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Mapping on Intuitionistic Fuzzy Soft Expert Sets

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Abstract – We introduce the mapping on intuitionistic fuzzy soft expert set and its operations are studied. The basic operations of mapping on intuitionistic fuzzy soft expert set theory are defined.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [16] whose basic component is only a degree of membership. Atanassov [10] generalized this idea to intuitionistic fuzzy set (IFS in short) using a degree of membership and a degree of non-membership, under the constraint that the sum of the two degrees does not exceed one. The conception of IFS can be viewed as an appropriate /alternative approach in case where available information is not sufficient to define the impreciseness by the conventional fuzzy set. A detailed theoretical study may be found in [10]. Later on, many hybrid structures with the concept of intuitionistic fuzzy sets appeared in [32, 33, 34, 35, 36, 37, 38].
Soft set theory [6] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. A soft set is in fact a set-valued map which gives an approximation description of objects under consideration based on some parameters. After Molodtsov's work, Maji et al.[29] introduced the concept of fuzzy soft set, a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties and also discussed their properties. Also, Maji et al.[30] devoted the concept of intuitionistic fuzzy soft sets by combining intuitionistic fuzzy sets with soft sets. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [22, 23, 27, 28], on intuitionistic fuzzy soft set theory [24, 25, 26, 30], on possibility fuzzy soft set [34], on generalized fuzzy soft sets [8,39], on generalized intuitionistic fuzzy soft [15, 31,43,44], on possibility intuitionistic fuzzy soft set [17], on possibility vague soft set [11] and so on. All these research aim to solve most of our real life problems in medical sciences, engineering, management, environment and social science which involve data

that are not crisp and precise. Moreover all the models created will deal only with one expert. To redefine this one expert opinion, Alkhazaleh and Salleh in 2011 [32] defined the concept of soft expert set in which the user can know the opinion of all the experts in one model and give an application of this concept in decision making problem. Also, they introduced the concept of the fuzzy soft expert set [40] as a combination between the soft expert set and the fuzzy set. Recently, Broumi and Smaranadache [42] introduced, a more generalized concept, the concept of the intuitionistic fuzzy soft expert set as a combination between the soft expert set and the intuitionistic fuzzy set and gave the application in decision making problem. The soft expert models are richer than soft set models since the soft set models are created with the help of one expert whereas but the soft expert models are made with the opinions of all experts. Later on, many researchers have worked with the concept of soft expert sets and their hybrid structures [1,2, 3, 7, 10, 11, 12, 16, 17, 19, 45].

The notion of mapping on soft classes are introduced by Kharal and Ahmad [4]. The same authors presented the concept of a mapping on classes of fuzzy soft sets [5] and studied the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets, and supported them with examples and counterexamples. In intuitionistic fuzzy environment, there is no study on mapping on the classes of intuitionistic fuzzy soft expert sets, so there is a need to develop a new mathematical tool called "Mapping on intuitionistic fuzzy soft expert set".

In this paper we introduce the notion of mapping on intuitionistic fuzzy oft expert classes and study the properties of intuitionistic fuzzy soft expert images and intuitionistic fuzzy soft expert inverse images of intuitionistic fuzzy soft expert sets. Finally, we give some examples of mapping on intuitionistic fuzzy soft expert.

2   Preliminaries

In this section, we will briefly recall the basic concepts of intuitionistic fuzzy sets, soft set, soft expert sets, fuzzy soft expert sets and intuitionistic fuzzy soft expert set.

Let U be an initial universe set of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects. Let P (U) denote the power set of U and A$\subseteq$ E.

