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NURSE SCHEDULING AND RESCHEDULING UNDER UNCERTAINTY

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Abstract: Nurse planning decisions play a critical role on hospital budgeting, quality of nursing services and job satisfaction of nurses. Nurse planning in a hospital is composed of four main phases: nurse budgeting, nurse scheduling (rostering), nurse rescheduling (staffing) and nurse assignment. In this study, the integrated problem of nurse scheduling and rescheduling under demand uncertainty is considered. The problem is formulated and solved as a two-stage stochastic integer program. The value of the stochastic solution is estimated using realistic problem instances over a monthly planning horizon generated based on the data provided by a private healthcare provider in Ankara, Turkey. The computational results show that the value of the stochastic solution can be more than 9% in some problem instances, and hence healthcare providers can benefit from using models that capture uncertainty rather than deterministic models.

Keywords: *Nurse scheduling; nurse rescheduling; two-stage stochastic programming.*

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BELİRSİZLİK ALTINDA HEMŞİRE ÇİZELGELEME PROBLEMİ

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Öz: Hemşire planlama, hastane bütçelemesinde, hastalara verilen hizmetin kalitesinde ve hemşirelerin iş memnuniyetinde kilit rol oynar. Bir hastanedeki hemşire planlama süreci temel olarak dört başlıktan oluşur: Bütçeleme, çizelgeleme, çizelgelerin güncellenmesi ve hemşire atama. Bu çalışmada, aylık çizelgelerin oluşturulması ve güncellenmesi problemi talep belirsizliği altında incelenmektedir. Bu entegre problem, iki aşamalı stokastik tamsayılı program olarak modellenmiş ve çözülmüştür. Aylık planlama ufkunu gözeten gerçekçi problem örnekleri kullanılarak gerçekleştirilen sayısal çalışmalarla belirsizliği modellemenin değeri ölçülmüştür. Elde edilen sonuçlar, belirsizliği modellemenin değerinin %9'u geçebileceğini ve bu nedenle stokastik modelleri kullanmanın oluşturulan çizelgelerin performansını önemli ölçüde iyileştirebileceğini göstermektedir.

Anahtar Sözcükler: *Hemşire çizelgeleme, hemşire çizelgelerinin güncellenmesi; iki aşamalı stokastik programlama.*

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INTRODUCTION

The annual increase in healthcare expenditures of Turkey between 2008 and 2011 is reported to be 13% on average (Memiş, 2012). Due to this significant increase, the planning of the use of limited resources in healthcare delivery systems has received ample attention of the practitioners and the researchers. Nurses are among the most important scarce resources in a hospital and nursing services have a big impact on hospital budgeting and the quality of service provided. According to Punnakitikashem (2007), nurse planning decisions can be classified into four categories. First step is *nurse budgeting*, which includes long-term decisions such as the number of nurses to be hired and annual budget for nursing services. The second step is *nurse scheduling* or *nurse rostering*, in which the volume of patient arrivals is estimated and the assignment of nurses to shifts is made accordingly. Decisions made in this stage are mid-term decisions. The third step is *nurse rescheduling*, which are short-term decisions such as the adjustments on the number of nurses available to meet the realized demand. The last step is *nurse assignment*, in which the assignment of nurses to patients is made. In this study, our focus is on the nurse scheduling and rescheduling decisions**.**

Nurse scheduling and rescheduling decisions are typically made by head nurses and require a great deal of their time and effort. These decisions are challenging because of the following reasons:

- Demand is stochastic; there are variations in staffing requirements between days and even between shifts.

- Maintaining an acceptable level of service at all times is compulsory.

- Nurses are limited resources.

- Nurses are of different skill and seniority levels and this should be considered when building the schedule.

- Maintaining equity between nurses about their working times and satisfying their special requests are important.

- There are legal rules about working times, which are compulsory to be considered.

A poor schedule has a negative impact on the quality of nursing services and the job satisfaction of nurses due to the following reasons:

- Excessive amount of adjustments made in the schedule (high rate of calling on-call nurses, working overtime, etc.) during the month, which results in unexpected changes in the planned working times of nurses.

- Not being able to meet special requests of nurses about working times.

- Not being able to provide equity between nurses in terms of their working hours.

Job dissatisfaction of nurses may cause high nurse turnover rate. Since qualification is very important in nursing services, training cost has an important role on budgeting and high turnover rate may significantly increase these costs.

