ISSN: **1304-7981** Number: **3**, Year: **2013**, Pages: **60-71**. http://jnrs.gop.edu.tr

Gaziosmanpasa University
Graduate School of Natural and Applied Sciences
Journal of New Results in Science

Received: 13.09.2013 *Editors-in-Chief*: *Naim Çağman*
Accepted: 18.11.2013 *Area Editor*: *Oktay Muhtaroğlu*

Slightly $\pi g\gamma$-Continuous Functions

O. Ravi[a,1] (siingam@yahoo.com)
S. Margaret parimalam[b] (margaret_parimalam@yahoo.com)
S. Murugesan[c] (satturmuruges1@gmail.com)
A. Pandi[d] (pandi2085@yahoo.com)

[a]*Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.*
[b]*Department of Mathematics, CSI College of Arts and Science for Women, Madurai, Tamil Nadu, India.*
[c]*Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur-626 203, Tamil Nadu, India.*
[d]*Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.*

Abstract − A new class of functions, called slightly π-generalized γ-continuous functions is introduced. Basic properties of slightly π-generalized γ-continuous functions are studied. The class of slightly π-generalized γ-continuous functions properly includes the class of slightly gγ-continuous functions and $\pi g\gamma$-continuous functions. Also, by using slightly π-generalized γ-continuous functions, some properties of domain/range of functions are characterized.

Keywords − *Slight $g\gamma$-continuity, slight $\pi g\gamma$-continuity, $\pi g\gamma$-continuity, $\pi g\gamma$-closed set, $\pi G\gamma O$-connectedness*

1 Introduction and Preliminaries

In the literature, γ-open sets are being studied by many authors. Andrijevic [6] introduced b-open sets in 1996 and Dontchev and Przemski [11] studied sp-open sets in that year and in 1997, El-Atik [16] introduced γ-open sets. Fujimoto et. al. [17] introduced γ-continuous functions. Slightly γ-continuous functions were introduced by Ekici and Caldas [15] in 2004. Al-Omari and Noorani [5] introduced the notion of $g\gamma$-continuous functions and investigated some of their basic properties. Ravi et.al. [23] introduced slightly $g\gamma$-continuous functions. Sreeja and Janki [25] introduced $\pi g\gamma$-closed sets and $\pi g\gamma$-continuous functions.

[1]Corresponding Author

In this paper, we define slightly π-generalized γ-continuous functions and show that the class of slightly π-generalized γ-continuous functions properly includes the class of slightly gγ-continuous functions and $\pi g\gamma$-continuous functions. Secondly we obtain some new results on πgγ-closed sets and investigate basic properties of slightly π-generalized γ-continuous functions concerning composition and restriction. Finally, we study the behaviours of some separation axioms and related properties, πGγO-compactness and πGγO-connectedness under slightly π-generalized γ-continuous functions. Relationships between slightly π-generalized γ-continuous functions and πGγO-connected spaces are investigated. In particular, it is shown that slightly π-generalized γ-continuous image of πGγO-connected spaces is connected.

Throughout this paper, (X, τ) and (Y, σ) (or X and Y) represents a topological space on which no separation axioms are assumed, unless otherwise mentioned. The closure and interior of A $\subseteq$ X will be denoted by cl(A) and int(A) respectively.

Definition 1.1.

1. *A subset A of a space X is called regular open [28] if A = int(cl(A)).*

2. *A subset A of a space X is called π-open [29] if the finite union of regular open sets. The complement of π-open set is called π-closed.*

3. *A subset A of a space X is called β-open [1] if A $\subseteq$ cl(int(cl(A))).*

4. *A subset A of a space X is called b-open [6] or sp-open [11] or γ-open [16] if A $\subseteq$ cl(int(A))$\cup$int(cl(A)) [16].*

   The complement of γ-open set is γ-closed [16].

   The intersection of all γ-closed sets containing A is called the γ-closure of A and is denoted by γcl(A) [16].

   A is said to be γ-clopen [16] if it is γ-open and γ-closed.

   The largest γ-open set contained in A (denoted by γint(A)) is called the γ-interior [16] of A.

5. *A subset A of a space X is said to be generalized closed [21] (briefly g-closed) if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is open in X.*

   The complement of g-closed set is called g-open.

6. *A subset A of a space X is said to be π-generalized closed [12] (briefly πg-closed) if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is π-open in X.*

   The complement of πg-closed set is πg-open.

7. *A subset A of a space X is said to be gγ-closed [13] if γcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is open in X.*

8. *A subset A of a space X is said to be gγ-open [13] if F $\subseteq$ γint(A) whenever F $\subseteq$ A and F is closed in X.*

   Also it is a complement of gγ-closed set.

