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Soft Closed Sets on Soft Bitopological Space

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Abstract - Soft set theory was introduced by Molodtsov as a general mathematical tool for dealing with problems that contain uncertainity. In this paper, on soft bitopological space, we define soft closed sets; soft α-closed, soft semi-closed, soft pre-closed, regular soft closed, soft g-closed and soft sg-closed. We also give related properties of these soft sets and compared their properties with each other.

Keywords - *Soft α-closed, soft semi-closed, soft pre-closed, regular soft closed, soft g-closed, soft sg-closed.*

1 Introduction

Soft set theory [17] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertainty. Recently, on soft sets, soft topological space has been studied increasingly. Shabir and Naz [26] defined the theory of soft topological space over an initial universe with a fixed set of parameters. Çağman et al. [7] introduced a topology on a soft set called "soft topology" and presented the foundations of the theory of soft topological spaces. Moreover, many authors studied soft topology and its applications (e.g. [2, 3, 13, 15, 16, 19, 24, 30]).

In 1963, Kelly [14] was defined bitopological space as an original and fundamental work by using two different topologies on a set. The notion of semi-open sets in bitopological spaces was initated by Ravi and Thivagar [21] in 2004. They also introduced the $(1,2)^*$ semi-generalised closed sets [22]. The concept of α-closed sets, semi-closed sets, g-closed sets and sg-closed sets have been introduced by many authors in bitopological space (e.g. [12, 21, 22, 23, 29]). Also, there are several theorical works (e.g. [5, 8, 9, 10, 11, 18]) and applications (e.g. [1, 4, 20, 22, 27]) on bitopological spaces.

Based on Çağman et al.[7]'s soft topology, Şenel and Çağman [28] define a bitopology on a soft set, called "soft bitopology". Then, they study its related properties and

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obtained some relations between soft topology and soft bitopology. In this paper, we define soft closed sets; soft α-closed, soft semi-closed, soft pre-closed, regular soft closed, soft g-closed and soft sg-closed on soft bitopological space. We also investigate related properties of these soft sets and compared their properties with each other.

2 Preliminary

In this section, we have presented the basic definitions and results of soft set theory, soft topology, bitopological space and soft bitopological space to use in the sequel.

Throughout this paper, U is an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subseteq E$.

Definition 2.1. *[6] A soft set F_A on the universe U is defined by the set of ordered pairs*

$$F_A = \{(x, f_A(x)) : x \in E\}$$

where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty, some may have nonempty intersection.

Note that the set of all soft sets over U will be denoted by $S(U)$.

Example 2.2. *[6] Suppose that there are six houses in the universe $U = \{h_l, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that $E = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of decision parameters. The x_i $(i = 1, 2, 3, 4, 5)$ stand for the parameters "expensive", "beautiful", "wooden", "cheap", and "in green surroundings" respectively.*

Consider the mapping f_A given by "houses (.)", where (.) is to be filled in by one of the parameters $x_i \in E$. For instance, $f_A(e_1)$ means "houses (expensive)", and its functional value is the set $\{h \in U : h$ is an expensive house$\}$.

Suppose that $A = \{x_1, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{h_2, h_4\}$, $f_A(x_3) = U$, and $f_A(x_4) = \{h_1, h_3, h_5\}$. Then we can view the soft set *F_A as consisting of the following* collection *of approximations,*

$$F_A = \{(x_1, \{h_2, h_4\}), (x_3, U), (x_4, \{h_1, h_3, h_5\})\}$$

Definition 2.3. *[6] Let $F_A \in S(U)$. Then,*

i. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty set, denoted by F_Φ.

ii. If $f_A(x) = U$ for all $x \in A$, then F_A is called A-universal soft set, denoted by $F_{\tilde{A}}$.

iii. If $A = E$, then the A-universal soft set is called universal soft set denoted by $F_{\tilde{E}}$.