In this study, we consider the nurse scheduling and rescheduling problems under demand uncertainty in an integrated framework and formulate this problem as a two-stage stochastic program where the objective is minimizing the expected deviations from the planned schedule, and hence the adjustments to be made, during the planning period. The first-stage decisions are monthly scheduling decisions and the second-stage decisions are the adjustment (rescheduling) decisions that are made when more information about the demand becomes available. We solve realistic instances constructed based on the data provided by a private healthcare provider in Ankara, Turkey and use our results to estimate the value of the stochastic solution and the expected value of perfect information.

1. BACKGROUND INFORMATION

Although nurse scheduling/rescheduling is a problem encountered in all hospitals and shares common aspects across all types of hospitals (public, private, general, specialty, teaching, etc.), there are also hospital-specific or even ward-specific details of the problem since each system is unique. In this study, we consider the intensive-care unit (ICU) nurse scheduling/rescheduling problem of a private hospital in Ankara, which has been in service since 2010. There are 25 active departments and the ICU includes 4 sub-units, which are Cardiovascular Surgery ICU, General ICU, Coronary ICU, and Neonatal ICU. Our focus is on the nurse scheduling process in the Cardiovascular Surgery ICU and the General ICU. There are 12 beds and 17 nurses in total. The total number of inpatients served during a month is 45 on average, and the average length of stay of a patient is 4.5 days.

According to the interviews made by the head nurse, there is a high turnover rate in nursing services due to the job dissatisfaction caused by undesirable schedules with high overtime levels and excessive amount of adjustments made in the monthly schedules.

The monthly scheduling decisions include the assignment of nurses to shifts and the determination of off-days for each nurse. There are three shifts in a day. The first shift includes the hours from 07:00 to 16:00, the second shift is from 15:00 to 24:00 and the third shift (night shift) is from 23:00 to 08:00.

There are four types of nurses in a shift:

- Scheduled nurses, who are assigned to that shift in the monthly schedule.

- On-call nurses are also assigned in the monthly schedule and they should be prepared to serve when they are called due to the overload in the ICU.

- Overtime nurses, who are assigned to the shift in the last minute (i.e., rescheduled to work) when the scheduled and on-call nurses are not enough to meet the workload.

- Undertime nurses, who are allowed to leave (i.e., rescheduled not to work) when the actual demand is less than the planned capacity.

A desirable schedule should:

- meet the legal working limits, rules and permissions,
- provide equity between nurses,
- be reasonably stable during the month,
- meet special requests of the nurses,
- and result in a low rate of calling on-call nurses, overtime and undertime hours.

The last hour of a shift overlaps with the first hour of the next shift. During this overlapping period, the incoming nurses are informed about the health status of the patients and the tasks to be performed in the new shift are determined. At the end of each shift, the actual demand for patient care for the next shift is observed (i.e., estimated with a higher level of accuracy) and the following rescheduling decisions are made accordingly:

- If there is a shortage, the on-call nurse is called to work.

- If the on-call nurse is not enough to meet the extra demand, overtime nurses are assigned.

- If there is redundant workforce, the excess number of nurses are considered as undertime nurses and allowed to leave.

In the current system, the demand (i.e., workload) is estimated based on the experience of the head nurse and the monthly scheduling decisions are prepared accordingly. The daily demand for patient care varies during a month and it may even change between consecutive shifts. Ignoring this variability is one of the main reasons which cause poor schedules.

The workload is measured by using the Therapeutic Intervention Scoring System (TISS) which is a commonly used and easily applicable task-based method for measurement of workload in ICUs. There are four categories of tasks performed in the considered ICU. Each category has a score from 1 to 4. For each patient, tasks to be performed during the considered period are checked on a check list and the TISS score is obtained as a weighted value where the weights are task-based coefficients. According to this scoring system a nurse should have a workload of 50 TISS score on average.

2. REVIEW OF THE RELATED WORK

In this section, we provide a brief review of the methodological tools used in this study and the nurse scheduling literature.

2.1. Stochastic Programming

Stochastic programming, which was first introduced by Dantzig (1955), is mathematical programming where the problem parameters are random variables. Since real world problems typically include uncertainty, stochastic programming has a wide range of application areas including finance, manufacturing, transportation, logistics, airline operations, capacity planning and telecommunications (Punnakitikashem, 2007).

A two-stage stochastic program is the simplest form of a stochastic program. In a two-stage stochastic program, decision variables are divided into two groups: first-stage decision variables and second-stage decision variables. First-stage decision variables are decided in the first stage before the realization of the random scenario. Second-stage variables are set once the actual values of random parameters become known.

As presented in Birge and Louveaux (1997), the general formulation of a two-stage stochastic program with recourse is as follows:

$$\min c^T x + E_{\zeta}[Q(x,\zeta)] \quad (1)$$
subject to
$$Ax = b \quad (2)$$
$$x \geq 0, \quad (3)$$

where $Q(x,\zeta) = \min\{q^T y | Wy = h - Tx, y \geq 0\}$; ζ is the random vector formed by the components of q, h, W, and T; and E_{ζ} represents the expectation with respect to ζ.

