
Accepted: *14 October 2019*
Available online: *25 March 2020*

Abstract

The aim of this paper is to give new parameter reduction methods according to Pythagorean fuzzy soft sets. The reason for the definition of these methods is to help decision-makers facilitate their decision-making processes. The algorithm of the first method defined is related to the selection of some parameters. The second method determines the parameters with less deviation than the other parameters. Further, numerical examples related to the new algorithms are examined.

1. Introduction

Fuzzy Sets (FSs) put forward by Zadeh [1] has influenced deeply all the scientific fields since the publication of his paper. It is seen that this concept, which is very important for real-life situations, had not enough solution to some problems in time. New quests for such problems have been coming up. Atanassov [2] initiated Intuitionistic fuzzy sets (IFSs) for such cases.

The SS theory, which contributed to the solution of uncertainties in non-parametric problems, was initiated by Molodtsov [3]. SS theory is an inherent extension of the FS theory and can, therefore, be easily applied to all branches of science and technology. This theory deals with a concentration of approximate illustration of objects. In the approximate illustration, there exists a set of predicate values and a set of approximate values. This theory is suitable and simply operative in performance due to the nonentity of restrictions on the approximate illustrations.

Yager [4] offered a new FS called Pythagorean fuzzy set(PFS). PFS has fascinated the care of great deal researchers in a little while time. The formulation of the negation for IFSs and PFSs is examined by Yager [5]. In [6], PF subsets and its relationship with IF subsets were debated and some set operations on PF subsets were defined. Peng et al. [7], given the definition of the Pythagorean fuzzy soft set(PFSS), investigated its properties. Kirisci [8] introduce Pythagorean fuzzy parametrized soft set(PFPSS) and examine some characteristics, operations. In addition, the answer of the decision-making(DM) problem with PFPSS and other related notions is presented in [8].

2. Preliminaries

Give the sets $\mathscr{U}$, P, $\rho(\mathscr{U})$ as an initial universe, parameters, the power set of $\mathscr{U}$ respectively, and $S \subset P$. Give the mapping $m : X \to \rho(\mathscr{U})$. Therefore, u_X is said to a soft set(SS) over U [3].

Choose the set $\mathscr{U} = \{x_i\}_{i=1}^{n}$. Let $\{S(j)\}_{j=1}^{k}$ be a set of parameters.
Let $\{\cup_{j=1}^{k} S(j)\} \subseteq P$ and every parameter set $S(k)$ indicate the kth class of parameters and the elements of $S(k)$ represents a exclusive attribute set.

The set

$$A = \{\langle x, u_A(x), v_A(x)\rangle : x \in \mathscr{U}\}$$

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is said to be an intuitionistic fuzzy set(IFS) A on $\mathscr{U}$, [2] where, $u_A, v_A : \mathscr{U} \to [0,1]$ such that $0 \leq u_A(x) + v_A(x) \leq 1$ for any $x \in \mathscr{U}$. The degree of indeterminacy $w_A = 1 - u_A(x) - v_A(x)$.

An Pythagorean fuzzy set(PFS) φ over $\mathscr{U}$ is given by

$$\varphi = \{\langle x, u_\varphi(x), v_\varphi(x)\rangle : x \in \mathscr{U}\},$$

where $u_\varphi, v_\varphi : \mathscr{U} \to [0,1]$ denote the degree of membership and the degree of non-membership of $x \in \mathscr{U}$ to φ, respectively, such that $0 \leq (u_\varphi(x))^2 + (v_\varphi(x))^2 \leq 1$ [4,5]. $\mathscr{I}_\varphi = \sqrt{1 - (u_\varphi(x)^2 - (v_\varphi(x))^2}$ represent the degree of indeterminacy.

Then, $\varphi(S)$ is called Pythagorean Fuzzy Soft Set(PFSS) on $\mathscr{U}$, if $\varphi(S)$ is mapping given by $\varphi(S) : P \to \rho(\mathscr{U})$ [7].

Remark 2.1. *It is easy to check that PFSSs generalize both IFSs and SSs. That is, all IF degrees are part of the PF degrees. In actual DM problems, the PFSS characterizes a larger membership space than the IFSS. That is, the PFSS a higher capability than the IFSS to model uncertainty in real DM problems.*

Let Pythagorean fuzzy numbers (PFNs) are denoted by $E = (u_S, v_S)$ [9]. Choose three PFNs $\theta = E(u,v), \theta_1 = \langle u_1, v_1\rangle, \theta_2 = \langle u_2, v_2\rangle$. We can give some basic operations as follows [4]:

- $\bar{\theta} = \langle u, v\rangle$;
- $\theta_1 \vee \theta_2 = \langle \max\{u_1, u_2\}, \min\{v_1, v_2\}\rangle$;
- $\theta_1 \wedge \theta_2 = \langle \min\{u_1, u_2\}, \max\{v_1, v_2\}\rangle$;
- $\theta_1 \oplus \theta_2 = \langle \sqrt{u_1^2 + u_2 2 - u_1^2 u_2^2}, v_1 v_2\rangle$;
- $\theta_1 \otimes \theta_2 = \langle u_1 u_2, \sqrt{v_1^2 + v_2 2 - v_1^2 v_2^2}\rangle$;
- $\alpha.\theta = \langle \sqrt{1 - (1 - u^2)^\alpha}, v^\alpha\rangle$;
- $\theta^\alpha = \langle u^\alpha, \sqrt{1 - (1 - v^2)^\alpha}, \rangle$;

for $\alpha > 0$.

Maji et al. [10] firstly gave IFSS. In [11], Q-IFSS defined and basic properties are investigated. Broumi et al. [12] are given new definitions for IFSS such as concentration, dilatation and normalization. In [13], first Zadeh's containment, IF conjunction, IF disjunction of two IFSSs are defined and some basic properties are examined. In [14], three new operations based on Second Zadeh's containment, conjunction and disjunction operations have been defined and studied. Maji [15] has been extended IFSS with new operations. In [16], a new approach to IFSS was presented with rough set for DM problems. In this study, we adopt the PFSS from idea of Ghosh and Das [17]. Kirisci [18] compared IFPSS and Riesz summability methods using medical real dataset.

3. Comparison of IFSs and PFSs

IFS, offered by Atanassov [2] is an extension of FS Theory [1]. IFS is characterized by a membership degree and a non-membership degree and therefore can indicate the fuzzy character of data in more detail comprehensively. The prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree with their sum equal to or less than 1. However, in some practical DM process, the sum of the membership degree and the non-membership degree to which an alternative satisfying a criterion provided by a decision maker may be bigger than 1, but their square sum is equal to or less than 1.

Therefore, Yager [4] proposed PFS characterized by a membership degree and a non-membership degree, which satisfies the condition that the square sum of its membership degree and non-membership degree is less than or equal to 1. Yager [19] gave an example to state this situation: a decision maker gives his support for membership of an alternative is $\frac{\sqrt{3}}{2}$ and his against membership is $\frac{1}{2}$. Owing to the sum of two values is bigger than 1, they are not available for IFS, but they are available for PFS since $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 \leq 1$. Obviously, PFS is more capable than IFS to model the vagueness in the practical multicriteria decision-making problems.

The main difference between PFNs and IFNs is their corresponding constraint conditions, which can be easily shown in Figure 3.1. Here, we observe that intuitionistic membership grades are all points under the line $x + y \leq 1$ and the Pythagorean membership grades are all points with $x^2 + y^2 \leq 1$.

One important implication of this is that it allows the use of the PFSs in situations in which we cannot use IFSs. An example of this would be a case in which a user indicates that their support for membership of x is $\frac{\sqrt{3}}{2}$ and their support against membership is $\frac{1}{2}$. As we noted these values are not allowable for intuitionistic membership grades but allowable as Pythagorean membership grades. Thus in this case, rather then requiring the user to change their information to satisfy the constraints of the IFS we can use a PFS.

4. Parameter Reduction Method

Take a complete lattice $(\mathscr{L}, \leq_{\mathscr{L}})$, such that $\mathscr{L} = \{(x,y) : x, y \in [0,1], x^2 + y^2 < 1\}$. The corresponding partial order $\leq_{\mathscr{L}}$ id defined by $(x,y) \leq_{\mathscr{L}} (a,b) \quad x \leq a, \quad y \geq b, \forall (x,y), (a,b) \in \mathscr{L}$. Any ordered pair $(x,y) \in \mathscr{L}$ is said to be Pythagorean fuzzy value(PFV) or Pyhtagorean

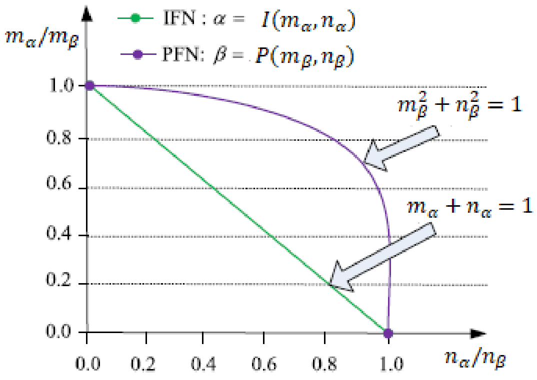

Figure 3.1: The PFNs and the IFNs

fuzzy number(PFN) [20].