2.1. Intuitionistic Fuzzy Set

Definition 2.1 [8]: Let U be an universe of discourse then the intuitionistic fuzzy set A is an object having the form A = {< x, $\mu_A(x)$, $\omega_A(x)$>,x $\in$ U},where the functions $\mu_A(x)$, $\omega_A(x)$: U→[0,1] define respectively the degree of membership, and the degree of non-membership of the element x $\in$ X to the set A with the condition.

$$0 \leq \mu_A(x) + \omega_A(x) \leq 1.$$

For two IFS,

A_{IFS}= {<x, $\mu_A(x)$, $\omega_A(x)$> | x $\in$ X }

and

B_{IFS}= {<x, $\mu_B(x)$, $\omega_B(x)$> | x $\in$ X }

Then,

    1. $A_{IFS} \subseteq B_{IFS}$ if and only if

$\mu_A(x) \leq \mu_B(x), \omega_A(x) \geq \omega_B(x)$

    2. $A_{IFS} = B_{IFS}$ if and only if,

$\mu_A(x) = \mu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$.

3. The complement of A_{IFS} is denoted by A_{IFS}^o and is defined by

$$A_{IFS}^o = \{<x, \omega_A(x), \mu_A(x) \| x \in X \}$$

4. $A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}>: x \in X \}$

5. $A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}>: x \in X \}$

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse U=$\{x_1, x_2, x_3, x_4\}$. It may be further assumed that the values of x_1, x_2, x_3 and x_4 are in [0, 1] Then, A is an intuitionistic fuzzy set (IFS) of U, such that,

A= $\{< x_1, 0.4, 0.6>, < x_2, 0.3, 0.7>, < x_3, 0.2, 0.8>, <x_4, 0.2, 0.8>\}$

2.2. Soft set

Definition 2.3 [4]
Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a nonempty set A, $A \subset E$. A pair (K, A) is called a soft set over U, where K is a mapping given by K : A $\rightarrow$ P(U).
As an illustration, let us consider the following example.
Example 2.4 . Suppose that U is the set of houses under consideration, say U = $\{h_1, h_2, \ldots, h_5\}$. Let E be the set of some attributes of such houses, say E = $\{e_1, e_2, \ldots, e_8\}$, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings", "moderate", , "cheap"," expensive", "wooden" and "very costly" respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:
A=$\{e_1, e_2, e_3, e_4, e_5\}$;
$K(e_1) = \{h_2, h_3, h_5\}$, $K(e_2) = \{h_2, h_4\}$, $K(e_3) = \{h_1\}$, $K(e_4) = U$, $K(e_5) = \{h_3, h_5\}$.

2.3 Intuitionistic fuzzy soft sets.
Definition 2.5 [28] Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IFS(U) denotes the set of all intuitionistic fuzzy subsets of U. The collection (F, A) is termed to be the intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \rightarrow IFS(U)$.
Example 2.6 Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a word or sentence involving intuitionistic fuzzy words. Consider $E =\{$beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive$\}$. In this case, to define a intuitionistic fuzzy soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \ldots, h_5\}$ and the set of parameters
$A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$ stands for `moderate'. Then the intuitionistic fuzzy set (F, A) is defined as follows:

$$(F,A) = \begin{cases} \left(e_1\left\{\dfrac{h_1}{(0.1,0.6)}, \dfrac{h_2}{(0.2,0.7)}, \dfrac{h_3}{(0.6,0.2)}, \dfrac{h_4}{(0.7,0.3)}, \dfrac{h_5}{(0.2,0.3)}\right\}\right) \\ \left(e_2\left\{\dfrac{h_1}{(0.3,0.5)}, \dfrac{h_2}{(0.2,0.4)}, \dfrac{h_3}{(0.1,0.2)}, \dfrac{h_4}{(0.1,0.3)}, \dfrac{h_5}{(0.3,0.6)}\right\}\right) \\ \left(e_3\left\{\dfrac{h_1}{(0.4,0.3)}, \dfrac{h_2}{(0.6,0.3)}, \dfrac{h_3}{(0.2,0.5)}, \dfrac{h_4}{(0.2,0.6)}, \dfrac{h_5}{(0.7,0.3)}\right\}\right) \\ \left(e_4\left\{\dfrac{h_1}{(0.1,0.6)}, \dfrac{h_2}{(0.3,0.6)}, \dfrac{h_3}{(0.6,0.4)}, \dfrac{h_4}{(0.4,0.2)}, \dfrac{h_5}{(0.5,0.3)}\right\}\right) \end{cases}$$

2.4. Soft expert sets

Definition 2.7[30] Let U be a universe set, E be a set of parameters and X be a set of experts (agents). Let O= {1=agree, 0=disagree} be a set of opinions. Let Z= E × X × O and A ⊆ Z

A pair (F, E) is called a soft expert set over U, where F is a mapping given by F : A → P(U) and P(U) denote the power set of U.