The extensive form of a two-stage stochastic program can be represented as given below, where K is the number of all possible realizations (i.e., scenarios), and p_k is the probability of occurrence of the k^{th} realization:

$$\min c^T x + \sum_{k=1}^{K} p_k \, q_k^T y_k \quad (4)$$
subject to
$$Ax = b \quad (5)$$
$$T_k x + W y_k = h_k \qquad k = 1, \dots, K \quad (6)$$
$$x \geq 0 \quad (7.a)$$
$$y_k \geq 0 \qquad k = 1, \dots, K \quad (7.b)$$

As the number of scenarios in a stochastic program increases, solving the extensive form becomes computationally impractical. Therefore, using decomposition-based methods such as the L-shaped method (Van Slyke and Wets, 1969) to solve stochastic programs is very common.

There are some important measures used to evaluate the impact of uncertainty in stochastic programming. The expected value of perfect information (EVPI) measures the amount of payment that a decision maker is willing to pay in return for perfect information about the future. In order to determine the EVPI, one needs to first solve a deterministic model for each realization of the random variables, and then find the expected value of optimal objective values of these solutions. This is called the wait-and-see solution (WS). EVPI is obtained by comparing the wait-and-see solution to the here-and-now solution, which is the optimal solution of the stochastic problem (also known as the recourse problem) (RP). For a stochastic program where an objective function is minimized: EVPI = RP – WS.

A heuristic solution for the stochastic program can be obtained by replacing the random problem parameters with their expected values and solving the resulting deterministic mathematical program, which is known as the expected value or mean value problem. The value of the stochastic solution (VSS) is defined as the possible gain obtained when the stochastic model is solved. VSS for a two-stage stochastic program with a minimization type of objective is VSS = EEV – RP where EEV represents the expected value of the expected value problem's optimal solution.

2.2. Nurse Scheduling

Nurse scheduling problems are widely studied in the operations research literature. We provide a review of the most relevant and recent studies in this section and refer the interested reader to the more extensive reviews available in Cheang *et al.* (2003), Burke *et al.* (2004), Ernst *et al.* (2004), and Tein and Ramli (2010).

2.2.1. Deterministic Nurse Scheduling

Bard and Purnomo (2004) study a nurse rescheduling problem where the aim is to reallocate the available resources in a way that the cost of shortfall is minimized while ensuring that each unit in the hospital has sufficient coverage. They formulate the problem as an integer program where the decisions are the overtime level, number of outside nurses and floaters.

Belien and Demeulemeester (2008) consider the integrated problem of operating room and nurse scheduling. They formulate the problem as an integer program where the objective is minimizing the number of required nurses. The constraints of the model guarantee that the coverage requirements (number of nurses with appropriate skills need to be scheduled for each demand period) and the collective agreement requirements (rules for an acceptable schedule in terms of workload, day-off and resting time between shifts) are met. They propose a column generation technique to solve the instances with reasonable sizes.

Glass and Knight (2010) formulate the nurse rostering problem as a mixed-integer program with two sets of constraints: 1) Staffing constraints that ensure the sufficiency of nurse levels for each duty at any particular time, 2) Schedule constraints that are related to the sequences and combinations of shifts to be worked by each nurse. In the problem setting considered by the authors, these two sets of constraints cannot be satisfied simultaneously. Therefore, they are treated as soft constraints and the proposed model minimizes the violation of these constraints.

Mobasher *et al.* (2011) study daily scheduling of nurses in operating suites. They formulate the problem as a multi-objective integer program where the objectives are minimizing the maximum demand deviation for any case, minimizing the maximum amount of overtime assigned to any nurse, and minimizing the maximum number of cases assigned to any nurse. The decisions include the nurse-surgery-task assignments in the time intervals over the considered planning horizon. The surgery durations and the number of required nurses for each task are the main parameters of the problem. Two methodologies are proposed to find a solution. The first method is based on generating a pool of solutions by solving multiple optimization problems and the second method is a modified goal programming approach.

Atmaca *et al.* (2012) study a multi-objective nurse scheduling problem where the objectives are minimizing the total number of working days of nurses, minimizing the difference between the total numbers of working days of nurses, and minimizing the number of assignments of nurses to consecutive shifts. They formulate and solve the problem as a pure binary integer program whose objective is to minimize the weighted sum of the deviations from these goals related to each objective.

