EXACT SOLUTIONS FOR AXIAL STATIC ANALYSIS OF NANORODS USING WEIGHTED RESIDUALS

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Keywords	Abstract
Axial Static, Nanorod, Weighted Residual Methods, Displacement Function, Exact Solution.	In the present work, axial static analysis of nanorods under triangular loading is presented via Eringen's nonlocal differential model. Three weighted residual methods (Subdomain, Galerkin and Least squares methods) are used to obtain the exact static deflection. These methods require that the integral of the error with different assumptions over the domain be set to zero. The number of equations have to be equal to unknown terms. A cubic displacement function has been chosen for three weighted residual methods. Subdomain, Galerkin and Least squares methods yield identical solution as the exact solution. The plots of the solution are shown for different number of unknown coefficients.

AĞIRLIKLI ARTIKLAR KULLANILARAK NANOÇUBUKLARIN EKSENEL STATİK ANALİZİ İÇİN KESİN ÇÖZÜMLER

Anahtar Kelimeler	Öz
Eksenel Statik, Nanoçubuk, Ağırlıklı Artık Yöntemleri, Yer Değiştirme Fonksiyonu, Gerçek Çözüm.	Bu çalışmada, Eringen'in yerel olmayan diferansiyel modeli kullanılarak; üçhen yayılı yüklenmiş nano çubukların eksenel statik analizi verilmiştir. Üç ağırlıklı artık tabanlı yöntem (Subdomain, Galerkin ve Least squares yöntemleri) gerçek statik deplasmanı elde etmek için kullanılmıştır. Bu yöntemler bölgenin tamamında integral hatalarını minimize etme varsayımına dayanmaktadır. Sistem denklemleri çözümü aranan bilinmeyenler ile aynı sayıda olmadır. Bu yüzden üç ağırlıklı artık yöntemi için de kübik polinomlar statik deplasmanı göstermek üzere seçilmiştir. Subdomain, Galerkin and Least squares yöntemleriyle gerçek çözümler ile aynı polinomlar olarak elde edilmiştir. Değişik sayıda bilinmeyen içeren sabitler ile grafikler çizdirilerek çözümler gösterilmiştir.

1. Introduction

The analytical and computer-based modeling of the dynamical or static behavior of nanobeams (carbon nanotubes) have been come the focal points of researches in computational material physics over the recent years. Moreover, many researhers have studied a structural mechanics and continuum approaches for more efficient and practical modeling. For this purpose, beam, plate and rod theories have been employed by several researchers. In the literature, there have been several number of researches, both theoretical and atomistic, of the mechanical properties of nanobeams. The mechanical analysis of nanosized beams has been implemented using several types of classic elasticity theories, namely based on Euler Bernoulli or Timoshenko beam theories, frame models and plate-shell theories. However, these classical mechanics theories do not account for different phenomena that appear at the small size, such as molecular interactions of atoms and their effects. To overcome these shortcomings, different methods of non-classical elasticity models based on nonlocal elasticity theories have been

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constructed and utilized to nanobeams in recent years. The classical continuum based models are not able to predict higher order effects due to the absence of the nonlocal parameter. To overcome these shortcomings, different nonlocal stress or strain gradient elasticity theories (modified couple stress theory, nonlocal elasticity theory, couple stress theory, strain gradient elasticity theory etc...) have been used to solve these non-classical problems.