Definition 2.8 [30] An agree- soft expert set $(F,A)_1$ over U, is a soft expert subset of (F,A) defined as :

$$(F,A)_1 = \{F(\alpha): \alpha \in E \times X \times \{1\}\}.$$

Definition 2.9 [30] A disagree- soft expert set $(F,A)_0$ over U, is a soft expert subset of (F,A) defined as :

$$(F,A)_0 = \{F(\alpha): \alpha \in E \times X \times \{0\}\}.$$

2.5.Fuzzy Soft expert sets

Definition 2.10 [31] A pair (F, A) is called a fuzzy soft expert set over U, where F is a mapping given by

$$F : A \to I^U, \text{ and } I^U \text{ denote the set of all fuzzy subsets of U.}$$

2.6.Intuitionitistic Fuzzy Soft expert sets

Definition 2.11 [40] Let U= $\{u_1, u_2, u_3, \ldots, u_n\}$ be a universal set of elements, E={ $e_1, e_2, e_3, \ldots, e_m$} be a universal set of parameters, X={ $x_1, x_2, x_3, \ldots, x_i$} be a set of experts (agents) and O= {1=agree, 0=disagree} be a set of opinions. Let Z= { E × X × Q } and A ⊆ Z. Then the pair (U, Z) is called a soft universe. Let $F: Z \to (I \times I)^U$ where $(I \times I)^U$ denotes the collection of all intuitionistic fuzzy subsets of U. Suppose $F: Z \to (I \times I)^U$ be a function defined as:

$$F(z) = F(z)(u_i), \text{ for all } u_i \in U.$$

Then $F(z)$ is called an intuitionistic fuzzy soft expert set (IFSES in short) over the soft universe (U, Z)

For each $z_i \in Z$. $F(z) = F(z_i)(u_i)$ where F(z_i) represents the degree of belongingness and non-belongingness of the elements of U in F(z_i). Hence $F(z_i)$ can be written as:

$$F(z_i) = \left\{\left(\frac{u_1}{F(z_1)(u_1)}\right), \ldots, \left(\frac{u_i}{F(z_i)(u_i)}\right)\right\}, \text{ for i=1,2,3,\ldots n}$$

where $F(z_i)(u_i) = \langle \mu_{F(z_i)}(u_i), \omega_{F(z_i)}(u_i) \rangle$ with $\mu_{F(z_i)}(u_i)$ and $\omega_{F(z_i)}(u_i)$ representing the membership function and non-membership function of each of the elements $u_i \in U$ respectively.

Sometimes we write F as (F, Z). If A ⊆ Z. we can also have IFSES (F, A).

3. Mapping on Intuitionistic Fuzzy Soft Expert Set.

In this paper, we introduce the notion of a mapping on intuitionistic fuzzy soft expert classes. Intuitionistic fuzzy soft expert classes are collections of an intuitionistic fuzzy soft expert sets. We also define and study the properties of an intuitionistic fuzzy soft expert images and an intuitionistic fuzzy soft expert inverse images of an intuitionistic fuzzy soft expert sets, and support them with example and theorems.