2.2.2. Stochastic Nurse Scheduling

Punnakitikashem *et al*. (2008) consider a nurse assignment problem under workload (i.e., demand) uncertainty and formulate this problem as a two-stage stochastic integer program. The first-stage decisions are the assignment of nurses to patients and the second-stage decisions are the realized workload for each nurse. The aim is to minimize excess workload on nurses. They solve the proposed stochastic program by using the Benders' Decomposition method where the scenario subproblems are further decomposed since the realized workload can be computed for each nurse separately.

Punnakitikashem (2007) builds a two-stage stochastic program to formulate the integrated problem of nurse staffing and nurse assignment. Workload on the nurses is uncertain and the objective is to minimize the expected excess workload on the nurses under a budget constraint. The first-stage decisions are the nurse-patient assignments and the second-stage decisions are the assignment of overtime/agency nurses or cancellation of the scheduled nurses. The authors propose three solution approaches for the problem: Benders' Decomposition, Lagrangian Relaxation with Benders Decomposition, and Nested Benders Decomposition. They also consider a multi-stage extension of the problem and use Nested Benders' Decomposition to solve the related multi-stage stochastic program.

Kim (2014) considers an integrated staffing and scheduling problem under demand uncertainty and formulates this problem as a two-stage stochastic integer program with mixed-integer recourse. The first-stage decisions are the number of nurses assigned to work in pre-generated scheduling patterns and the second-stage decisions are adjustment decisions including the amount of overstaffing and understaffing. The problem size is large due to not only the integrated structure of the problem but also the large number of possible scheduling patterns (i.e., shift combinations). The L-shaped method is used to solve the problem and several enhancements are considered such as defining mixed-integer rounding cuts for the second-stage problem and exploring heuristic approaches for aggregating cuts.

3. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

We formulate the nurse scheduling and rescheduling problem of the system described in Section 2 as a two-stage stochastic program that minimizes the expected cost of rescheduling activities. The first-stage decisions in our model are the assignment of regular and on-call nurses to the shifts and the second-stage decisions are the adjustments made in the schedule including calling the on-call nurse and using overtime/undertime nurses.

We use the following notation in our formulation:

Indices:

i: nurse index (i=1,2,…,I)

j: day index (j=1,2,…,J)

k: shift index (k=1,2,3)

s: scenario index (s=1,2,…,S)

Parameters:

$$m_i = \begin{cases} 1 & \text{if nurse } i \text{ is a senior nurse} \\ 0 & \text{otherwise} \end{cases}$$

$$w_{ij} = \begin{cases} 1 & \text{if nurse } i \text{ is pregnant or has breast} - \text{feeding permission on day } j \\ 0 & \text{otherwise} \end{cases}$$

$$a_{ijk} = \begin{cases} 1 & \text{if nurse } i \text{ has a request not to work on day } j \text{ at shift } k \\ 0 & \text{otherwise} \end{cases}$$

d_{jk}^s: required number of nurses on day j at shift k under scenario s

a: per shift cost of overtime nurses

b: per shift cost of on-call nurses

c: per shift cost of undertime nurses

In our numerical analysis, in order to see the sensitivity of the solution to the changes in these problem parameters, we consider two settings about the objective function coefficients a, b, and c:

- $a = c > b > 0$. The least expensive adjustment is calling the on-call nurse. The cost of using overtime and undertime is the same. The cost of assigning overtime (a) and undertime (c) nurses is taken as 4, and the cost of calling on-call nurses (b) is taken as 2.

- $a > c > b > 0$. The least expensive adjustment is calling the on-call nurse. Using overtime has a higher unit cost than that of undertime. The cost of assigning overtime nurses (a) is taken as 6, the cost of assigning undertime nurses (c) is taken as 4, and the cost of calling on-call nurses (b) is taken as 2.

First Stage Decision Variables:

$$x_{ijk} = \begin{cases} 1 & \text{if nurse } i \text{ works on day } j \text{ at shift } k \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1 & \text{if nurse } i \text{ is an on} - \text{call nurse on day } j \text{ at shift } k \\ 0 & \text{otherwise} \end{cases}$$

$$f_{ij} = \begin{cases} 1 & \text{if nurse } i \text{ uses a day} - \text{off on day } j \\ 0 & \text{otherwise} \end{cases}$$

y_i: total number of day shifts for nurse i during the month

n_i: total number of night shifts for nurse i during the month

p_i: total number of on − call assignments for nurse i during the month

Second Stage Decision Variables:

o_{jk}^s: required number of overtime nurses on day j at shift k under scenario s

$$c_{jk}^s = \begin{cases} 1 & \text{if on} - \text{call nurse is called on day } j \text{ at shift } k \text{ under scenario } s \\ 0 & \text{otherwise} \end{cases}$$

r_{jk}^s: number of nurses who will be permitted to go on day j at shift k under scenario s

Note that the first-stage decisions are scheduling decisions (i.e., decisions about the construction of the monthly schedules) whereas the second-stage decisions are rescheduling decisions (i.e., decisions about the daily modifications).