Definition 2.4. *[6] Let $F_A, F_B \in S(U)$. Then,*

i. F_A is a soft subset of F_B, denoted by $F_A \tilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

ii. F_A and F_B are soft equal, denoted by $F_A = F_B$, if and only if $f_A(x) = f_B(x)$ for all $x \in E$.

Definition 2.5. *[6] Let $F_A, F_B \in S(U)$. Then, soft union $F_A \tilde{\cup} F_B$ and soft intersection $F_A \tilde{\cap} F_B$ of F_A and F_B are defined by the approximate functions, respectively,*

$$f_{A\tilde{\cup}B}(x) = f_A(x) \cup f_B(x), \quad f_{A\tilde{\cap}B}(x) = f_A(x) \cap f_B(x)$$

and the soft complement $F_A^{\tilde{c}}$ of F_A is defined by the approximate function

$$f_{A^{\tilde{c}}}(x) = f_A^c(x)$$

where $f_A^c(x)$ is complement of the set $f_A(x)$, that is, $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $(F_A^{\tilde{c}})^{\tilde{c}} = F_A$ and $F_\Phi^{\tilde{c}} = F_{\tilde{E}}$

Definition 2.6. *[7] Let $F_A \in S(U)$. Power soft set of F_A is defined by*

$$\tilde{P}(F_A) = \{F_{A_i} \tilde{\subseteq} F_A : i \in I\}$$

and its cardinality is defined by

$$|\tilde{P}(F_A)| = 2^{\sum_{x\in E} |f_A(x)|}$$

where $|f_A(x)|$ is cardinality of $f_A(x)$.

Example 2.7. *[7] Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then*

$$\begin{aligned}
F_{A_1} &= \{(x_1, \{u_1\})\},\\
F_{A_2} &= \{(x_1, \{u_2\})\},\\
F_{A_3} &= \{(x_1, \{u_1, u_2\})\},\\
F_{A_4} &= \{(x_2, \{u_2\})\},\\
F_{A_5} &= \{(x_2, \{u_3\})\},\\
F_{A_6} &= \{(x_2, \{u_2, u_3\})\},\\
F_{A_7} &= \{(x_1, \{u_1\}), (x_2, \{u_2\})\},\\
F_{A_8} &= \{(x_1, \{u_1\}), (x_2, \{u_3\})\},\\
F_{A_9} &= \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\},\\
F_{A_{10}} &= \{(x_1, \{u_2\}), (x_2, \{u_2\})\},\\
F_{A_{11}} &= \{(x_1, \{u_2\}), (x_2, \{u_3\})\},\\
F_{A_{12}} &= \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\},\\
F_{A_{13}} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\},\\
F_{A_{14}} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\},\\
F_{A_{15}} &= F_A,\\
F_{A_{16}} &= F_\Phi
\end{aligned}$$

are all soft subsets of F_A. So $|\tilde{P}(F_A)| = 2^4 = 16$.

Definition 2.8. *[7] Let $F_A \in S(U)$. A soft topology on F_A, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_A having following properties:*

i. F_Φ and F_A belong to $\tilde{\tau}$

ii. Union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

iii. Intersection of two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$

The pair $(F_A, \tilde{\tau})$ is called a soft topological space.

Example 2.9. *[7] In Example 2.7, $F_{A_2} = \{(x_1, \{u_2\})\}$, $F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$ and $F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$ are soft subsets of F_A. Hence, $\tilde{\tau}_1 = \{F_\Phi, F_A\}$, $\tilde{\tau}_2 = \tilde{P}(F_A)$, $\tilde{\tau}_3 = \{F_\Phi, F_A, F_{A_2}, F_{A_{11}}, F_{A_{13}}\}$ are soft topologies on F_A.*

Definition 2.10. *[7] Let $(F_A, \tilde{\tau})$ be a soft topological space. Then, every element of $\tilde{\tau}$ is called soft open set. Clearly, F_Φ and F_A are soft open sets.*