Definition 3.1: Let $\widetilde{(U,Z)} and (Y,\widetilde{Z'})$ be an intuitionistic fuzzy soft expert classes. Let r: U→Y and

s: $Z \to Z'$ be mappings. Then a mapping $f:\widetilde{(U,Z)} \to (Y,\widetilde{Z'})$ is defined as follows :
For an intuitionistic fuzzy soft expert set (F, A) in $\widetilde{(U,Z)}$, f (F, A) is an intuitionistic fuzzy soft expert set in $(\widetilde{Y,Z'})$

$$f(F, A)(\beta)(y) = \begin{cases} V_{x\in r^{-1}(y)}(V_\alpha F(\alpha)) \ if \ r^{-1}(y) \ and \ s^{-1}(\beta) \cap A \neq \emptyset, \\ 0 \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

for $\beta \in s(Z) \subseteq Z'$, y ∈ Y and $\forall \alpha \in s^{-1}(\beta) \cap A$, f (F, A) is called an intuitionistic fuzzy soft expert image of the intuitionistic fuzzy soft expert set (F, A).

Definition 3.2 : Let $\widetilde{(U,Z)} and (Y,\widetilde{Z'})$ be an intuitionistic fuzzy soft expert classes. Let r: U→Y and

s: $Z \to Z'$ be mappings. Then a mapping $f^{-1}:(Y,\widetilde{Z'}) \to \widetilde{(U,Z)}$ is defined as follows :
For an intuitionistic fuzzy soft expert set (G, B) in $(Y,\widetilde{Z'})$, $f^{-1}(G, B)$ is an intuitionistic fuzzysoft expert set in $\widetilde{(U,Z)}$

$$f^{-1}(G, B)(\alpha)(u) = \begin{cases} G(s(\alpha))(r(u)) \qquad\qquad ,if s(\alpha) \in B \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

For $\alpha \in s^{-1}(\beta) \subseteq Z$ and u ∈ U. $f^{-1}(G, B)$ is called an intuitionistic fuzzy soft expert inverse image of the an intuitionistic fuzzy soft expert set (F, A).

Example 3.3. Let U=$\{u_1, u_2, u_3\}$, Y=$\{y_1, y_2, y_3\}$ and let A ⊆ Z = $\{(e_1, p, 1), (e_2, p, 0), (e_3, p, 1)\}$, and $A' \subseteq Z' = \{(e_1', p', 1), (e_2', p', 0), (e_1', q', 1)\}$.

Suppose that $\widetilde{(U,A)} and (Y,\widetilde{A'})$ are an intuitionistic fuzzy soft expert classes. Define r : U → Y and s: : A → A' as follows :
r$(u_1) = y_1$, r$(u_2) = y_3$, r$(u_3) = y_2$,
s $(e_1, p, 1) = (e_2', p', 0)$, s $(e_2, p, 0) = (e_1', p', 1)$, s $(e_3, p, 1) = (e_1', q', 1)$,
Let (F, A) and (G, A') be two an intuitionistic fuzzy soft experts over U and Y respectively such that.

(F, A) = $\left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{(0.4,0.6)}, \frac{u_2}{(0.3,0.4)}, \frac{u_3}{(0.3,0.5)} \right\} \right), \left((e_3, p, 1), \left\{ \frac{u_1}{(0.3,0.2)}, \frac{u_2}{(0.5,0.4)}, \frac{u_3}{(0.6,0.3)} \right\} \right), \right.$

$\left. \left((e_2, p, 0), \left\{ \frac{u_1}{(0.5,0.3)}, \frac{u_2}{(0.5,0.2)}, \frac{u_3}{(0.6,0.1)} \right\} \right) \right\},$

(G, A') = $\left\{ \left((e_1', p', 1), \left\{ \frac{y_1}{(0.3,0.1)}, \frac{y_2}{(0.5,0.4)}, \frac{y_3}{(0.3,0.1)} \right\} \right), \left((e_1', q', 1), \left\{ \frac{y_1}{(0.5,0.4)}, \frac{y_2}{(0.5,0.3)}, \frac{y_3}{(0.6,0.1)}, \right\} \right), \right.$

$\left. \left((e_2', p', 0), \left\{ \frac{y_1}{(0.3, 0.4)}, \frac{y_2}{(0.1, 0.5)}, \frac{y_3}{(0.1, 0.2)}, \right\} \right) \right\}$