Mechanical behavior of rods and nono rods is one of the most frequently studied topics (Arda and Aydogdu 2014; Arda and Aydogdu 2016; Aydogdu and Arda 2016; Arda and Aydogdu 2017; Arda and Aydogdu 2018; Akbaş 2019; Akgöz 2019; Uzun and Yaylı 2020a, Uzun et al. 2020a). More appropriate elasticity theories rather than local or classical shell, plate and beam theories are needed in investigating the size effect in nano-structures. So far several higher order elasticity models have been proposed to understand the small size effect (Li, 2014; Li et al., 2015; Li et al., 2017; Civalek and Demir, 2016; Liu et al., 2017; Kounadis et al., 2006; Akgöz and Civalek, 2013; ; Akgöz and Civalek, 2015; Ansari et al., 2013; Thai et al. 2018; Ebrahimi et al. 2019; Jalaei and Civalek 2019; Arefi et al. 2020; Demir and Civalek 2013; Dastjerdi et al. 2020, Uzun and Yaylı 2020b, Uzun et al. 2020b, c). From the academic studies in the literature, the mechanical methods adopted to explore the dynamical and atomistic behavior of nanobeams can be divided in to three main categories: the atomistic approaches, higher order elasticity theories (gradient elasticity, nonlocal elasticity, modified couple stress theory. etc.) and the classical continuum mechanics approaches. Since the several classical models can not predict the static deflection or dynamical behaviour of nanobeams, different higher elasticity theories have been recently presented to predict the material properties of nanotubes (nanobeams, nanorods) in recent years (Ece and Aydoğdu, 2007; Şimşek, 2010; Aydoğdu, 2009; Reddy and Pang, 2008; Aydoğdu, 2009; Yoon et al., 2003; Wang and Liew, 2007; Yaylı, 2017; Yaylı, 2016a; Yaylı et al., 2015; Yaylı, 2016b; Yaylı, 2014, Civalek et al. 2020, Uzun et al. 2020d, e).

In this work, with regard to the importance of analyzing the axial deflection of nanorods and their vast range of applications, the axial static analsis with linear loading conditions is investigated to predict the impact of small scale parameter on axial deflections of nanorods by wighted residuals. In this study therefore, based on the Eringen's nonlocal elasticity theory, the static problem of axial deformations of a nanorod is tackled and investigated with the weighted residuals methods (Galerkin, subdomain, least squares methods). Moreover, the effect of the number of terms used in the polynomial and the nonlocal parameter of axial deflection of the nanorod has been addressed. A number of graphical results for the axial deflections of a nanorod by using weighted results are presented.

2. Nonlocal Elasticity Theory

For homogenous-isotropic elastic solids, Eringen's nonlocal elasticity theory is defined by the following equations (Eringen and Edelen, 1972; Eringen, 1983):

$$\sigma_{kl,l} + \rho(f_l - \frac{\partial^2 u_l}{\partial t^2}) = 0 \tag{1}$$

$$\sigma_{kl}(x) = \int_V \alpha(|x-x'|, X)T_{kl}(x')dV(x') \tag{2}$$

$$T_{kl}(x') = \lambda\varepsilon_{mm}(x')\delta_{kl} + 2\mu\varepsilon_{kl}(x') \tag{3}$$

$$\varepsilon_{kl}(x') = \frac{1}{2}(\frac{\partial u_k(x')}{\partial x'_l} + \frac{\partial u_l(x')}{\partial x'_k}) \tag{4}$$

in which, σ_{kl} is the nonlocal stress tensor, ρ is the mass density of the body, u_l is the displacement vector, $\alpha|x|$ can be displayed by a linear differential operator, f_l is the applied force density, $\tau_{kl}(x')$ is the Cauchy stress tensor at any point x', $\varepsilon_{kl}(x')$ is the strain tensor, μ and λ are Lame constants, V is the volume occupied by the body, t denotes the time $\alpha|x-x'|$ is the distance form of Euclidean. The following relation can be used in nonlocal elasticity (Eringen and Edelen, 1972; Eringen, 1983):

$$\Gamma\alpha(|x-x'|) = \delta(|x-x'|) \tag{5}$$

where δ[x] is the nonlocal distance. The following operator has been derived from equation (2):

$$\Gamma\alpha_{kl} = T_{kl} \tag{6}$$

In which α[x] is the nonlocal kernel. Furthermore, following relation can be derived from equation (1):