Definition 2.11. *[14] Let $X \neq \emptyset$, τ_1 and τ_2 be two different topologies on X. Then (X, τ_1, τ_2) is called a bitopological space. Throughout this paper (X, τ_1, τ_2) [or simply X] denote bitopological space on which no seperation axioms are assumed unless explicitly stated.*

Definition 2.12. *[14] A subset S of X is called $\tau_1\tau_2$-open if $S = H \cup K$ such that $H \in \tau_1$ and $K \in \tau_2$ and the complement of $\tau_1\tau_2$ open is $\tau_1\tau_2$ closed.*

Example 2.13. *[14] Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}\}$. The sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ are called $\tau_1\tau_2$ open and the sets in $\{\emptyset, X, \{b, c\}, \{a, c\}, \{c\}\}$ are called $\tau_1\tau_2$ closed.*

Definition 2.14. *[14] Let S be a subset of X. Then,*

i. The $\tau_1\tau_2$-closure of S, denoted by $\tau_1\tau_2 cl(S)$, is defined by

$$\bigcap\{F : S \subseteq F, F \ \ \textit{is a} \ \ \tau_1\tau_2\textit{-closed}\}$$

ii. The $\tau_1\tau_2$-interior of S, denoted by $\tau_1\tau_2 int(S)$, is defined by

$$\bigcup\{A : A \subseteq S, A \ \textit{is a} \ \tau_1\tau_2\textit{-open}\}$$

Definition 2.15. *[28] Let F_A be a nonempty soft set on the universe U, $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be two different soft topologies on F_A. Then, $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ is called a soft bitopological space.*

Definition 2.16. *[28] Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space and $F_B \tilde{\subset} F_A$. Then, F_B is called $\tilde{\tau}_1\tilde{\tau}_2$-soft open if $F_B = F_C \tilde{\cup} F_D$, where $F_C \in \tilde{\tau}_1$ and $F_D \in \tilde{\tau}_2$.*
The complement of $\tilde{\tau}_1\tilde{\tau}_2$-soft open set is called $\tilde{\tau}_1\tilde{\tau}_2$-soft closed.

Definition 2.17. *[28] Let F_B be a soft subset of F_A. Then,*

i. $\tilde{\tau}_1\tilde{\tau}_2$-soft closure of F_B, denoted by $\tilde{\tau}_1\tilde{\tau}_2 cl(F_B)$, is defined by

$$\tilde{\tau}_1\tilde{\tau}_2 cl(F_B) = \tilde{\bigcap}\{F_K : F_B \tilde{\subseteq} F_K, F_K \ \textit{is} \ \tilde{\tau}_1\tilde{\tau}_2\textit{-soft closed}\}$$

ii. The $\tilde{\tau}_1\tilde{\tau}_2$-soft interior of F_B, denoted by $\tilde{\tau}_1\tilde{\tau}_2 int(F_B)$, is defined by

$$\tilde{\tau}_1\tilde{\tau}_2 int(F_B) = \tilde{\bigcup}\{F_C : F_C \tilde{\subseteq} F_B, F_C \ \textit{is} \ \tilde{\tau}_1\tilde{\tau}_2\textit{-soft open}\}$$

Note that $\tilde{\tau}_1\tilde{\tau}_2 int(F_B)$ is the biggest $\tilde{\tau}_1\tilde{\tau}_2$-soft open set that contained by F_B and $\tilde{\tau}_1\tilde{\tau}_2 cl(F_B)$ is the smallest $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set that containing F_B.

Example 2.18. *[28] Refer example 2.7 $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_2}\}$ and $\tilde{\tau}_2 = \{F_\Phi, F_A, F_{A_1}, F_{A_4}\}$.*
The sets in $\{F_\Phi, F_A, F_{A_2}, F_{A_1}, F_{A_4}, F_{A_3}\}$ are called $\tilde{\tau}_1\tilde{\tau}_2$-soft open and the sets in $\{F_\Phi, F_A, F_{A_1}, F_{A_2}, F_{A_5}$ are called $\tilde{\tau}_1\tilde{\tau}_2$-soft closed.