Mathematical Formulation:

$$\text{Min } Q(x) \tag{8}$$

subject to:

1. There must be at least one senior nurse in each shift.

$$\sum_{i=1}^{I} x_{ijk} m_i \geq 1 \qquad \forall j,k \tag{9}$$

2. Nurses who are pregnant or have breast-feeding permission cannot be assigned to the night shift.

$$w_{ij} + x_{ij3} + z_{ij3} \leq 1 \qquad \forall i,j \tag{10}$$

3. Working shifts of a nurse should be at least two shifts apart from each other.

$$x_{ij2} + x_{ij3} + x_{i(j+1)1} \leq 1 \qquad \forall i, j < J \tag{11.a}$$

$$x_{ij3} + x_{i(j+1)1} + x_{i(j+1)2} \leq 1 \qquad \forall i, j < J \tag{11.b}$$

4. Exactly one on-call nurse must be assigned to each shift.

$$\sum_{i=1}^{I} z_{ijk} = 1 \qquad \forall j,k \tag{12}$$

5. A nurse cannot be assigned as a regular nurse on her/his day-off. (She/he cannot be assigned as an on-call nurse as well, and this is guaranteed by constraint 9.)

$$\left(\sum_{k=1}^{3} x_{ijk}\right) + f_{ij} = 1 \qquad \forall i,j \tag{13}$$

6. Each nurse must use two off days during each week.

$$\sum_{j=j'}^{j'+6} f_{ij} = 2 \qquad \forall i, j' = 1,8,15,22 \tag{14}$$

7. A nurse can work at most three night shifts in a row.

$$x_{ij3} + x_{i(j+1)3} + x_{i(j+2)3} + x_{i(j+3)3} \leq 3 \quad \forall i, j < J-3 \tag{15}$$

8. The difference between the number of night shifts, total regular shifts and number of assignments as an on-call nurse of nurses cannot be more than two.

$$y_i = \sum_{k=1}^{3} \sum_{j=1}^{J} x_{ijk} \qquad \forall i \qquad (16.a)$$

$$n_i = \sum_{j=1}^{J} x_{ij3} \qquad \forall i \qquad (16.b)$$

$$p_i = \sum_{k=1}^{3} \sum_{j=1}^{J} z_{ijk} \qquad \forall i \qquad (16.c)$$

$$p_i - p_{i'} \leq 2 \qquad \forall i, i' \neq i \qquad (16.d)$$

$$n_i - n_{i'} \leq 2 \qquad \forall i, i' \neq i \qquad (16.e)$$

$$y_i - y_{i'} \leq 2 \qquad \forall i, i' \neq i \qquad (16.f)$$

9. The on-call nurse for the first shift is chosen among the nurses who will work in the second shift on same day.

$$z_{ij1} \leq x_{ij2} \qquad \forall i, j \qquad (17.a)$$

The on-call nurse for the third shift is chosen among the nurses who have worked in the second shift on same day.

$$z_{ij3} \leq x_{ij2} \qquad \forall i, j \qquad (17.b)$$

The on-call nurse for the second shift is chosen among the nurses either who have worked in the first shift or will work in the third shift on same day.

$$z_{ij2} \leq x_{ij1} + x_{ij3} \qquad \forall i, j \qquad (17.c)$$

10. If nurse i has a request not to work on day j at shift k, then this request should be considered.

$$a_{ijk} + x_{ijk} + z_{ijk} \leq 1 \qquad \forall i, j, k \qquad (18)$$

11. Sign constraints

$$x_{ijk}, z_{ijk} \in \{0,1\} \qquad \forall i, j, k \qquad (19.a)$$

$$f_{ij} \in \{0,1\} \qquad \forall i, j \qquad (19.b)$$

$$p_i, n_i, y_i \geq 0 \qquad \forall i \qquad (19.c)$$

where $Q(x) = E_s\big(Q(x,s)\big)$ and

$$Q(x,s) = \min \sum_{j=1}^{J} \sum_{k=1}^{3} (a * o_{jk}^{s} + b * c_{jk}^{s} + c * r_{jk}^{s}) \qquad (20)$$

subject to:

12. There must be required number of nurses at each shift to satisfy the demand, and there could be at most one on-call nurse.

$$\sum_{i=1}^{t} x_{ijk} + o_{jk}^{s} + c_{jk}^{s} - r_{jk}^{s} = d_{jk}^{s} \qquad \forall j,k \qquad (21.a)$$

$$c_{jk}^{s} \leq 1 \qquad \forall j,k \qquad (21.b)$$

13. Sign constraints

$$c_{jk}^{s}, o_{jk}^{s}, r_{jk}^{s} \geq 0 \qquad \forall j,k \qquad (22)$$

4. COMPUTATIONAL RESULTS

In this section, we first give some information on the data set we used to generate the problem instances for our computational experiments. Next, we present our numerical results and some important managerial insights about the problem.