9. *A subset A of a space X is said to be $\pi g\gamma$-closed [25] if $\gamma cl(A) \subseteq U$, whenever $A \subseteq U$ and U is π-open in X.*

10. *A subset A of a space X is said to be $\pi g\gamma$-open [25] if $F \subseteq \gamma int(A)$ whenever $F \subseteq A$ and F is π-closed in X.*

    Also it is a complement of $\pi g\gamma$-closed set.

    If A is both $\pi g\gamma$-closed and $\pi g\gamma$-open, then it is said to be $\pi g\gamma$-clopen.

In this paper, the family of all open (resp. πg-open, $\pi g\gamma$-open, clopen) sets of a space X is denoted by O(X) (resp. $\pi GO(X)$, $\pi G\gamma O(X)$, CO(X)) and the family of $\pi g\gamma$-open (resp. clopen) sets of X containing x is denoted by $\pi G\gamma O(X, x)$ (resp. CO(X, x)).

Definition 1.2. *A function $f : X \to Y$ is called:*

1. *π-irresolute [14] if $f^{-1}(F)$ is π-closed in X for every π-closed set F of Y .*
2. *$\pi g\gamma$-continuous [25] if $f^{-1}(F)$ is $\pi g\gamma$-closed in X for every closed set F of Y .*
3. *slightly continuous [24] (resp. slightly $g\gamma$-continuous [23]) if for each $x \in X$ and each clopen set V of Y containing f(x), there exists an open (resp. $g\gamma$-open) set U such that $f(U) \subseteq V$.*
4. *$\pi g\gamma$-irresolute [25] if $f^{-1}(F)$ is $\pi g\gamma$-closed in X for every $\pi g\gamma$-closed set F of Y.*
5. *strongly γ-closed [13] if the image of each γ-closed set in X is γ-closed in Y.*
6. *$\pi g\gamma$-homeomorphism if it is bijective, $\pi g\gamma$-irresolute and its inverse f^{-1} is $\pi g\gamma$-irresolute.*

Definition 1.3. *[23] A function $f : X \to Y$ is called slightly $g\gamma$-continuous if the inverse image of every clopen set in Y is $g\gamma$-open (resp. $g\gamma$-closed, $g\gamma$-clopen) in X.*

Proposition 1.4. *[6] The intersection of an open and a γ-open set is a γ-open set.*

Lemma 1.5. *[25] A subset A of a space X is $\pi g\gamma$-open if and only if $F \subseteq \gamma int(A)$ whenever F is π-closed and $F \subseteq A$.*

Lemma 1.6. *[10] In any space (X, τ), the intersection of an open set and πg-open set in X is a πg-open set in X.*

2 Slightly π-generalized γ-continuous functions

Definition 2.1. *A function $f : X \to Y$ is called slightly π-generalized γ-continuous (briefly slightly $\pi g\gamma$-continuous) if the inverse image of every clopen set in Y is $\pi g\gamma$-open in X.*

The proof of the following Theorem is straightforward and hence omitted.

Theorem 2.2. *For a function $f : X \to Y$, the following statements are equivalent:*

1. *f is slightly $\pi g\gamma$-continuous.*

2. *Inverse image of every clopen subset of Y is $\pi g\gamma$-open in X.*

3. *Inverse image of every clopen subset of Y is $\pi g\gamma$-closed in X.*

4. *Inverse image of every clopen subset of Y is $\pi g\gamma$-clopen in X.*

Obviously, slight $g\gamma$-continuity implies slight $\pi g\gamma$-continuity and $\pi g\gamma$-continuity implies slight $\pi g\gamma$-continuity. The following Examples show that the implications are not reversible.

Example 2.3. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Then the function $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = a$, $f(b) = b$ and $f(c) = c$ is slightly $\pi g\gamma$-continuous but not slightly $g\gamma$-continuous.*

Example 2.4. *Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is slightly $\pi g\gamma$-continuous but not $\pi g\gamma$-continuous.*

A space is called locally discrete if every open subset is closed [9]. Also, a space is called as $g\gamma$-$T_{1/2}$ if every $\pi g\gamma$-closed subset of it is $g\gamma$-closed.

The next two Theorems are immediate of the definitions of a locally discrete and $g\gamma$-$T_{1/2}$ space.