3 Soft semi-generalised closed sets

In this section, we introduce α-closed, semi-closed, pre-closed, regular closed, g-closed and sg-closed sets in a soft bitopological space $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$.

Definition 3.1. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space and $F_B \tilde{\subseteq} F_A$. Then,*

i. If $F_B \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(F_B)))$ then F_B is called soft α-open, denoted by $\widetilde{(1,2)}\alpha$-open.

ii. If $F_B \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(F_B))$ then F_B is called soft semi-open, denoted by $\widetilde{(1,2)}$-semi-open.

iii. If $F_B \tilde{\subseteq} \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B))$, then F_B is called soft pre-open, denoted by $\widetilde{(1,2)}$-pre-open.

iv. If $F_B = \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B))$, then F_B is called regular soft-open, denoted by regular $\widetilde{(1,2)}$-open.

Definition 3.2. *Let $(F_A, \tilde{\tau}_1, \tilde{\tau}_2)$ be a soft bitopological space and $F_B \tilde{\subseteq} F_A$. Then,*

i. If $\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B))) \tilde{\subseteq} F_B$, then F_B is called soft α-closed, denoted by $\widetilde{(1,2)}\alpha$-closed.

ii. If $\tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B)) \tilde{\subseteq} F_B$ then F_B is called soft semi-closed, denoted by $\widetilde{(1,2)}$-semi-closed.

iii. If $\tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(F_B)) \tilde{\subseteq} F_B$, then F_B is called soft pre-closed, denoted by $\widetilde{(1,2)}$-pre-closed.

iv. If $F_B = \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(F_B))$, then F_B is called regular soft-closed, denoted by regular $\widetilde{(1,2)}$-closed.

Note that the families of all $\widetilde{(1,2)}\alpha$-open, $\widetilde{(1,2)}$-semi-open, $\widetilde{(1,2)}$-pre-open and regular $\widetilde{(1,2)}$-open sets of F_A are denoted by $\widetilde{(1,2)}\alpha O(F_A)$, $\widetilde{(1,2)}SO(F_A)$, $\widetilde{(1,2)}PO(F_A)$ and $\widetilde{(1,2)}RO(F_A)$ respectively. The family of all regular $\widetilde{(1,2)}$-closed sets of F_A is denoted by $\widetilde{(1,2)}RC(F_A)$.

Definition 3.3. *Let F_B be a soft subset F_A. Then,*

i. $\widetilde{(1,2)}$-semi-closure of F_B, denoted by $\widetilde{(1,2)}scl(F_B)$, is defined by

$$\widetilde{(1,2)}scl(F_B) = \tilde{\bigcap}\{F_K : F_B \tilde{\subseteq} F_K, F_K \textit{ is } \widetilde{(1,2)}\textit{-semi closed}\}$$

ii. $\widetilde{(1,2)}$-semi-interior of F_B, denoted by $\widetilde{(1,2)}sint(F_B)$, is defined by

$$\widetilde{(1,2)}sint(F_B) = \tilde{\bigcup}\{F_C : F_C \tilde{\subseteq} F_B, F_C \textit{ is } \widetilde{(1,2)}\textit{-semi open}\}$$

Theorem 3.4. *Let F_A, F_B be two soft sets and $F_B \tilde{\subseteq} F_A$. Then, F_B is a $\widetilde{(1,2)}$-semi-closed if and only if $\widetilde{(1,2)}scl(F_B) = F_B$.*

Proof. The proof is trivial.

Theorem 3.5. *Let F_A, F_B be two soft sets and $F_B \tilde{\subseteq} F_A$. Then,*

i. $\widetilde{(1,2)}scl(F_B) = F_B \tilde{\cup} \tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B))$,

ii. $\widetilde{(1,2)}sint(F_B) = F_B \tilde{\cap} \tilde{\tau}_1\tilde{\tau}_2 cl(\tilde{\tau}_1\tilde{\tau}_2 int(F_B))$

Proof. Proof is clear.