Nine months of historical data provided by the private healthcare provider described in Section 2 is used to generate realistic problem instances. To forecast the demand, we considered and compared several univariate time series models including the autoregressive model (AR), the moving average model (MA), and the autoregressive moving average model (ARMA) (NIST, 2015; Rachev *et al.*, 2010). Based on our analysis, we concluded that an autoregressive model with order of one (AR(1)) is the most appropriate and efficient model for our purposes. We constructed the confidence intervals for each day in the planning horizon by using AR(1) and we generated the random demand values within these intervals.

When creating the instances for our computational experiments, the number of nurses working throughout the considered planning horizon is determined based on the historical data, the required number of nurses (i.e., demand) for each shift is generated within the confidence intervals constructed by using AR(1), and the other parameters (such as the number of senior nurses, special requests, and breast-feeding permissions) are set based on the interviews made by the head nurse.

We created two problem sets each of which includes 9 instances. To accurately represent the decision making timeline in practice, we consider a monthly (four-week) planning horizon. For the instances in the first problem set, we generated 100 scenarios by randomly sampling over the confidence intervals for the daily demand forecasts. We use the second problem set to strongly reflect the time series connection between two consecutive days. To achieve this, we generated 3 scenarios for each instance based on the minimum, average, and maximum values of the confidence intervals for the daily demand forecasts. Table 1 summarizes the important parameters for the instances we consider in our main experiments.

Table 1. Problem Parameters for the Instances Used in the Main Experiments

Problem Set	**Instance**	***J/I/S***	Number of senior nurses	Number of breast-feeding permissions (number of days)	Number of special requests (number of shifts)	Average monthly demand (number of nurses)
Set 1	**1**	28/17/100	17	3	5	520.41
	2	28/17/100	17	3	5	468.39
	3	28/17/100	17	3	5	481.53
	4	28/17/100	17	3	5	418.92
	5	28/17/100	17	3	5	475.65
	6	28/17/100	17	3	5	315.96
	7	28/17/100	17	3	5	395.07
	8	28/17/100	17	3	5	618.45
	9	28/17/100	17	3	5	674.28
Set 2	**1**	28/17/3	17	3	5	517.50
	2	28/17/3	17	3	5	465.00
	3	28/17/3	17	3	5	478.50
	4	28/17/3	17	3	5	418.50
	5	28/17/3	17	3	5	477.00
	6	28/17/3	17	3	5	312.00
	7	28/17/3	17	3	5	393.00
	8	28/17/3	17	3	5	616.50
	9	28/17/3	17	3	5	673.50

All of our computational experiments were executed using CPLEX 12.6.1 on a 3.10 GHz computer with 16.0 GB memory and 64-bit Windows 7 operating system. Our preliminary experiments indicated that the L-shaped method is outperformed by solving the extensive form for our problem. Therefore, our main experiments are based on solving the extensive form. In our experiments, we imposed a time limit of 3 hours and for the instances that cannot be solved within this time limit, the best available solution is reported (and marked with "*"). In Table 2., we report the solution times (in CPU seconds) and uncertainty related measures for monthly schedules. As can be observed from this table, VSS ranges between 0.3% and 9.9% and is higher on average for the instances with strong time series connection, which indicates that capturing uncertainty is particularly important for such problems. EVPI is between 6.3% and 89.8% and is higher on average when time series connection is weak, which is an expected result.