Then we define the mapping from $f:\widetilde{(U,Z)} \to (Y,\widetilde{Z'})$ as follows :
For an intuitionistic fuzzy soft expert set (F, A) in (U, Z), (f (F, A), K) is an intuitionistic fuzzy soft expert set in (Y, Z') where
K= s(A)=$\{(e_1', p', 1), (e_2', p', 0), (e_1', q', 1)\}$. and is obtained as follows:
f (F, A) $(e_1', p', 1)(y_1) = V_{x\in r^{-1}(y_1)}(V_\alpha F(\alpha)) = V_{x\in \{u_1\}}\left(V_{\alpha\in\{(e_2,p,0),(e_3,p,1)\}} F(\alpha) \right)$
$$= (0.5, 0.3) \cup (0.3, 0.2)$$
$$= (0.5, 0.2)$$

$$f\,(F,A)\,(e_1',p,1)\,(y_2) = V_{x\in r^{-1}(y_2)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_2,p,0),(e_2,p,1)\}}F(\alpha)\right)$$
$$= (0.6,0.1)\cup(0.6,0.3)$$
$$= (0.6,0.1)$$

$$f\,(F,A)\,(e_1',p,1)\,(y_3) = V_{x\in r^{-1}(y_3)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_2,p,0),(e_2,p,1)\}}F(\alpha)\right)$$
$$=(0.5,0.2)\cup(0.5,0.4)$$
$$= (0.5,0.2)$$

Then

$$f\,(F,A)\,(e_1',p,1) = \left\{\frac{y_1}{(0.5,0.2)},\frac{y_2}{(0.6,0.1)},\frac{y_3}{(0.5,0.2)}\right\}$$

$$f\,(F,A)\,(e_2',p,0)\,(y_1) = V_{x\in r^{-1}(y_1)}(V_\alpha F(\alpha)) = V_{x\in\{u_1\}}\left(V_{\alpha\in\{(e_1,p,1)\}}F(\alpha)\right)$$
$$= (0.4,0.6)$$

$$f\,(F,A)\,(e_2',p,0)\,(y_2) = V_{x\in r^{-1}(y_2)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_1,p,1)\}}F(\alpha)\right)$$
$$= (0.3,0.5)$$

$$f\,(F,A)\,(e_2',p,0)\,(y_3) = V_{x\in r^{-1}(y_3)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_1,p,1)\}}F(\alpha)\right)$$
$$= (0.3,0.4)$$

Then

$$f\,(F,A)\,((e_2',p,0)= \left\{\frac{y_1}{(0.4,0.6)},\frac{y_2}{(0.3,0.5)},\frac{y_3}{(0.3,0.4)}\right\}$$

$$f\,(F,A)\,(e_1',q,1)\,(y_1) = V_{x\in r^{-1}(y_1)}(V_\alpha F(\alpha)) = V_{x\in\{u_1\}}\left(V_{\alpha\in\{(e_2,p,1)\}}F(\alpha)\right)$$
$$= (0.3,0.2)$$

$$f\,(F,A)\,(e_1',q,1)\,(y_2) = V_{x\in r^{-1}(y_2)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_2,p,1)\}}F(\alpha)\right)$$
$$= (0.6,0.3)$$

$$f\,(F,A)\,(e_1',q,1)\,(y_3) = V_{x\in r^{-1}(y_3)}(V_\alpha F(\alpha)) = V_{x\in\{u_2\}}\left(V_{\alpha\in\{(e_2,p,1)\}}F(\alpha)\right)$$
$$= (0.5,0.4)$$

Then

$$f\,(F,A)\,((e_1',q,1)= \left\{\frac{y_1}{(0.3,0.2)},\frac{y_2}{(0.6,0.3)},\frac{y_3}{(0.5,0.4)}\right\}$$