Definition 3.6. *Let F_A, F_B be two soft sets and $F_B \tilde{\subseteq} F_A$. Then, F_B is called a $\widetilde{(1,2)}$ generalized closed set, denoted by $\widetilde{(1,2)}$-g-closed, if and only if $\tilde{\tau}_1\tilde{\tau}_2 cl(F_B) \tilde{\subseteq} F_C$ whenever $F_B \tilde{\subseteq} F_C$ and F_C is $\tilde{\tau}_1\tilde{\tau}_2$ soft open.*

Remark 3.7. *The intersection of two $\widetilde{(1,2)}$-g-closed set is generally not a $\widetilde{(1,2)}$-g-closed set as seen in the following example.*

Example 3.8. *Consider Example 2.7, $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$ and $F_A = \{F_{A_2}, F_{A_3}, F_{A_5}, \{(x_1, \{u_1, u_3\})\}, \{(x_1, \{u_2, u_3\})\}\}$. Let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ soft topologies be $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_2}\}$ and $\tilde{\tau}_2 = \{F_\Phi, F_A\}$. Where, the set of $\{F_\Phi, F_A, F_{A_2}\}$ $\tilde{\tau}_1\tilde{\tau}_2$-soft open and the set of $\{F_\Phi, F_A, \{(x_1, \{u_1, u_3\})\}\}$ $\tilde{\tau}_1\tilde{\tau}_2$-soft closed sets. Clearly F_{A_3} and $\{(x_1, \{u_2, u_3\})\}$ are $\widetilde{(1,2)}$-g-closed sets but $F_{A_3} \tilde{\cap} \{(x_1, \{u_2, u_3\})\} = F_{A_2}$ is not $\widetilde{(1,2)}$-g-closed since $\tilde{\tau}_1\tilde{\tau}_2 cl(F_{A_2}) = F_A \tilde{\nsubseteq} (F_{A_2})$ whenever F_{A_2} is $\tilde{\tau}_1\tilde{\tau}_2$ soft open.*

Definition 3.9. *Let F_A, F_B be two soft sets and $F_B \tilde{\subseteq} F_A$. Then, F_B is called a $\widetilde{(1,2)}$ semi-generalized closed set, denoted by $\widetilde{(1,2)}$-sg-closed, if and only if $\tilde{\tau}_1\tilde{\tau}_2 scl(F_B) \tilde{\subseteq} F_C$ whenever $F_B \tilde{\subseteq} F_C$ and F_C is $\widetilde{(1,2)}$ semi-open set.*

Remark 3.10. *The following example shows that the union of two $\widetilde{(1,2)}$-sg-closed set is not, in general, $\widetilde{(1,2)}$-sg-closed.*

Example 3.11. *Refer example 2.7, $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$ and $F_A = \{F_{A_1}, F_{A_2}, F_{A_3}, F_{A_5}, F_{A_6}\}$. Let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ soft topologies be $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_1}, F_{A_2}, F_{A_3}\}$ and $\tilde{\tau}_2 = \{F_\Phi, F_A\}$. Clearly F_{A_1} and F_{A_2} are $\widetilde{(1,2)}$-sg-closed sets. But $F_{A_1} \tilde{\cup} F_{A_2} = F_{A_3}$ is not $\widetilde{(1,2)}$-sg-closed since $\tilde{\tau}_1\tilde{\tau}_2 scl(F_{A_3}) = F_A \tilde{\nsubseteq} F_{A_3}$ whenever $F_{A_3} \tilde{\subseteq} F_{A_3}$ and $F_{A_3} \in \widetilde{(1,2)} - SO(F_A)$.*

Theorem 3.12. *Let F_A, F_B be two soft sets and $F_B \tilde{\subseteq} F_A$. Then, the following conditions hold:*

i. The complement of $\widetilde{(1,2)}$-g-closed set is $\widetilde{(1,2)}$ g-open.

ii. The complement of $\widetilde{(1,2)}$-semi-generalized closed set is $\widetilde{(1,2)}$-semi-generalized open.

iii. The intersection of two $\widetilde{(1,2)}$-sg-closed set is $\widetilde{(1,2)}$-sg-closed.

