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Some Properties of Contra gb-continuous Functions

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Abstract

We introduce some properties of functions called contra gb-continuous function which is a generalization of contra b-continuous functions [3]. Some characterizations and several properties concerning contra $gb-$ continuous functions are obtained.

Keywords: *g*$-$*open, g-continuity, contra gb-continuity.*

1 Introduction

In 1996, Donthev [16] introduced the notion of contra continuous functions. In 2007, Caldas, Jafari, Noiri and Simoes [10] introduced a new class of functions called generalized contra continuous (contra g-continuous) functions. They defined a function $f : X \to Y$ to be contra $g-$ continuous if preimage every open subset of Y is $g-$closed in X. New types of contra generalized continuity such as contra αg-continuity [23] and contra gs-continuity [17] have been introduced and investigated. Recently, Nasef [30] introduced and studied so-called contra b-continuous functions. After that in 2009, Omari and

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Noorani [4] have studied further properties of contra b-continuous functions. The purpose of the present paper is to introduce some propeties of notion of contra generalized $b-$continuity (contra $gb-$ continuity) via the concept of $gb-$open sets in [3] and investigate some of the fundamental properties of contra $gb-$continuous functions. It turns out that contra $gb-$continuity is stronger than contra $g\beta-$continuity and weaker than both contra $gp-$continuity and contra $gs-$continuity [17].

2 Preliminaries

Throughout the paper, the space X and Y (or (X,τ) and (Y,σ)) stand for topological spaces with no separation axioms assumed unless otherwise stated. Let A be a subset of a space X. The closure and interior of A are denoted by $cl(A)$ and $int(A)$, respectively.

Definition 2.1. *A subset A of a space X is said to be:*
(a) regular open [33] *if $A = int(cl(A))$*
(b) $\alpha-$open [31] *if $A \subset int(cl(int(A)))$*
(c) semi-open [24] *if $A \subset cl(int(A))$*
(d) pre-open [28] *or nearly open* [19] *if $A \subset int(cl(A))$*
(e) β-open [1] *or semi-preopen* [6] *if $A \subset cl(int(cl(A)))$*
(f) b-open [7] *or sp-open* [18] *or γ-open* [19] *if $A \subset cl(int(A)) \cup int(cl(A))$.*

The family of all semi-open (resp. preopen, α-open, $\beta-$open, $\gamma-$open) sets of (X,τ) will be denoted by $SO(X,\tau)$ (resp. $PO(X,\tau), \alpha O(X,\tau), \beta O(X,\tau), \gamma O(X,\tau)$). It is shown in [31] that $\alpha O(X,\tau)$ is a topology denoted by τ^{α} and it is stronger than the given topology on X. The complement of a regular-open (resp. semi-open, preopen, α-open, β-open, γ-open) set is said to be regular closed (resp. semi-closed, preclosed, α-closed, β-closed, γ-closed). The collection of all closed subsets of X will be denoted by $C(X)$. We set $C(X,x) = \{V \in C(X) : x \in V\}$ for $x \in X$. We define similarly $\gamma O(X,x)$.

The complement of b-open set is said to be b-closed [7]. The intersections of all b-closed sets of X containing A is called the $b-$closure of A and is denoted by $bcl(A)$. The union of all b-open sets X contained in A is called b-interior of A and is denoted by $bint(A)$.

Definition 2.2. [30] *A function $f : (X,\tau) \to (Y,\sigma)$ is called contra b-continuous if the preimage of every open subset of Y is b-closed in X.*

Definition 2.3. [21] *Let X be a space. A subset A of X is called a generalized b-closed set (simply; gb-closed set) if $bcl(A) \subset U$ whenever $A \subset U$ and U is open.*

The complement of a generalized b-closed set is called generalized b-open (simply; gb-open). Every b-closed set is gb-closed, but the converse is not true. And the collection of all gb-closed (resp. gb-open) subsets of X is denoted by $gbC(X)$ (resp. $gbO(X)$).

Example 2.4. [5] *Let $X = \{a,b,c\}$ and let $\tau = \{\emptyset, \{a\}, X\}$, then the family of all b-closed set of X is $bC(X) = \{\emptyset, \{b\}, \{c\}, \{b,c\}, X\}$ but the family of all gb-closed set of X is $gbC(X) = \{\emptyset, \{b\}, \{c\}, \{b,c\}, \{a,b\}, \{a,c\}, X\}$ then it is clear that $\{a,c\}$ is gb-closed but not b-closed in X.*

Lemma 2.5. *Let (X, τ) be a topological space.*
(a) The intersections of a b-open set and a gb-open set is a gb-open set.
(b) The union of any family of gb-open sets is a gb-open set.