Theorem 2.5. *If $f : X \to Y$ is slightly $\pi g\gamma$-continuous and Y is locally discrete, then f is $\pi g\gamma$-continuous.*

Theorem 2.6. *If $f : X \to Y$ is slightly $\pi g\gamma$-continuous and X is $g\gamma$-$T_{1/2}$ space, then f is slightly $g\gamma$-continuous.*

Definition 2.7. *[27] A topological space X is called hyperconnected if every open set is dense.*

Remark 2.8. *The following Example shows that slightly $\pi g\gamma$-continuous surjection do not necessarily preserve hyperconnectedness.*

Example 2.9. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is slightly $\pi g\gamma$-continuous surjective. Also (X, τ) is hyperconnected. But (X, σ) is not hyperconnected.*

3 Basic properties of slightly $\pi g\gamma$-continuous functions

Definition 3.1. *The intersection of all $\pi g\gamma$-closed sets containing a set A is called $\pi g\gamma$-closure of A and is denoted by $\pi g\gamma cl(A)$ [3].*

Remark 3.2. *It is obvious that $\pi g\gamma cl(A)$ is $\pi g\gamma$-closed and A is $\pi g\gamma$-closed if and only if $\pi g\gamma cl(A) = A$.*

Lemma 3.3. *Let A be a $\pi g\gamma$-open set and B be any set in X. If $\cap B = \emptyset$, then $A \cap \pi g\gamma cl(B) = \emptyset$.*

Proof: Suppose that A ∩ πgγcl(B) ≠ ∅ and x ∈ A ∩ πgγcl(B). Then x ∈ A and x ∈ πgγcl(B) and from the definition of πgγcl(B), A∩B ≠ ∅. This is contrary to hypothesis.

Proposition 3.4. *Let A be πgγ-open in X and B an open in X. Then A ∩ B is πgγ-open in X.*

Proof: Let F be any π-closed subset of X such that F ⊆ A∩B ⊆ A. By Lemma 1.5 F ⊆ γint(A) = ∪{G ⊆ X : G is γ-open in X and G ⊆ A}. Then F ⊆ (∪G)∩B = ∪(G∩B). Since G∩B is a γ-open set in X and G∩B ⊆ A∩B for each γ-open set G ⊆ A, F ⊆ ∪(A∩B) ⊆ γint(A∩B) and by Lemma 1.5 A∩B is πgγ-open in X.

For a subset A of space X, the π-kernel of A [4], denoted by π-ker(A), is the intersection of all π-open supersets of A.

Proposition 3.5. *A subset A of X is πgγ-closed if and only if γcl(A) ⊆ π-ker(A).*

Proof: Since A is πgγ-closed, γcl(A) ⊆ U for any π-open set U with A ⊆ U and hence γcl(A) ⊆ π-ker(A). Conversely, let U be any π-open set such that A ⊆ U. By hypothesis, γcl(A) ⊆ π-ker(A) ⊆ U and hence A is πgγ-closed.

The union of two πgγ-closed sets is generally not a πgγ-closed set and the intersection of two πgγ-open sets is generally not a πgγ-open set.

Example 3.6. *Let X = {a, b, c} and τ = {∅, {b}, {c}, {b, c}, X}. Then A = {a, b} and B = {a, c} are πgγ-open sets but their intersection A∩B = {a} is not a πgγ-open set.*

Example 3.7. *Let X = {a, b, c} and τ = {∅, {b}, {c}, {b, c}, X}. Then A = {b} and B = {c} are πgγ-closed sets but their union A∪B = {b, c} is not a πgγ-closed set.*

Proposition 3.8. *Let f : (X, τ) → (Y, σ) be a function. If f is slightly πgγ-continuous, then for each point x ∈ X and each clopen set V containing f(x), there exists a πgγ-open set U containing x such that f(U) ⊆ V.*

Proof: Let x ∈ X and V be a clopen set such that f(x) ∈ V. Since f is slightly πgγ-continuous, $f^{-1}(V)$ is πgγ-open set in X. If we put U = $f^{-1}(V)$, we have x ∈ U and f(U) ⊆ V.

Let (X, τ) be a topological space. The quasi-topology on X is the topology having as base all clopen subsets of (X, τ). The open (resp. closed) subsets of the quasi-topology are said to be quasi-open (resp. quasi-closed). A point x of a space X is said to be quasi closure of a subset A of X, denoted by cl_q A, if A ∩ U ≠ ∅ for every clopen set U containing x. A subset A is said to be quasi closed if and only if A = cl_qA [26]. If the closure of A in topological space coincides with πgγcl(A), then it is denoted by (X, μ).

Proposition 3.9. *Let f : (X, τ) → (Y, σ) be a function. Then the following are equivalent:*

1. *For each point x ∈ X and each clopen set V containing f(x), there exists a πgγ-open set U containing x such that f(U) ⊆ V.*

2. *For every subset A of X, f($\pi g\gamma cl$(A)) $\subseteq$ cl_q(f(A)).*

3. *The map f : (X, μ) $\rightarrow$ (Y, σ) is slightly continuous.*

Proof: (1) $\Rightarrow$ (2). Let y $\in$ f(πgγcl(A)) and V be any clopen nbd of y. Then there exists a point x $\in$ X and a πgγ-open set U containing x such that f(x) = y, x $\in$ πgγcl(A) and f(U) $\subseteq$ V. Since x $\in$ πgγcl(A), U $\cap$ A $\neq \emptyset$ holds and hence V $\cap$ f(A)$\neq \emptyset$. Therefore we have y = f(x) $\in$ cl_q(f(A)).