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$$T_{kl,l} + \Gamma(f_l - \rho \ddot{u}_k) = 0 \qquad (7)$$

the differential operator has been expressed as the following compact form:

$$\Gamma = (1 - (e_0 a)^2 \nabla^2) \qquad (8)$$

in which e_0a is the nonlocal parameter, ∇^2 is the Laplacian. Then constitutive equation in Eringen's nonlocal elasticity may be expressed in terms of nonlocal parameter

$$\left[1 - (e_0 a)^2 \nabla^2 \right] \sigma_{kl} = T_{kl} \qquad (9)$$

3. Governing Equation

By using the relation in (9), the governing equation in terms of the axial deflection as follows (Aydoğdu, 2009):

$$EA \frac{d^2 u(x)}{dx^2} + q(x) - \mu \frac{d^2 q(x)}{dx^2} = 0 \qquad (10)$$

$$(e_0 a)^2 = \mu \qquad (11)$$

where $u(x)$ denotes the axial deflection, E is the modulus of elasticity, A is the cross sectional area. Eq. (10) is the governing differential equation for the static deflection of nanorod. Integrating the above equation with respect to x:

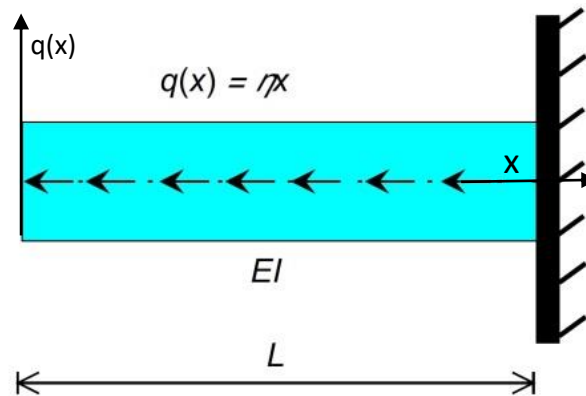$$EA \frac{du(x)}{dx} - \mu \frac{dq(x)}{dx} = \int_0^x q(x)dx \qquad (12)$$

Figure 1. Axially loaded cantilever nanorod

Only one strict boundary condition was used in our study. When x = L, u = 0. For the other boundary condition, scientific debate continues that du / dx is zero when x = 0. It is clear that the derivative is not necessarily zero, as the force is the boundary condition here. In this study, the cantilever nanorod in Figure 1 carries a triangular axial load:

$$q(x) = \eta x \qquad (13)$$

where η is a coefficient which represents the slope of triangular line load.

4. Weighted Residuals Methods

4.1. Least squares method

The method of least squares requires the integral over the domain residual function have to be minimized with respect to unknown terms:

$$\frac{\partial}{\partial c_i}(\int_0^L R^2 dx) = 0, \qquad i = 1, 2, ..., N. \tag{14}$$

The above equation can be written as follows;

$$(\int_0^L R\frac{\partial R}{\partial c_i}dx) = 0 \tag{15}$$

Taking the slope $\eta=10$, the following relations can be written by using the Eq. 15:

$$\int_0^L AE(\ AE(\ 3c_3(\ x - L\ )^2 + 2c_2(\ x - L\ ) + c_1\ ) \\ -10\mu + 5x^2\ )dx = 0 \tag{16}$$

$$\int_0^L AE2(\ x - L\ )(\ AE(\ 3c_3(\ x - L\ )^2 + 2c_2(\ x - L\ ) \\ +c_1\ ) - 10\mu + 5x^2\ )dx = 0 \tag{17}$$

$$\int_0^L AE3(\ x - L\ )^2(\ AE(\ 3c_3(\ x - L\ )^2 + 2c_2(\ x - L\ ) \\ +c_1\ ) - 10\mu + 5x^2\ )dx = 0 \tag{18}$$

Taking the definite integrals; following systems of linear equations are obtained:

$$A^2 c_3 E^2 L^3 - A^2 c_2 E^2 L^2 + A^2 c_1 E^2 L \\ +\frac{5}{3}AEL^3 - 10AE\mu L = 0 \tag{19}$$

$$-\frac{3}{2}A^2 c_3 E^2 L^4 + \frac{4}{3}A^2 c_2 E^2 L^3 - A^2 c_1 E^2 L^2 \\ -\frac{5}{6}AEL^4 + 10AE\mu L^2 = 0 \tag{20}$$

$$\frac{9}{5}A^2 c_3 E^2 L^5 - \frac{3}{2}A^2 c_2 E^2 L^4 + A^2 c_1 E^2 L^3 \\ +\frac{1}{2}AEL^5 - 10AE\mu L^3 = 0 \tag{21}$$