Table 2. Solution Time, EVPI and VSS Values

Problem Set	Instance	Solution Time	Objective Value of Stochastic Solution	EEV	WS	VSS (%)	EVPI (%)
Set 1	**1**	4.5	600.2	642.1	496.4*	6.5	17.3
	2	131.5	528.6	544.2	346.2*	2.9	34.5
	3	4538.9	562.5	585.3*	410.5*	3.9	27.0
	4	47.7	494.2	506.3	235.5*	2.4	52.4
	5	679.1	523.3	535.8	368.4*	2.3	29.6
	6	4.9	444.7	446.2	45.2*	0.3	89.8
	7	4.5	508.6	524.3	208.4*	3.0	59.0
	8	507.8	957.7	1017	886.2*	5.8	7.5
	9	4.8	1239.0	1276	1153.0	3.0	7.0
	Avg.	***658.2***	***651.0***	***675.3***	***461.1***	***3.4***	***36.0***
	Max.	***4538.9***	***1239.0***	***1276***	***1153.0***	***6.5***	***89.8***
Set 2	**1**	2.3	683.0	740.3	596.0	7.7	12.7
	2	7200.0	618.8*	636.3	465.3	2.8	24.8
	3	2534.5	664.9	696.4*	528.7	4.5	20.5
	4	2.8	606.7	622.7	373.3	2.6	38.5
	5	2.6	637.0	654.3	504.7	2.7	20.8
	6	2.6	558.7	562.7	177.7	0.7	68.2
	7	3.1	602.0	620.7	333.7	3.0	44.6
	8	4.4	1003.3	1113.3	940.0	9.9	6.3
	9	2.0	1269.7	1357.0	1188.0	6.4	6.4
	Avg.	***85.6***	***738.2***	***778.2***	***567.5***	***4.5***	***27.3***
	Max	***664.9***	***2534.5***	***1357.0***	***1188.0***	***9.9***	***68.2***

To see the impact of objective function coefficients on the uncertainty related measures, we also solved our instances under the setting where overtime is more valuable than undertime ($a = 6$, $b = 2$ and $c = 4$). The results are presented in Table 3. As revealed by Table 3., VSS and EVPI values are slightly lower on average but still significant when the unit cost of overtime is higher.

Table 3. Solution Time, EVPI and VSS Values under the High Overtime Cost Setting

Problem Set	Instance	Solution Time	Objective Value of Stochastic Solution	EEV	WS	VSS (%)	EVPI (%)
Set 1	**1**	4.6	820.7	871.6	669.4*	5.9	18.4
	2	360.7	691.3	715.7	458.3*	3.4	33.7
	3	5130.8	749.2	768.6	549.5*	2.5	26.6
	4	647.3	610.3	624.4	310.5*	2.3	49.1
	5	460.4	692.5	708.3	485.9*	2.2	29.8
	6	4.2	488.5	492.9	52.0*	0.9	89.0
	7	4.7	622.6	652.5	276.3*	4.6	55.6
	8	47.4	1349.2	1431.6	1254.9*	5.8	7.0
	9	5.0	1757.0	1801.3	1663.0	2.5	5.4
	Avg.	***740.6***	***864.6***	***896.3***	***635.5***	***3.3***	***35.0***
	Max.	***5130.8***	***1757.0***	***1801.0***	***1663.0***	***5.9***	***89.0***
Set 2	**1**	2.1	930.7	986.3	840.0	5.6	9.7
	2	7219.0	807.3*	830.7	648.3	2.8	19.7
	3	7213.0	881.3*	920.3	742.7	4.2	15.7
	4	2.9	755.7	775.7	519.0	2.6	31.3
	5	3.5	841.0	860.7	705.7	2.3	16.1
	6	2.5	630.3	646.0	239.0	2.4	62.1
	7	4.0	742.7	770.3	463.3	3.6	37.6
	8	5.2	1412.0	1554.0	1344.3	9.1	4.8
	9	1.8	1802.0	1905.0	1714.0	5.4	4.9
	Avg.	***1606.0***	***978.1***	***1028.0***	***801.8***	***4.2***	***22.4***
	Max.	***7219.0***	***1802.0***	***1905.0***	***1714.0***	***9.1***	***62.1***

We also observe from Table 2. and Table 3. that, the optimal WS solution cannot be obtained within the time limit for most of the instances in Set 1 due to the high number of scenarios for this setting. Strengthening the formulation or using parallel computing would be helpful in obtaining solutions with higher quality (i.e., lower objective function values) in such cases.

Besides solving the mean value problem, we consider three other heuristic methods to solve the problem. These approaches are based on solving a deterministic model by using the first quartile, median and the third quartile values instead of the average demand values. We refer to the gap between the optimal solution (i.e., the solution of the stochastic program) and the solutions returned by these heuristic method as VSS_{25}, VSS_{50}, and VSS_{75}, respectively. We solve our problem instances using these methods and report the related VSS values in Table 4. According to the average and the worst-case performance, solving the mean value problem outperforms the other heuristic methods. However, using the first quartile yields a better solution for some of the instances. As can be seen from Table 1, instances 6 and 7 are the instances with the lowest average monthly demand and instances 8 and 9 are the ones with the highest average monthly demand in both sets of problem instances. Therefore, based on the results given in Table 4., we conclude that considering the average values and the first quartile values is the best strategy when average demand is low and high, respectively.