(2) $\Rightarrow$ (1). Let x $\in$ X and V be a clopen set with f(x) $\in$ V. Let A = f^{-1}(Y\V), then x $\notin$ A. Since f(πgγcl(A)) $\subseteq$ cl_q(f(A)) $\subseteq$ cl_q(Y\V) = Y\V, it is shown that πgγcl(A) = A. Then since x $\notin$ πgγcl(A), there exists a πgγ-open set U containing x such that U $\cap$ A = $\emptyset$ and hence f(U) $\subseteq$ f(X\A) $\subseteq$ V.

(2) $\Rightarrow$ (3). Suppose that (2) holds and let V be any clopen subset of Y. Since f(πgγcl(f^{-1}(V))) $\subseteq$$cl_q$(f($f^{-1}$(V))) $\subseteq$$cl_q$(V) = V, it is shown that πgγcl(f^{-1}(V)) = f^{-1}(V) and hence we have f^{-1}(V) is πgγ-closed in (X, τ) and hence f^{-1}(V) is closed in (X, μ).

(3) $\Rightarrow$ (2). Let y $\in$ f(πgγcl(A)) and V be any clopen nbd of y. Then there exists a point x $\in$ X such that f(x) = y and x $\in$ πgγcl(A). Since f is slightly continuous, f^{-1}(V) is open in (X, μ) and so πgγ-open set containing x. Since x $\in$ πgγcl(A), f^{-1}(V) $\cap$ A $\neq$ $\emptyset$ holds and hence V $\cap$ f(A) $\neq \emptyset$. Therefore, we have y = f(x) $\in$ cl_q(f(A)).

Now we investigate some basic properties of slightly πgγ-continuous functions concerning composition and restriction. The proofs of the first three results are straightforward and hence omitted.

Theorem 3.10. *If f : X $\rightarrow$ Y is $\pi g\gamma$-irresolute and g : Y $\rightarrow$ Z is slightly $\pi g\gamma$-continuous, then g o f : X $\rightarrow$ Z is slightly $\pi g\gamma$-continuous.*

Theorem 3.11. *If f : X $\rightarrow$ Y is slightly $\pi g\gamma$-continuous and g : Y $\rightarrow$ Z is continuous, then g o f : X $\rightarrow$ Z is slightly $\pi g\gamma$-continuous.*

Corollary 3.12. *Let $\{X_i : i \in I\}$ be any family of topological spaces. If f : X $\rightarrow$ ΠX_i is slightly $\pi g\gamma$-continuous mapping, then P_i o f : X $\rightarrow$ X_i is slightly $\pi g\gamma$-continuous for each $i \in I$, where P_i is the projection of ΠX_i onto X_i.*

Lemma 3.13. *Let f : X $\rightarrow$ Y be bijective, π-irresolute and strongly γ-closed. Then for every $\pi g\gamma$-closed set A of X, f(A) is $\pi g\gamma$-closed in Y.*

Proof: Let A be any πgγ-closed set of X and V a π-open set of Y containing f(A). Since f^{-1}(V) is a π-open set of X containing A, γcl(A) $\subseteq$ f^{-1}(V) and hence f(γcl(A)) $\subseteq$ V. Since f is strongly γ-closed and γcl(A) is γ-closed in X, f(γcl(A)) is γ-closed in Y. Since γcl(f(A)) $\subseteq$ γcl(f(γcl(A))) $\subseteq$ V, γcl(f(A)) $\subseteq$ V. Therefore f(A) is πgγ-closed in Y.

Theorem 3.14. *Let f : X $\rightarrow$ Y and g : Y $\rightarrow$ Z be functions. If f is bijective, π-irresolute and strongly γ-closed and if g o f : X $\rightarrow$ Z is slightly $\pi g\gamma$-continuous, then g is slightly $\pi g\gamma$-continuous.*

Proof: Let V be a clopen subset of Z. Then (g o f)$^{-1}$(V) = f^{-1}(g^{-1}(V)) is πgγ-closed in X. Then by the above Lemma, g^{-1}(V) = f(f^{-1}(g^{-1}(V))) is πgγ-closed in Y.

Combining Theorems 3.10 and 3.14, we obtain the following result.