Solving the above equations, we obtain the unknown coefficients as follows:

$$c_1 = -\frac{5(\ L^2 - 2\mu\ )}{AE} \tag{22}$$

$$c_2 = -\frac{5L}{AE} \tag{23}$$

$$c_3 = -\frac{5}{3AE} \tag{24}$$

Substituting the symbolic values for the coefficients given in deflection function:

$$\bar{U}_{LSM} = \frac{5(\ L^3 - 6\mu L - x^3 + 6\mu x\ )}{3AE} \tag{25}$$

4.2. Subdomain Method

Subdomain method require the integral over the some selected domains residual function be set zero:

$$\int_0^{L/3} Rdx = \int_0^{L/3} \frac{EA(\ 3c_3(\ x - L\ )^2 + 2c_2(\ x - L\ )}{+c_1\ ) - 10\mu + 5x^2 dx} = 0 \tag{26}$$

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$$\int_{L/3}^{2L/3} R dx = \int_{L/3}^{2L/3} \begin{matrix} EA(3c_3(x-L)^2 + 2c_2(x-L) \\ +c_1) - 10\mu + 5x^2 dx = 0 \end{matrix} \tag{27}$$

$$\int_{2L/3}^{L} R dx = \int_{2L/3}^{L} \begin{matrix} EA(3c_3(x-L)^2 + 2c_2(x-L) \\ +c_1) - 10\mu + 5x^2 dx = 0 \end{matrix} \tag{28}$$

Calculating the above integrals gives the following systems of equations;

$$\frac{19}{27} Ac_3 EL^3 - \frac{5}{9} Ac_2 EL^2 + \frac{1}{3} Ac_1 EL$$
$$+ \frac{5L^3}{81} - \frac{10\mu L}{3} = 0 \tag{29}$$

$$\frac{7}{27} Ac_3 EL^3 - \frac{1}{3} Ac_2 EL^2 + \frac{1}{3} Ac_1 EL$$
$$+ \frac{35L^3}{81} - \frac{10\mu L}{3} = 0 \tag{30}$$

$$\frac{1}{27} Ac_3 EL^3 - \frac{1}{9} Ac_2 EL^2 + \frac{1}{3} Ac_1 EL$$
$$+ \frac{95L^3}{81} - \frac{10\mu L}{3} = 0 \tag{31}$$

By combining these three equations as a system, these can be written in matrix form as follows:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} -\dfrac{5L^3}{81} + \dfrac{10\mu L}{3} \\ -\dfrac{35L^3}{81} + \dfrac{10\mu L}{3} \\ -\dfrac{95L^3}{81} + \dfrac{10\mu L}{3} \end{pmatrix} \tag{32}$$

We have obtained the unknown coefficient expressions as same as the Eqs. 22, 23 and 24. Substituting these coefficients in deflection function then the same relation obtained from Eq. 25.

4.3. Galerkin Method

The method of Galerkin requires the residual function to be ortogonal to other weighting functions:

$$\left(\int_0^L RW_i dx \right) = 0, \qquad i = 1, 2, ..., N. \tag{33}$$

In this study, the weighting function formulas are selected to be a part of deflection function. The title problem of this letter have three unknown coefficients (c_1, c_2, c_3), therefore following three weighting functions are chosen;

$$W_1 = x - L \tag{34}$$
$$W_2 = (x - L)^2 \tag{35}$$
$$W_3 = (x - L)^3 \tag{36}$$

Using the above approximation functions, the following relations are generated.

$$\int_0^L \begin{matrix} AE(AE(3c_3(x-L)^2 + 2c_2(x-L) + c_1) \\ -10\mu + 5x^2)(x-L)dx = 0 \end{matrix} \tag{37}$$

$$\int_0^L \begin{matrix} AE2(x-L)(AE(3c_3(x-L)^2 + 2c_2(x-L) \\ +c_1) - 10\mu + 5x^2)(x-L)^2 dx = 0 \end{matrix} \tag{38}$$

$$\int_0^L AE3(x-L)^2(AE(3c_3(x-L)^2+2c_2(x-L) \\ +c_1)-10\mu+5x^2)(x-L)^3dx=0 \qquad (39)$$