Hence

$(f\,(F,A),K)$
$$=\left\{\left((e_1',p,1),\left\{\frac{y_1}{(0.5,0.2)},\frac{y_2}{(0.6,0.1)},\frac{y_3}{(0.5,0.2)}\right\}\right),\left((e_2',p,0),\left\{\frac{y_1}{(0.4,0.6)},\frac{y_2}{(0.3,0.5)},\frac{y_3}{(0.3,0.4)}\right\}\right),\right.$$
$$\left.\left((e_1',q,1),\left\{\frac{y_1}{(0.3,0.2)},\frac{y_2}{(0.6,0.3)},\frac{y_3}{(0.5,0.4)}\right\}\right)\right\}$$

Next, for the intuitionistic fuzzy soft expert set inverse images, we have the following:
For an intuitionistic fuzzy soft expert set (G,A') in (Y,Z), $(f^{-1}(G,A'),D)$ is an intuitionistic fuzzy soft expert set in (U,Z) where

$D=s^{-1}(A')=\{(e_1,p,1),(e_2,p,0),(e_3,p,1)\}$. and is obtained as follows:

$$f^{-1}(G,B)\,(e_1,p,1)\,(u_1) = G(s(e_1,p,1))(r(u_1)) = G((e_2',p,0))(y_1)=(0.3,0.4) \quad f^{-1}(G,$$
$$B)\,(e_1,p,1)\,(u_2) = G(s(e_1,p,1))(r(u_2)) = G((e_2',p,0))(y_3)=(0.1,0.2)$$

$$f^{-1}(G,B)\,(e_1,p,1)\,(u_3) = G(s(e_1,p,1))(r(u_3)) = G((e_2',p,0))(y_2)=(0.1,0.5) \quad \text{Then}$$

$$f^{-1}(G,B)\,(e_1,p,1) = \left\{\frac{u_1}{(0.3,0.4)},\frac{u_2}{(0.1,0.2)},\frac{u_3}{(0.1,0.5)}\right\}$$

$$f^{-1}(G,B)\,(e_2,p,0)\,(u_1) = G(s(e_2,p,0))(r(u_1)) = G((e_1',p,1))(y_1)=(0.3,0.1)$$
$$f^{-1}(G,B)\,(e_2,p,0)\,(u_2) = G(s(e_2,p,0))(r(u_2)) = G((e_1',p,1))(y_3)=(0.3,0.1)$$
$$f^{-1}(G,B)\,(e_2,p,0)\,(u_3) = G(s(e_2,p,0))(r(u_3)) = G((e_1',p,1))(y_2)=(0.5,0.4)$$

Then

$$f^{-1}(G,B)\,(e_2,p,0)= \left\{\frac{u_1}{(0.3,0.1)},\frac{u_2}{(0.3,0.1)},\frac{u_3}{(0.5,0.4)}\right\}$$

f^{-1} (G, B) $(e_3, p, 1)$ $(u_1) = G(s(e_3, p, 1))(r(u_1)) = G((e'_1, q', 1))(y_1) = (0.5, 0.4)$ f^{-1} (G, B) $(e_3, p, 1)$ $(u_2) = G(s(e_3, p, 1))(r(u_2)) = G((e'_1, q', 1))(y_3) = (0.6, 0.1)$
f^{-1} (G, B) $(e_3, p, 1)$ $(u_3) = G(s(e_3, p, 1))(r(u_3)) = G((e'_1, q', 1))(y_2) = (0.5, 0.3)$

Then

$$f^{-1} \text{ (G, B) } (e_3, p, 1) = \left\{ \frac{u_1}{(0.5, 0.4)}, \frac{u_2}{(0.6, 0.1)}, \frac{u_3}{(0.5, 0.4)} \right\}$$