Corollary 3.15. *Let $f : X \to Y$ be a bijective $\pi g\gamma$-homeomorphism and let $g : Y \to Z$ be a function. Then $g\ o\ f : X \to Z$ is slightly $\pi g\gamma$-continuous if and only if g is slightly $\pi g\gamma$-continuous.*

Lemma 3.16. *[25] Let $A \subseteq X$. If A is open, then $\pi O(A, \tau/A) = V \cap A$ such that $V \in \pi O(X, \tau)$.*

Let B $\subseteq$ A $\subseteq$ X. Then we say that B is πgγ-closed relative to A if $\gamma cl_A(B) \subseteq U$ where B $\subseteq$ U and U is π-open in A.

Theorem 3.17. *Let $B \subseteq A \subseteq X$ where A is $\pi g\gamma$-closed and π-open set. Then B is $\pi g\gamma$-closed relative to A if and only if B is $\pi g\gamma$-closed in X.*

Proof: We first note that B $\subseteq$ A and A is both πgγ-closed and π-open set, then $\gamma cl(A) \subseteq A$ and $\gamma cl(B) \subseteq \gamma cl(A) \subseteq A$. Now from the fact that $A \cap \gamma cl(B) = \gamma cl_A(B)$, we have $\gamma cl(B) = \gamma cl_A(B) \subseteq A$. If B is πgγ-closed relative to A and U is π-open subset of X such that B $\subseteq$ U, then $B = B \cap A \subseteq U \cap A$ where $U \cap A$ is π-open in A. Hence as B is πgγ-closed relative to A, $\gamma cl(B) = \gamma cl_A(B) \subseteq U \cap A \subseteq U$. Therefore B is πgγ-closed in X.

Conversely, if B is πgγ-closed in X and U is a π-open subset of A such that B $\subseteq$ U, then $U = V \cap A$ for some π-open subset V of X. As B $\subseteq$ V and B is πgγ-closed in X, $\gamma cl(B) \subseteq V$. Thus $\gamma cl_A(B) = \gamma cl(B) \cap A \subseteq V \cap A = U$. Therefore B is πgγ-closed relative to A.

Theorem 3.18. *If f: $X \to Y$ is slightly $\pi g\gamma$-continuous and A is $\pi g\gamma$-closed and π-open subset of X, then $f/A : A \to Y$ is slightly $\pi g\gamma$-continuous.*

Proof: Let V be a clopen subset of Y. Then $(f/A)^{-1}(V) = f^{-1}(V) \cap A \subseteq f^{-1}(V)$. Since $f^{-1}(V)$ is πgγ-closed in X, $(f/A)^{-1}(V)$ is πgγ-closed in X . Since $(f/A)^{-1}(V) = f^{-1}(V) \cap A \subseteq A \subseteq X$, by Theorem 3.17, $(f/A)^{-1}(V)$ is πgγ-closed in the relative topology of A. Therefore f/A is slightly πgγ-continuous.

4 Some application theorems

Definition 4.1. *A space is called*

1. *$\pi g\gamma$-T_2 (resp. ultra Hausdorff or UT_2 [24]) if every two distinct points of X can be separated by disjoint $\pi g\gamma$-open (resp. clopen) sets.*

2. *$\pi G\gamma O$-compact (resp. mildly compact [26]) if every $\pi g\gamma$-open (resp. clopen) cover has a finite subcover.*

Example 4.2. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, X\}$. Then (X, τ) is $\pi g\gamma$-T_2.*

The following Theorem gives a characterization of πgγ-T_2 spaces and is an analogous to that in general topology, hence its proof is omitted.

Theorem 4.3. *A space X is $\pi g\gamma$-T_2 if and only if for every point x in X, $\{x\} = \cap\{F : F$ is $\pi g\gamma$-closed nbd of $x\}$.*

Theorem 4.4. *If $f : X \to Y$ is slightly $\pi g\gamma$-continuous injection and Y is UT_2, then X is $\pi g\gamma$-T_2.*

Proof: Let $x_1, x_2 \in X$ and $x_1 \neq x_2$. Then since f is injective and Y is UT_2, $f(x_1) \neq f(x_2)$ and there exist $V_1, V_2 \in CO(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Since f is slightly $\pi g\gamma$-continuous, $x_i \in f^{-1}(V_i) \in \pi G\gamma O(X)$ for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus X is $\pi g\gamma$-T_2.

Theorem 4.5. *If $f : X \to Y$ is slightly $\pi g\gamma$-continuous surjection, and X is $\pi G\gamma O$-compact, then Y is mildly compact.*

Proof: Let $\{V_\alpha : V_\alpha \in CO(Y), \alpha \in I\}$ be a cover of Y. Since f is slightly $\pi g\gamma$-continuous, $\{f^{-1}(V_\alpha): \alpha \in I\}$ be $\pi g\gamma$-open cover of X so there is a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, $Y = \cup\{V_\alpha : \alpha \in I_0\}$ since f is surjective. Thus Y is mildly compact.