Computing the above integrals; the following systems equations are derived:

$$-\frac{3}{4}Ac_3EL^4+\frac{2}{3}Ac_2EL^3-\frac{1}{2}Ac_1EL^2 \\ -\frac{5L^4}{12}+5\mu L^2=0 \qquad (40)$$

$$\frac{3}{5}Ac_3EL^5-\frac{1}{2}Ac_2EL^4+\frac{1}{3}Ac_1EL^3 \\ +\frac{L^5}{6}-\frac{10\mu L^3}{3}=0 \qquad (41)$$

$$-\frac{1}{2}Ac_3EL^6+\frac{2}{5}Ac_2EL^5-\frac{1}{4}Ac_1EL^4 \\ -\frac{L^6}{12}+\frac{5\mu L^4}{2}=0 \qquad (42)$$

We can write the above relations in a matrix form as follows:

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}=\begin{pmatrix} \dfrac{5L^4}{12}-5\mu L^2 \\ -\dfrac{L^5}{6}+\dfrac{10\mu L^3}{3} \\ \dfrac{L^6}{12}-\dfrac{5\mu L^4}{2} \end{pmatrix} \qquad (43)$$

Substituting the symbolic values for the coefficients given in deflection function:

$$\bar{U}_{GM}=\frac{5(L^3-6\mu L-x^3+6\mu x)}{3AE} \qquad (44)$$

4.4. Two Parameter Solution

In this subsection two parameter solutions have been performed in order to assess the effects of residuals of each method. Following relations can be obtained for Sub-domain method:

$$\int_0^{L/2}Rdx=\int_0^{L/2}EA(2c_2(x-L)+c_1)-10\mu+5x^2dx=0 \qquad (45)$$

$$\int_{L/2}^L Rdx=\int_{L/2}^L EA(2c_2(x-L)+c_1)-10\mu+5x^2dx=0 \qquad (46)$$

c_1 and c_2 coefficients can be found from above relations as follows:

$$c_1=-\frac{5(5L^2-12\mu)}{6AE} \qquad (47)$$

$$c_2=-\frac{5L}{2AE} \qquad (48)$$

and the axial deflection function is derived:

$$\bar{U}_{SDM}=\frac{5(L-x)(-12\mu+2L^2+3Lx)}{6AE} \qquad (49)$$

Galerkin method, which minimize the residual, together with boundary conditions, defines the following definite integrals for two terms:

$$\int_0^L \begin{matrix} AE(\ AE(\ 2c_2(\ x-L\ )+c_1\ ) \\ -10\mu+5x^2\ )(\ x-L\ )dx=0 \end{matrix} \tag{50}$$

$$\int_0^L \begin{matrix} AE2(\ x-L\ )(\ AE(\ 2c_2(\ x-L\ ) \\ +c_1\ )-10\mu+5x^2\ )(\ x-L\ )^2 dx=0 \end{matrix} \tag{51}$$

The unknown terms can be found as follows:

$$c_1 = -\frac{7L^2-20\mu}{2AE} \tag{52}$$

$$c_2 = -\frac{2L}{AE} \tag{53}$$

The following axial deflection function can be written by using the above equations

$$\bar{U}_{GM} = \frac{(\ L-x\ )(-20\mu+3L^2+4Lx\ )}{2AE} \tag{54}$$

In least squares method, following integral equations can be written by using the approximation function:

$$\int_0^L AE(\ AE(\ 2c_2(\ x-L\ )+c_1\ )-10\mu+5x^2\ )dx=0 \tag{55}$$

$$\int_0^L \begin{matrix} AE2(\ x-L\ )(\ AE(\ 2c_2(\ x-L\ )+c_1\ ) \\ -10\mu+5x^2\ )dx=0 \end{matrix} \tag{56}$$