Hence

$(f^{-1}(G, A'), D)$
$$= \left\{ \left((e_1, p, 1), \left\{ \frac{u_1}{(0.3, 0.4)}, \frac{u_2}{(0.1, 0.2)}, \frac{u_3}{(0.1, 0.5)} \right\} \right), \left((e_2, p, 0), \left\{ \frac{u_1}{(0.3, 0.1)}, \frac{u_2}{(0.3, 0.1)}, \frac{u_3}{(0.5, 0.4)} \right\} \right), \right.$$
$$\left. \left((e_3, p, 1), \left\{ \frac{u_1}{(0.5, 0.4)}, \frac{u_2}{(0.6, 0.1)}, \frac{u_3}{(0.5, 0.4)} \right\} \right) \right\}.$$

Definition 3.4. Let f: $\widetilde{(U, Z)} \to \widetilde{(Y, Z')}$ be a mapping and (F, A) and (G, B) a intuitionistic fuzzy soft expert sets in $\widetilde{(U, E)}$. Then for $\beta \in Z', y \in Y$ the union and intersection of intuitionistic fuzzy soft expert images (F, A) and (G, B) are defined follows :

$$\left(f(F, A) \widetilde{\vee} f(G, B) \right)(\beta)(y) = f(F, A)(\beta)(y) \widetilde{\vee} f(G, B)(\beta)(y).$$

$$\left(f(F, A) \widetilde{\wedge} f(G, B) \right)(\beta)(y) = f(F, A)(\beta)(y) \widetilde{\wedge} f(G, B)(\beta)(y).$$

Definition 3.5. Let f: $\widetilde{(U, Z)} \to \widetilde{(Y, Z')}$ be a mapping and (F, A) and (G, B) a intuitionistic fuzzy soft expert sets in $\widetilde{(U, E)}$. Then for $\alpha \in Z, u \in U$, the union and intersection of intuitionistic fuzzy soft expert inverse images (F, A) and (G, B) are defined follows :

$$\left(f^{-1}(F, A) \widetilde{\vee} f^{-1}(G, B) \right)(\alpha)(u) = f^{-1}(F, A)(\alpha)(u) \widetilde{\vee} f^{-1}(G, B)(\alpha)(u).$$

$$\left(f^{-1}(F, A) \widetilde{\wedge} f^{-1}(G, B) \right)(\alpha)(u) = f^{-1}(F, A)(\alpha)(u) \widetilde{\wedge} f^{-1}(G, B)(\alpha)(u).$$

Theorem 3.6 Let f: $\widetilde{(U, Z)} \to \widetilde{(Y, Z')}$ be a mapping. Then for intuitionistic fuzzy soft expert sets (F, A) and (G, B) in the intuitionistic fuzzy soft expert class $\widetilde{(U, Z)}$.

1. $f(\emptyset) = \emptyset$
2. $f(Z) \subseteq Y$.
3. $f\left((F, A) \widetilde{\vee} (G, B) \right) = f(F, A) \widetilde{\vee} f(G, B)$
4. $f\left((F, A) \widetilde{\wedge} (G, B) \right) = f(F, A) \widetilde{\wedge} f(G, B)$
5. If $(F, A) \subseteq (G, B)$, Then $f(F, A) \subseteq f(G, B)$.

Proof For (1), (2) and (5) the proof is trivial , so we just give the proof of (3) and (4). For $\beta \in Z'$ and $y \in Y$, we want to prove that

$$\left(f(F, A) \widetilde{\vee} f(G, B) \right)(\beta)(y) = f(F, A)(\beta)(y) \widetilde{\vee} f(G, B)(\beta)(y)$$

For left hand side, consider $f\left((F, A) \widetilde{\vee} (G, B) \right)(\beta)(y) = f(H, A \cup B)(\beta)(y)$. Then

$$f(H, A \cup B)(\beta)(y) = \begin{cases} \vee_{x \in r^{-1}(y)}(\vee_\alpha H(\alpha)) & \text{if } r^{-1}(y) \text{ and } s^{-1}(\beta) \cap (A \cup B) \neq \emptyset, \\ 0 & \text{otherwise} \end{cases} \quad (1,1)$$