Table 4. Optimality Gap (VSS) Values for the Proposed Heuristics

Problem Set	Instance	VSS (%)	VSS_{25} (%)	VSS_{50} (%)	VSS_{75} (%)
Set 1	**1**	6.5	1.1	12.9	25.9
	2	2.9	11.9	6.0	23.5
	3	3.9	3.2	9.9	27.5
	4	2.4	21.3	5.3	21.4
	5	2.3	4.8	7.3	24.5
	6	0.3	33.3	4.5	8.7
	7	3.0	24.9	4.5	18.8
	8	5.8	2.1	9.3	17.5
	9	3.0	2.3	4.7	9.8
	Avg.	***3.4***	***11.6***	***7.2***	***19.7***
	Max.	***6.5***	***33.3***	***12.9***	***27.5***
Set 2	**1**	7.7	8.3	7.7	30.6
	2	2.8	18.8	2.8	27.3
	3	4.5	13.6	4.5	28.6
	4	2.6	26.7	2.6	26.1
	5	2.7	10.0	2.7	26.9
	6	0.7	36.5	0.7	18.4
	7	3.0	24.9	3.0	26.1
	8	9.9	6.8	9.9	21.6
	9	6.4	3.0	6.4	15.3
	Avg.	***4.5***	***16.5***	***4.5***	***24.6***
	Max.	***9.9***	***36.5***	***9.9***	***30.6***

We also use these heuristics to solve our problem instances under the setting where the unit cost of overtime is higher and present the results in Table 5. As can be observed, our inferences remain valid under this setting.

Table 5. Optimality Gap (VSS) Values for the Proposed Heuristics under the High Overtime Cost Setting

Problem Set	Instance	VSS (%)	VSS_{25} (%)	VSS_{50} (%)	VSS_{75} (%)
Set 1	**1**	5.9	1.8	14.0	27.5
	2	3.4	11.6	4.4	25.4
	3	2.5	4.9	9.9	23.8
	4	2.3	25.5	6.2	18.4
	5	2.2	6.7	7.4	23.9
	6	0.9	36.2	4.7	7.5
	7	4.6	25.2	6.1	15.1
	8	5.8	2.2	7.7	16.8
	9	2.5	3.2	5.0	9.1
	Avg.	***3.3***	***13.0***	***7.3***	***18.6***
	Max.	***5.9***	***36.2***	***14.0***	***27.5***
Set 2	**1**	5.6	7.9	5.6	27.0
	2	2.8	19.9	2.8	26.8
	3	4.2	13.3	4.2	24.3
	4	2.6	32.1	2.6	22.9
	5	2.3	12.2	2.3	25.4
	6	2.4	45.0	2.4	15.1
	7	3.6	26.9	3.6	22.5
	8	9.1	4.0	9.1	19.9
	9	5.4	5.2	5.4	14.7
	Avg.	***4.2***	***18.5***	***4.2***	***22.0***
	Max.	***9.1***	***45.0***	***9.1***	***27.0***

CONCLUSION

Nurse scheduling and rescheduling problem is a challenging and critical problem since nurses are scarce resources and a poor schedule would cause low quality of nursing services and job dissatisfaction for nurses. In this study, we consider the integrated problem of scheduling and rescheduling of nurses in an ICU under demand uncertainty with a particular focus on the Cardiovascular Surgery and General ICUs of a private hospital in Ankara, Turkey. We formulate the problem as a two-stage stochastic programming model which minimizes the expected cost of rescheduling activities, which are using on-call workforce, overtime and undertime.

We consider two different sets of problem instances in our computational experiments. The first problem set includes instances where the demand values are generated by randomly sampling over the confidence intervals for the daily demand forecasts. And the second problem set includes instances where the demand values are generated based on the minimum, average, and maximum values of the confidence intervals for the daily demand forecasts in order to reflect the time series connection between two consecutive days.

We estimate uncertainty related measures such as VSS and EVPI based on these two sets of realistic problem instances. Our results indicate that VSS values are higher and EVPI values are lower on average when time series connection is strong. Therefore, using stochastic models is particularly important for such problems. Our results also indicate that VSS and EVPI values slightly decrease on average but still remain significant when using overtime becomes more expensive than using undertime.

We also consider other heuristics than solving the mean value problem and provide important insights about the performance of these heuristics under different problem settings. Our results indicate that using a deterministic model based on the average values and the first quartile values is the best strategy when average demand is low and high, respectively.

As indicated by our results, considering the nurse scheduling and rescheduling problem under demand uncertainty in an integrated framework would significantly decrease the amount of adjustments made in the schedule while maintaining an acceptable level of service at all times, meeting the legal requirements and special requests of nurses about working times, and providing equity between nurses in terms of working hours.

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