We shall continue to work by generalizing the well known theorems in general topology.

Recall that a space X is submaximal if every dense set is open and it is said to be extremally disconnected if the closure of every open set is open.

Lemma 4.6. *If X is submaximal and extremally disconnected, then every β-open set in X is open [18]. That is, every γ-open set in X is open.*

Remark 4.7. *By Lemma 4.6 and the following diagram, we can say that every $\pi g\gamma$-open set in X is πg-open.*

$$\begin{array}{ccc} \gamma\text{-open} & \longrightarrow & \text{open} \\ \downarrow & & \downarrow \\ g\gamma\text{-open} & \longrightarrow & g\text{-open} \\ \downarrow & & \downarrow \\ \pi g\gamma\text{-open} & \longrightarrow & \pi g\text{-open} \end{array}$$

Theorem 4.8. *If $f : X \to Y$ is a slightly continuous, $g : X \to Y$ is a slightly $\pi g\gamma$-continuous, Y is UT_2, X is submaximal and extremally disconnected, then $A = \{x \in X : f(x) = g(x)\}$ is πg-closed.*

Proof: Let $x \notin A$, then $f(x) \neq g(x)$. Since Y is UT_2, there exist $V_1, V_2 \in CO(Y)$ such that $f(x) \in V_1$ and $g(x) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Since f is slightly continuous and g is slightly $\pi g\gamma$-continuous, $f^{-1}(V_1)$ is open in X and $g^{-1}(V_2)$ is $\pi g\gamma$-open and hence πg-open in X since X is submaximal and extremally disconnected (Remark 4.7) with $x \in f^{-1}(V_1) \cap g^{-1}(V_2)$. Let $U = f^{-1}(V_1) \cap g^{-1}(V_2)$. Then U is a πg-open set (Lemma 1.6). If $z \in U$, then $z \in f^{-1}(V_1)$ and $z \in g^{-1}(V_2)$. Hence $f(z) \neq g(z)$. This shows that $U \subseteq A^c$ and therefore A is πg-closed.

Definition 4.9. *A subset of a space X is said to be $\pi g\gamma$-dense if its $\pi g\gamma$-closure equals X.*

The next Corollary is a generalization of the well known principle of extension of the identity.

Corollary 4.10. *Let f, g be slightly $\pi g\gamma$-continuous from a space X into a UT_2-space Y. If f and g agree on $\pi g\gamma$-dense set of X, then f = g everywhere.*

Definition 4.11. *Let A be a subset of X. A mapping $r : X \to A$ is called slightly $\pi g\gamma$-continuous retraction if r is slightly $\pi g\gamma$-continuous and the restriction $r|_A$ is the identity mapping on A.*

Theorem 4.12. *Let A be a subset of X and $r : X \to A$ be a slightly $\pi g\gamma$-continuous retraction. If X is UT_2, then A is $\pi g\gamma$-closed set of X.*

Proof: Suppose that A is not πgγ-closed. Then there exists a point x in X such that x $\in$ πgγcl(A) but x $\notin$ A. It follows that r(x) $\neq$ x because r is slightly πgγ-continuous retraction. Since X is UT_2, there exist disjoint clopen sets U and V such that x $\in$ U and r(x) $\in$ V. Since r(x) $\in$ A, r(x) $\in$ V $\cap$ A and V $\cap$ A is clopen set in A. Now let W be arbitrary πgγ-nbhd of x. Then W $\cap$ U is a πgγ-nbhd of x. Since x $\in$ πgγcl(A), (W $\cap$ U) $\cap$ A $\neq \emptyset$. Therefore, there exists a point y in W $\cap$ U $\cap$ A. Since y $\in$ A, we have r(y) = y $\in$ U and hence r(y) $\notin$ V $\cap$ A. This implies r(W) $\nsubseteq$ V $\cap$ A because y $\in$ W. This is contrary to slight πgγ-continuity of r from Proposition 3.8. Hence A is πgγ-closed.

Definition 4.13. *A space X is called $\pi G\gamma O$-connected provided X is not the union of two disjoint, non-empty $\pi g\gamma$-open sets.*

Example 4.14. *Let X = {a, b, c} and τ = {$\emptyset$, {c}, {a, b}, X}. Then (X, τ) is not $\pi G\gamma O$-connected.*

Theorem 4.15. *If $f : X \to Y$ is slightly $\pi g\gamma$-continuous surjection, and X is $\pi G\gamma O$-connected, then Y is connected.*

Proof: Assume that Y is disconnected. Then there exist disjoint, non-empty clopen sets U and V for which Y = U $\cup$ V. Therefore, X = f^{-1}(U) $\cup$ f^{-1}(V) is the union of two disjoint, πgγ-open nonempty sets and hence is not πGγO-connected.