Proof. It can be proved clearly from Definition 3.6.

4 Comparison of soft closed sets

In this section, we study the relation between these classes of soft sets as in the following diagram:

$$
\begin{tikzcd}
\tilde{\tau}_1\tilde{\tau}_2\text{-closed} \arrow[r, shift left] \arrow[d, shift right] & \widetilde{(1,2)}\text{-g-closed} \arrow[l, shift left, "/" marking] \arrow[d, shift right, "/" marking] \\
\widetilde{(1,2)}\text{-semi-closed} \arrow[u, shift right, "/" marking] \arrow[r, shift left] & \widetilde{(1,2)}\text{-sg-closed} \arrow[l, shift left, "/" marking] \arrow[u, shift right, "/" marking]
\end{tikzcd}
$$

Theorem 4.1. *Every $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set is $\widetilde{(1,2)}$-semi-closed.*

Proof: Let F_B be $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set in F_A. Thus, $\tilde{\tau}_1\tilde{\tau}_2 cl(F_B) = F_B$. Since $\tilde{\tau}_1\tilde{\tau}_2 int(F_B) \tilde{\subseteq} F_B$, $\tilde{\tau}_1\tilde{\tau}_2 int(\tilde{\tau}_1\tilde{\tau}_2 cl(F_B)) \tilde{\subseteq} F_B$. Then, F_B is $\widetilde{(1,2)}$-semi-closed.

Remark 4.2. *The following example shows that $\widetilde{(1,2)}$-semi-closed set need not be $\tilde{\tau}_1\tilde{\tau}_2$-soft closed.*

Example 4.3. *Refer example 2.7, $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$ and $F_A = \{F_{A_1}, F_{A_3}, F_{A_5}, \{(x_2, \{u_1\})\}\}$. Let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ soft topologies be $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_1}\}$ and $\tilde{\tau}_2 = \{F_\Phi, F_A\}$. Clearly F_{A_2} is $\widetilde{(1,2)}$-semi-closed set but not $\tilde{\tau}_1\tilde{\tau}_2$-soft closed.*

Theorem 4.4. *Every $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set is $\widetilde{(1,2)}$-g-closed.*

Proof. Let F_B be $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set in F_A. Therefore $\tilde{\tau}_1\tilde{\tau}_2 cl(F_B) = F_B \tilde{\subseteq} F_A$ whenever $F_B \tilde{\subseteq} F_A$ and F_A is $\tilde{\tau}_1\tilde{\tau}_2$-soft open. It implies F_B is $\widetilde{(1,2)}$-g-closed.

Remark 4.5. *$\widetilde{(1,2)}$-g-closed set is not, in general, $\tilde{\tau}_1\tilde{\tau}_2$-soft closed as is illustrated in the following example.*

Example 4.6. *Refer example 2.7, $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$, $A = \{x_1, x_2\} \tilde{\subseteq} E$ and $F_A = \{F_{A_1}, \{(x_2, \{u_1, u_2\})\}, \{(x_1, \{u_2, u_3\})\}\}$. Let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ soft topologies be $\tilde{\tau}_1 = \{F_\Phi, F_A, \{(x_2, \{u_1\})\}\}$ and $\tilde{\tau}_2 = \{F_\Phi, F_A\}$. Clearly $\{(x_2, \{u_1, u_2\})\}$ is $\widetilde{(1,2)}$-g-closed set but not $\tilde{\tau}_1\tilde{\tau}_2$-soft closed.*

Theorem 4.7. *Every $\widetilde{(1,2)}$-semi-closed set is $\widetilde{(1,2)}$-sg-closed.*

Proof. Since F_B is $\tilde{\tau}_1\tilde{\tau}_2$-soft closed set in F_A, $\widetilde{(1,2)}scl(F_B) = F_B \tilde{\subseteq} F_A$ whenever $F_B \tilde{\subseteq} F_A$ and $F_A \in \widetilde{(1,2)}SO(F_A)$. It implies that F_B is $\widetilde{(1,2)}$-sg-closed.