Let $E=(u_E,v_E)$ be a Pythagorean fuzzy number (PFN). The mapping $\mathscr{SF}_E : L \to [-1,1]$ is called *score function*, if $\mathscr{SF}_E = u_E^2 - v_E^2$ for all $E \in L$ [8], [9].

For any two PFNs E,F

$$E \prec F, if \mathscr{SF}(E) < \mathscr{SF}(F),$$
$$E \succ F, if \mathscr{SF}(E) > \mathscr{SF}(F),$$
$$E \sim F, if \mathscr{SF}(E) = \mathscr{SF}(F).$$

Algorithm 1:

For each i,j, score values(SVs) of each of the entries of the PFSS $\varphi(S)$ denoted by $\phi = \mathscr{SF}_E$. Define the aggregated score as $\psi = \overline{\mathscr{SF}_E}(x_i) = \sum_{j=1}^{m} \mathscr{SF}_E(x_j)$. Consider $\lambda = \{x_i^*\}_{i=1}^{r} \subset P$ and $\mu = \{x_i^{**}\}_{i=1}^{r} \subset P$.

i. Calculate ϕ
ii. Compute ψ.
iii. Select the sets $\lambda, \mu \subset P$, where $\lambda, \mu \neq \emptyset$, $\lambda \cap \mu = \emptyset$, $\lambda \cup \mu \neq E$.
iv. Compute $\psi_{\lambda \subset P}$, $\psi_{\mu \subset P}$ $(\forall x_i \in \mathscr{U})$.
v. Choose the reduction parameter set of the PFSS ϕ as $P-(\lambda \cup \mu)$, if $\psi_{\lambda \subset P} - \psi_{\mu \subset P} = 0$.

Example 4.1. *Take the set of blouses $U=\{y_1,y_2,y_3,y_4,y_5,y_6\}$. Parameters can be defined as p_1:bright, p_2:cheap, p_3:costly, p_4:very costly, p_5:colourful, p_6:cotton, p_7:polystyrene, p_8:long sleeve. The tabular representation of PFFS φ is given Table 1.*

Table 2 shows score of each entry and object for the PFSS φ (for $i=1,2,\cdots,6$).

The order of the blouses is found as y_5,y_6,y_4,y_1,y_2,y_3. Now we choose the sets λ,μ such as $\lambda,\mu \subset P$ and $\lambda \cup \mu \neq P$. Consider the set $\lambda=\{p_1,p_4\}$ and $\mu=\{p_5,p_7\}$. It can be easily seen that reduced parameter set $P^*=\{p_2,p_3,p_6,p_8\}$ is obtained, when $\psi(y_i)$ of each object y_i are computed. The table which include the parameter set P^* is given Table 3. The object ordering in Table 2 and Table 3 appears to be the same.

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
y_1	(0.5, 0.3)	(0.4, 0.3)	(0.5, 0.4)	(0.1, 0.6)	(0.3, 0.5)	(0.4, 0.2)	(0.6, 0.1)	(0.1, 0.5)
y_2	(0.2, 0.5)	(0.4, 0.1)	(0.5, 0.2)	(0.1, 0.2)	(0.5, 0.2)	(0.2, 0.3)	(0.2, 0.1)	(0.4, 0.6)
y_3	(0.4, 0.6)	(0.2, 0.5)	(0.4, 0.3)	(0.5, 0.3)	(0.6, 0.4)	(0.3, 0.4)	(0.3, 0.5)	(0.5, 0.4)
y_4	(0.5, 0.4)	(0.3, 0.5)	(0.3, 0.6)	(0.6, 0.2)	(0.2, 0.6)	(0.6, 0.2)	(0.4, 0.5)	(0.4, 0.3)
y_5	(0.3, 0.6)	(0.4, 0.2)	(0.6, 0.3)	(0.6, 0.3)	(0.5, 0.3)	(0.7, 0.1)	(0.3, 0.5)	(0.5, 0.5)
y_6	(0.2, 0.5)	(0.6, 0.2)	(0.6, 0.1)	(0.5, 0.2)	(0.1, 0.7)	(0.3, 0.5)	(0.7, 0.1)	(0.4, 0.5)

Table 1: Tabular representation for PFSS φ

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	ψ
y_1	0.16	0.07	0.09	-0.35	-0.16	0.12	0.35	-0.24	0.04
y_2	-0.21	0.15	0.21	-0.03	0.21	-0.05	0.03	-0.2	0.11
y_3	-0.2	-0.21	0.07	0.16	0.2	-0.07	-0.16	0.09	-0.12
y_4	0.09	-0.16	-0.27	0.32	-0.32	0.32	-0.09	0.07	-0.04
y_5	-0.27	0.12	0.27	0.27	0.16	0.48	-0.16	0	0.87
y_6	-0.21	0.32	0.35	0.21	-0.48	-0.16	0.48	-0.09	0.42