Slight πgγ-continuity turns out to be a very natural tool for relating πGγO-connected spaces to connected spaces. In Theorem 4.15, we have seen that the slightly πgγ-continuous image of a πGγO-connected space is connected but that a slightly πgγ-continuous function is not necessarily a πGγO-connected function which is defined below.

Definition 4.16. *A function $f : X \to Y$ is called $\pi G\gamma O$-connected if the image of every $\pi G\gamma O$-connected subset of X is a connected subset of Y.*

The following Example shows that a slightly $\pi g\gamma$-continuous function is not necessarily $\pi G\gamma O$-connected.

Example 4.17. *Let X = Y = {a, b, c}, τ = {$\emptyset$, {a, c}, X} and σ = {$\emptyset$, {a}, {b}, {a, b}, Y}. Let $f : (X, \tau) \to (Y, \sigma)$ be defined by f(a) = f(c) = a and f(b) = b. Then f is slightly $\pi g\gamma$-continuous but it is not $\pi G\gamma O$-connected.*

Next we show by the Example that a πGγO-connected function need not be slightly πgγ-continuous.

Example 4.18. *Let* $X = \{\frac{1}{n} : n \in N\} \cup \{0\}$ *and* τ *be the usual relative topology on X. Let* $Y = \{0, 1\}$ *and* σ *be the discrete topology on Y. Define* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(\frac{1}{n}) = 0$ *for every* $n \in N$ *and* $f(0) = 1$*. It can be seen that the* $\pi g\gamma$*-open sets in* (X, τ) *are precisely the open sets. It then follows that f is* $\pi G\gamma O$*-connected but not slightly* $\pi g\gamma$*-continuous.*

Thus we have established that slight πgγ-continuity and πGγO-connectedness are independent.

Definition 4.19. *A space X is said to be* $\pi G\gamma O$*-connected between the subsets A and B of X provided there is no* $\pi g\gamma$*-clopen set F for which* $A \subseteq F$ *and* $F \cap B = \emptyset$*.*

Example 4.20. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$*. Take* $A = \{a, b\}$ *and* $B = \{b, c\}$*. Then* (X, τ) *is* $\pi G\gamma O$*-connected between the subsets A and B.*

Definition 4.21. *A function* $f : X \to Y$ *is said to be set* $\pi G\gamma O$*-connected if whenever X is* $\pi G\gamma O$*-connected between subsets A and B of X, then f(X) is connected between f(A) and f(B) with respect to the relative topology on f(X).*

Example 4.22. *Let* $X = \{a, b, c\}$*,* $\tau = \{\emptyset, \{c\}, \{b, c\}, X\}$ *and* $\sigma = \{\emptyset, \{c\}, X\}$*. Let* $f : (X, \tau) \to (X, \sigma)$ *be the identity function. By the above Example,* (X, τ) *is* $\pi G\gamma O$*-connected between the subsets A and B. It implies that f(X) is connected between f(A) and f(B) with respect to the relative topology on f(X).*

Theorem 4.23. *A function* $f : X \to Y$ *is set* $\pi G\gamma O$*-connected if and only if* $f^{-1}(F)$ *is* $\pi g\gamma$*-clopen in X for every clopen set F of f(X) (with respect to the relative topology on f(X)).*

Proof: The proof is obtained by following similar arguments as in ([8, Theorem 7]) or ([30, Theorem 20]) or ([7, Theorem 4.9]) or ([10, Theorem 3.23]).

Obviously, every slightly πgγ-continuous surjective function is set πGγO-connected. On the other hand, it can be easily shown that every set πGγO-connected function is slightly πgγ-continuous. Thus we have seen that in the class of surjective functions, slight πgγ-continuity and set πGγO-connectedness coincide. The following Example shows that in general slight πgγ-continuity is not equivalent to set πGγO-connectedness.

Example 4.24. *Let* $\Re$ *be real numbers with the usual topology and let f:* $\Re \to \Re$ *by* $f(x) = -1$ *if* $x < 0$ *and* $f(x) = 1$ *if* $x \geq 0$*. Since* $\Re$ *is connected, f is slightly* $\pi g\gamma$*-continuous. However,* $\{1\}$ *is clopen in* $f(\Re)$ *but* $f^{-1}(\{1\}) = [0, +\infty)$ *is not* $\pi g\gamma$*-open i.e., f is not set* $\pi G\gamma O$*-connected.*

5 Conclusion

General topology is important in many fields of applied sciences as well as branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc.