The same relations in Eqs. 47, 48 and 49 are obtained from the results of these integrals. Exact solution can be found as;

$$\bar{U}_{Exact} = \frac{5(\ L^3-6\mu L-x^3+6\mu x\ )}{3AE} \tag{25}$$

5. Results and Discussions

Based on the formulations obtained above with the nonlocal rod model, the axial deflections of nanorod are discussed and investigated here. A computer code is developed based on the different weighted residuals methods. Sufficient number polynomial terms are employed to predict accurate nonlocal axial deflection results in the analysis.
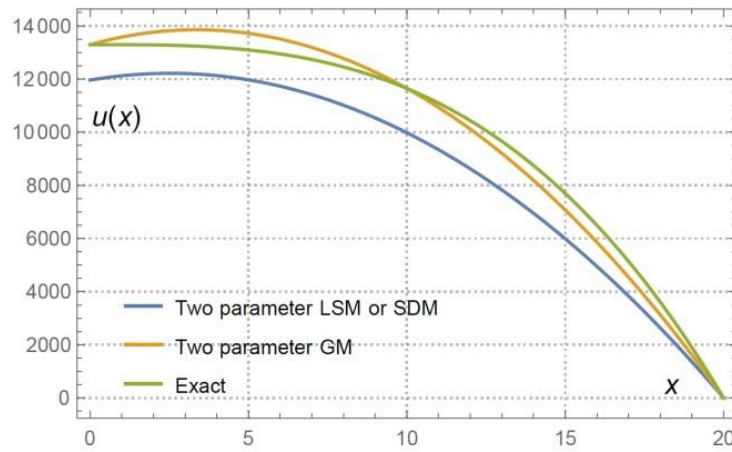
Figure 2. Weighted residuals method with exact solution using two parameters for *L* = 20 *nm*

Figure 2 and 3 depict the variation of the axial deflection of nanorods with two different values of lengths. For this study modulus of elasticity is supposed to 1 *nN/nm²*. Cross sectional area *A* = 1 *nm²*, nonlocal parameter *μ* = 0.2 *nm²* are considered in the analysis.
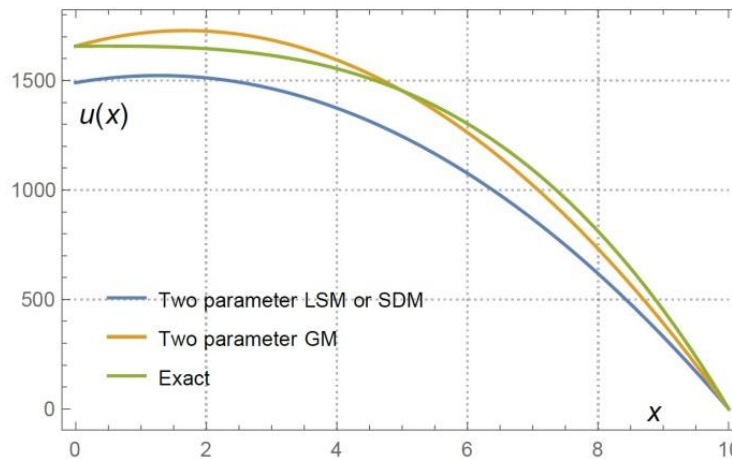

Figure 3. Weighted residuals method with exact solution using two parameters for *L* = 10 *nm*

It is apparent from Figures 4 and 5 that, the computed results with minimal effort match closely with that of the exact approach for three terms of polynomials.
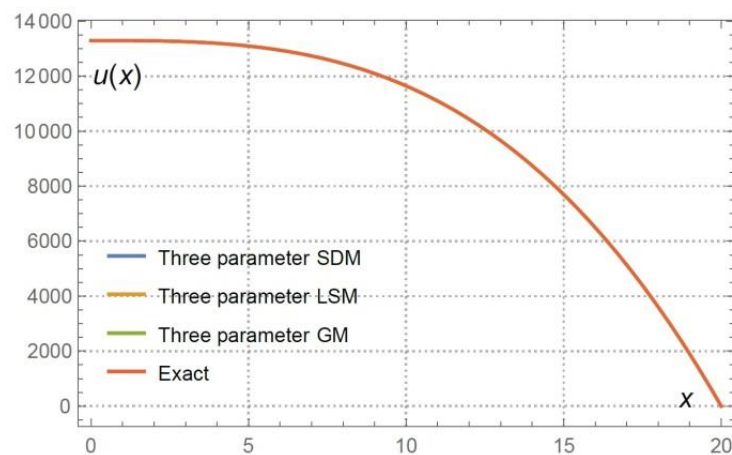