Table 2: Score of each object and entry for the PFSS φ

	p_2	p_3	p_6	p_8	ψ
y_1	0.07	0.09	0.12	-0.24	0.04
y_2	0.15	0.21	-0.05	-0.2	0.11
y_3	-0.21	0.07	-0.07	0.09	-0.12
y_4	-0.16	-0.27	0.32	0.07	-0.04
y_5	0.12	0.27	0.48	0	0.87
y_6	0.32	0.35	-0.16	-0.09	0.42

Table 3: Reduced parameter set

	p_1	p_2	p_3	p_4	p_5
y_1	(0.65, 0.42)	(0.7, 0.4)	(0.55, 0.12)	(0.9, 0.15)	(0.85, 0.2)
y_2	(0.45, 0.75)	(0.6, 0.4)	(0.33, 0.88)	(0.5, 0.5)	(0.52,0.5)
y_3	(0.6, 0.3)	(0.8, 0.05)	(0.7, 0.5)	(0.6, 0.3)	(0.7, 0.45)
y_4	(0,35, 0.65)	(0.4, 0.2)	(0.25, 0.75)	(0.4, 0.5)	(0.55, 0.45)

Table 4: Tabular representation of PFSS φ

	p_1	p_2	p_3	p_4	p_5	ψ
y_1	0.2461	0.33	0.2881	0.7875	0.6825	2.3342
y_2	-0.36	0.2	-0.6655	0	0.0204	-0.8051
y_3	0.27	0.6375	0.24	0.27	0.2875	1.705
y_4	-0.3	0.12	-0.5	-0.09	0.1	-0.67

Table 5: Score values for the PFSS φ

	p_1	p_3	p_4	p_5	ψ
y_1	0.2461	0.2881	0.7875	0.6825	2.0042
y_2	-0.36	-0.6655	0	0.0204	-1.0051
y_3	0.27	0.24	0.27	0.2875	1.0675
y_4	-0.3	-0.5	-0.09	0.1	-0.79

Table 6: The set of reduced parameter set and score values

Algorithm 2:

For $i \neq j$, the maximum score deviation denoted by $\sigma = \min\{|\overline{\mathscr{S}\mathscr{F}_N}(a_i) - \overline{\mathscr{S}\mathscr{F}_N}(a_j)|\}$, for $i, j = 1, 2, \cdots n$.

i. Calculate ϕ
ii. Compute ψ.
iii. Calculate σ with $P = \{p_1, p_2, \cdots, p_m\}$.
iv. Select the parameter $p_k \in P$ for the following situations for $i, j = 1, 2, \cdots, m$
   a) For $j \neq k$, $\mathscr{S}\mathscr{F}_N(a_i, p_k) \leq \mathscr{S}\mathscr{F}_N(a_i, p_j)$ $(\forall i)$.
   b) $\forall i$ and for $j \neq k$, $\max|\mathscr{S}\mathscr{F}_N(a_i, p_k) - \mathscr{S}\mathscr{F}_N(a_i, p_j)| < \sigma$.
v. Take the maximal number of parameter set A which fulfill the preceding step.
vi. Calculate the set $P - A$. This set is a reduced version of the parameters of PFSS.

Example 4.2. *Consider the information as given in Table 4. Firstly, compute SVs of* y_i, $\quad (i = 1,2,3,4)$. *Obtained SVs are corresponding to the parameters P, which is defined as* $\psi = \overline{\mathscr{S}\mathscr{F}_N}(a_i) = \sum_{j=1}^{m} \mathscr{S}\mathscr{F}_N(a_i, p_j)$.*Therefore the objects are ordered with SVs and accuracy values. The order is shown as* $\{h_1, h_3, h_4, h_2\}$.

The maximum deviation is computed as $\sigma = \min\{3.1393, 0.6292, 3.0042, -2.5101, -0.1351, 2.375\} = -2.5101$. *We consider the parameter* $p_2 \in P$, *where* $\mathscr{S}\mathscr{F}_N(a_i, p_k) \leq \mathscr{S}\mathscr{F}_N(a_i, p_j)$ *for all* i, j *and* $\max|\mathscr{S}\mathscr{F}_N(a_i, p_k) - \mathscr{S}\mathscr{F}_N(a_i, p_j)| < \sigma$ *for all* i, j *(Table 5). Thus the reduction of the parameter* p_2 *will have no effect in the ordering of objects, which is shown in Table 6. So, we get the reduced set of parameters as* $P - \{p_2\}$.

5. Conclusion

Parameter reduction is primarily a method used to avoid redundant parameters. The results obtained with the minimum subset of parameters are the same as the entire set of parameters. Parameter reduction is essentially based on this idea. It is proposed new parameter reduction methods for PFSSs. Examples are given to illustrate the applicability of these methods.

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