The notions of sets and functions in topological spaces and fuzzy topological spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all topological functions defined in this paper will have many possibilities of applications in digital topology and computer graphics.

Acknowledgement

The authors thank the referees for their valuable comments and suggestions for improvement of this paper.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud, β-open sets and β-continuous mappings, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.

[2] M. Akdag and A. Ozkan, Some properties of contra gb-continuous functions, Journal of New Results in Science, 1(2012), 40-49.

[3] S. C. Akgun and G. Ashm, On πgb-closed sets and related topics, International Journal of Mathematical Archive, 3(5)(2012), 1873-1884.

[4] A. K. Al-Obiadi, π-generalized b-closed sets in topological spaces, IBN AL-HAITHAM J. FOR PURE & APPL. SCi., 24(3)(2011),

[5] A. Al-Omari and M. S. M. Noorani, Generalized b-closed sets, Bull. Malays. Math. Sci. Soc., (2)32(1)(2009), 19-30.

[6] D. Andrijevic, On b-open sets, Mat. Vesnik, 48(1-2)(1996), 59-64.

[7] S. C. Arora and S. Tahiliani, Slightly generalized β-continuous functions, Sarajevo J. Math., 9(21)(2013), 143-152.

[8] C. W. Baker, Slightly precontinuous functions, Acta Math. Hungar., 94(1-2)(2002), 45-52.

[9] J. Cao, M. Ganster and I. Reilly, On sg-closed sets and gα-closed sets, Mem. Fac. Sci. Kochi Univ. Math., 20(1999), 1-5.

[10] C. Carpintero, N. Rajesh and E. Rosas, On slightly πg-continuous functions, Journal of Advanced Research in Pure Mathematics, 3(3)(2011), 115-127.

[11] J. Dontchev and M. Przemski, On the various decompositions of continuous and some weakly continuous functions, Acta Math. Hungar., 71(1-2)(1996), 109-120.

[12] J. Dontchev and T. Noiri, Quasi-normal spaces and πg-closed sets, Acta Math. Hungar., 89(3)(2000), 211-219.

[13] E. Ekici, On γ-normal spaces, Bull. Math. Soc. Sci. Math. Roumanie (N. S), 50(98)(2007), 259-272.

[14] E. Ekici and C. W. Baker, πg-closed sets and continuity, Kochi J. Maths., 2(2007), 35-42.

[15] E. Ekici and M. Caldas, Slighly γ-continuous functions, Bol. Soc. Parana. Math., (3)22(2)(2004), 63-74.

[16] A. A. El-Atik, A study on some types of mappings on topological spaces, Master's Thesis, Tanta University, Egypt, 1997.

[17] M. Fujimoto, S. Takigawa, J. Dontchev, T. Noiri and H. Maki, The topological structure and groups of digital n-spaces, Kochi J. Math., 1(2006), 31-55.

[18] M. Ganster and D. Andrijevic, On some questions concerning semi-preopen sets, J. Inst. Math. Compu. Sci. Math., 1(1988), 65-75.

[19] A. Keskin and T. Noiri, On bD-sets and associated separation axioms, Bulletin of the Iranian Mathematical Society, 35(1)(2009), 179-198.

[20] R. M. Latif, Characterizations and applications of γ-open sets, Soochow J. Math., 32(3)(2006), 1-10.

[21] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[22] H. Maki, Generalized $\wedge$-sets and the associated closure operator, The special issue in commemoration of Prof. Kazusda IKEDA's Retirement (1986), 139-146.

[23] O. Ravi, G. Selvi, S. Murugesan and A. Pandi, Slightly $g\gamma$-continuous functions, submitted.

[24] A. R. Singal and R. C. Jain, Slightly continuous mappings, J. Indian Math. Soc., 64(1997), 195-203.

[25] D. Sreeja and C. Janaki, On πgb-closed sets in topological spaces, International Journal of Mathematical Archive, 2(8)(2011), 1314-1320.

[26] R. Staum, The algebra of a bounded continuous functions into a nonarchimedian field, Pacific J. Math., 50(1974), 169-185.

[27] L. A. Steen and J. A. Seebach Jr, Counterexamples in topology, Holt, Rinenhart and Winston, New York, 1970.

[28] M. H. Stone, Applications of the theory of Boolean rings to general topology, Trans. Am. Math. Soc., 41(1937), 375-381.

[29] V. Zaitsev, On certain classes of topological spaces and their bicompactifications, Dokl. Akad. Nauk. SSSR 178(1986), 778-779.

[30] I. Zorlutuna, M. Kucuk and Y. Kucuk, Slightly generalized continuous functions, Kochi J. Math., 3(2008), 99-108.