Remark 4.8. *The converse of Theorem 4.7 is false as seen from the following example.*

Example 4.9. *Refer example 2.7,* $U = \{u_1, u_2, u_3\}$*,* $E = \{x_1, x_2\}$*,* $A = \{x_1, x_2\} \tilde{\subseteq} E$ *and* $F_A = \{F_{A_1}, F_{A_3}, F_{A_5}, \{(x_1, \{u_2, u_3\})\}\}$*. Let* $\tilde{\tau}_1$ *and* $\tilde{\tau}_2$ *soft topologies be* $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_3}\}$ *and* $\tilde{\tau}_2 = \{F_\Phi, F_A, \{(x_1, \{u_2, u_3\})\}\}$*. Clearly* $\{(x_1, \{u_1, u_3\})\}$ *is* $\widetilde{(1,2)}$*-sg-closed set but not* $\widetilde{(1,2)}$*-semi-closed.*

Example 4.10. *Refer example 2.7,* $U = \{u_1, u_2, u_3\}$*,* $E = \{x_1, x_2\}$*,* $A = \{x_1, x_2\} \tilde{\subseteq} E$ *and* $F_A = \{F_{A_2}, F_{A_3}, F_{A_5}, \{(x_2, \{u_1\})\}\}$*. Let* $\tilde{\tau}_1$ *and* $\tilde{\tau}_2$ *soft topologies be* $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_1}\}$ *and* $\tilde{\tau}_2 = \{F_\Phi, F_A\}$*. Clearly* F_{A_3} *is* $\widetilde{(1,2)}$*-g-closed set but not* $\widetilde{(1,2)}$*-sg-closed since* $\widetilde{(1,2)}scl(F_{A_3}) = F_A \tilde{\nsubseteq} F_{A_3}$ *whenever* $F_{A_3} \tilde{\subseteq} F_{A_3}$ *and* $F_{A_3} \in \widetilde{(1,2)} - SO(F_A)$*.*

Example 4.11. *Refer example 2.7,* $U = \{u_1, u_2, u_3\}$*,* $E = \{x_1, x_2\}$*,* $A = \{x_1, x_2\} \tilde{\subseteq} E$ *and* $F_A = \{F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, \{(x_1, \{u_3\})\}, \{(x_2, \{u_1, u_2\})\}\}$*. Let* $\tilde{\tau}_1$ *and* $\tilde{\tau}_2$ *soft topologies be* $\tilde{\tau}_1 = \{F_\Phi, F_A, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, \{(x_1, \{u_3\})\}\}$ *and* $\tilde{\tau}_2 = \{F_\Phi, F_A\}$*. Clearly* F_{A_1} *is* $\widetilde{(1,2)}$*-sg-closed set but it is not* $\widetilde{(1,2)}$*-g-closed since* $\tilde{\tau}_1\tilde{\tau}_2 cl(F_{A_1}) = \{(x_1, \{u_1, u_3\})\} \tilde{\nsubseteq} F_{A_3}$ *whenever* $F_{A_1} \tilde{\subseteq} F_{A_3}$ *and* F_{A_3} *is* $\tilde{\tau}_1\tilde{\tau}_2$*-soft open.*

Remark 4.12. *Examples 4.10 and 4.11 show that* $\widetilde{(1,2)}$*-g-closed and* $\widetilde{(1,2)}$*-sg-closed sets are, in general, independent.*

5 Conclusion

In this work, soft closed sets in the soft bitopological space are defined and developed. We then presented their properties and compared their relations with each other. In the future, using these sets, various classes of mappings on soft bitopological space can be studied.

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