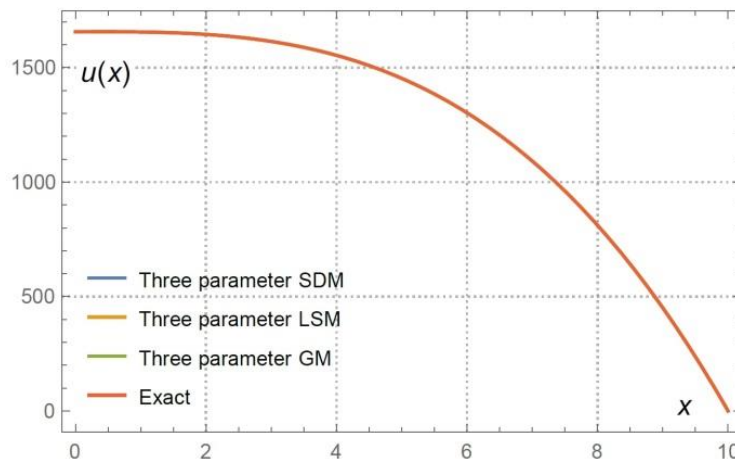**Figure 4.** Weighted residuals method with exact solution using three parameters for *L* = 20 *nm*

Figure 5. Weighted residuals method with exact solution using three parameters for $L = 10$ nm

In Figure 6, axial deflections of a nanorod are obtained by applying three different weighted residual methods and the nonlocal parameter are taken as different values. For the second example similar values are used; cross sectional area $A = 1$ nm^2, modulus of elasticity is 1 nN/nm^2, nonlocal parameter $\mu = 0.2$ nm^2 are utilized. It can be seen from the Figure 6 that lower elongation amounts are obtained when the non-local elasticity theory is used. The reason for this can be shown as the classical elasticity force boundary condition, which is used controversially in the literature, has never been used.
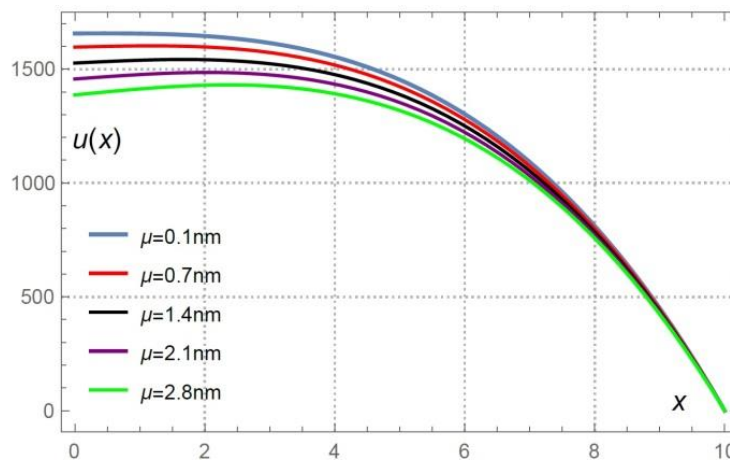

Figure 6. The effect of small scale parameter on the axial deflection for $L = 10$ nm

The results exhibited certain difference in axial deflection response predictions between nonlocal elasticity and local elasticity, which becomes predominant when the length of nanorod is smaller than 4 nm^2. One of the chief contributions of present paper is the derivation of an exact solution using the weighted residuals.

6. Conclusion

On the basis of the nonlocal elasticity theory, axial static analysis of nanorods under triangular axial loading conditions is investigated. A polynomial function that defines the axial deflection along the longitudinal axis of the continuum is proposed which satisfied the boundary conditions. The governing ordinary differential equation of the nanorod is solved with the weighted residuals methods (Galerkin, Subdomain, Least squares methods). Numerical values, graphical plots and the influence of the parameters in the polynomials on the axial deflection of the nanorod are presented.

Conflict of Interest

No conflict of interest was declared by the authors.

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