Such that $H(\alpha) = F(\alpha) \widetilde{\cup} G(\alpha)$ where $\widetilde{\cup}$ denotes intuitionistic fuzzy union. Considering only the non-trivial case, Then equation 1.1 becomes:

$$f(H, A \cup B)(\beta)(y) = \vee_{x \in r^{-1}(y)}(\vee(F(\alpha) \widetilde{\cup} G(\alpha))) \quad \textbf{(1,2)}$$

For right hand side and by using Definition 3.4, we have

$$\left(f(F, A) \widetilde{\vee} f(G, B) \right)(\beta)(y)$$
$$= f(F, A)(\beta)(y) \vee f(G, B)(\beta)(y)$$
$$= \left(\vee_{x \in r^{-1}(y)} \left(\vee_{\alpha \in s^{-1}(\beta) \cap A} F(\alpha) \right)(x) \right) \vee \left(\vee_{x \in r^{-1}(y)} \left(\vee_{\forall \alpha \in s^{-1}(\beta) \cap B} F(\alpha) \right)(x) \right)$$
$$= \vee_{x \in r^{-1}(y)} \quad \vee_{\alpha \in s^{-1}(\beta) \cap (A \cup B)} (F(\alpha) \vee G(\alpha))$$

$$=V_{x\in r^{-1}(y)}(V(F(\alpha) \, \tilde{\cup} \, G(\alpha)))(1,3)$$

From equation (1.1) and (1.3) we get (3)

4. For $\beta \in Z'$ and $y \in Y$, and using Definition 3.4, we have

$$f((F,A) \, \tilde{\wedge} \, (G,B))(\beta)(y)$$

$$=f(H, A \cup B)(\beta)(y)$$

$$=V_{x\in r^{-1}(y)}(V_{\alpha\in s^{-1}(\beta)\cap(A\cup B)} H(\alpha))(x)$$

$$=V_{x\in r^{-1}(y)}(V_{\alpha\in s^{-1}(\beta)\cap(A\cup B)} F(\alpha) \, \tilde{\cap} \, G(\alpha))(x)$$

$$=V_{x\in r^{-1}(y)}(V_{\alpha\in s^{-1}(\beta)\cap(A\cup B)} F(\alpha)(x) \, \tilde{\cap} \, G(\alpha)(x))$$

$$\subseteq \left(\bigvee_{x\in r^{-1}(y)} \left(\bigvee_{\alpha\in s^{-1}(\beta)\cap A} F(\alpha) \right) \right) \wedge \bigvee_{x\in r^{-1}(y)} \left(\bigvee_{\alpha\in s^{-1}(\beta)\cap B} G(\alpha) \right)$$

$$= f((F,A)(\beta)(y) \wedge (G,B)(\beta)(y))$$

$$=(f(F,A) \, \tilde{\wedge} \, f(G,B))(\beta)(y)$$

This gives (4)

Theorem 3.7. Let $f^{-1}: \widetilde{(U,Z)} \to \widetilde{(Y,Z')}$ be a mapping. Then for intuitionistic fuzzy soft expert sets (F, A) and (G, B) in the intuitionistic fuzzy soft expert class $\widetilde{(U,Z)}$.

$f^{-1}(\emptyset)=\emptyset$

$f^{-1}(X)\subseteq X$.

$f^{-1}((F,A)\tilde{V}(G,B))=f^{-1}(F,A)\tilde{V}f^{-1}(G,B)$

$f^{-1}((F,A) \, \tilde{\wedge} \, (G,B))=f^{-1}(F,A) \, \tilde{\wedge} \, f^{-1}(G,B)$

If $(F,A) \subseteq (G,B)$, Then $f^{-1}(F,A) \subseteq f^{-1}(G,B)$.

Proof. The proof is straightforward**.**

6. Conclusion

In this paper, we studied a mapping on intuitionistic fuzzy soft expert classes and its properties. We give some illustrative examples of mapping intuitionistic fuzzy soft expert set. We hope these fundamental results will help the researchers to enhance and promote the research on intuitionistic fuzzy soft set theory.

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