Proof. The statements are proved by using the same method as in proving the corresponding results for the class of $b-$open sets(see [7]). □

3 Contra *gb*-continuous functions

In this section, we introduce some properties of continuity called contra gb-continuity which is weaker than both of contra gs-continuity and contra gp-continuity and stronger than contra $g\beta$-continuity.

Definition 3.1. [3] *A function $f : (X, \tau) \to (Y, \sigma)$ is called contra gb-continuous if the preimage of every open subset of Y is gb-closed in X.*

Corollary 3.2. *If a functionf $: (X, \tau) \to (Y, \sigma)$ is contra b-continuous, then f is contra gb-continuous.*

Proof. Obviuous □

Note that the converse of the above is not necessary true as shows by the following example:

Example 3.3. *Let $X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is contra gb-continuous but not contra b-continuous, since $A = \{a, c\} \in \sigma$ but A is not b-closed in (X, τ).*

Definition 3.4. *Let A be a subset of a space (X, τ).*
(a) The set $\cap\{U \in \tau : A \subset U\}$ is called the kernel of A [29] *and is denoted by $ker(A)$. In* [25] *the kernel of A is called the $\Lambda-$set.*
(b) The set $\cap\{F \subset X : A \subset F, F$ is gb-closed$\}$ is called the gb-closure of A and is denoted by $gbcl(A)$ [21].
(c) The set $\cup\{G \subset X : G \subset A, G$ is gb-open$\}$ is called the gb-interior of A and is denoted by $gbint(A)$ [21].

Lemma 3.5. *For an $x \in X, x \in gbcl(A)$ if and only if $U \cap A \neq \emptyset$ for every gb-open set U containing x.*

Proof. (Necessity) Suppose there exists a gb-open set U containing x such that $U \cap A = \emptyset$. Since $A \subset X - U$, $gbcl(A) \subset X - U$. This implies $x \notin gbcl(A)$, a contradiction.
(Sufficiency) Suppose $x \notin gbcl(A)$. Then there exists a gb-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and $X - F$ is gb-open also $(X - F) \cap A = \emptyset$, a contradiction. □

Lemma 3.6. [22] *The following properties hold for subsets A, B of a space X:*
(a) $x \in ker(A)$ if and only if $A \cap F \neq \emptyset$ for any $F \in C(X, x)$.
(b) $A \subset ker(A)$ and $A = ker(A)$ if A is open in X.
(c) If $A \subset B$, then $ker(A) \subset ker(B)$.

Theorem 3.7. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following continuons are equivalent:*
(a) f is contra gb-continuous;
(b) For every closed subsets F of Y, $f^{-1}(F) \in gbO(X, x)$;
(c) For each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in gbO(X, x)$ such that $f(U) \subset F$;
(d) $f(gbcl(A)) \subset ker(f(A))$ for every subset A of X;
(e) $gbcl(f^{-1}(B)) \subset f^{-1}(ker(B))$ for every subset B of Y.

Proof. The implications $(a) \Leftrightarrow (b)$ and $(b) \Rightarrow (c)$ are obvious.
$(c) \Rightarrow (b)$: Let F be any closed set of Y and $x \in f^{-1}(F)$. Then $f(x) \in F$ and there exists $U_x \in gbO(X, x)$ such that $f(U_x) \subset F$. Therefore, we obtain $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$ which is gb-open in X.
$(b) \Rightarrow (d)$: Let A be any subset of X. Suppose that $y \notin ker(f(A))$. Then by Lemma 3.6 there exists $F \in C(Y, y)$ such that $f(A) \cap F = \emptyset$. Thus, we have $A \cap f^{-1}(F) = \emptyset$ and $gbcl(A) \cap f^{-1}(F) = \emptyset$. Therefore, we obtain $f(gbcl(A)) \cap F = \emptyset$ and $y \notin f(gbcl(A))$. This implies that $f(gbcl(A)) \subset ker(f(A))$.
$(d) \Rightarrow (e)$: Let B be any subset of Y. By (d) and Lemma 3.6, we have $f(gbcl(f^{-1}(B))) \subset ker(f(f^{-1}(B))) \subset ker(B)$ and $gbcl(f^{-1}(B)) \subset f^{-1}(ker(B))$.
$(e) \Rightarrow (a)$: Let V be any open set of Y. Then, by Lemma 3.6, we have $gbcl(f^{-1}(V)) \subset f^{-1}(ker(V)) = f^{-1}(V)$ and $gbcl(f^{-1}(V)) = f^{-1}(V)$. This shows that $f^{-1}(V)$ is gb-closed in X. □

Definition 3.8. [4] *A function $f : (X, \tau) \to (Y, \sigma)$ is called $gb-$continuous if the preimage of every open subset of Y is gb-open in X.*

Remark 3.9. *The following two examples will show that the concept of gb-continuity and contra gb-continuity are independent from each other.*

Example 3.10. *Let $X = \{a, b\}$ be the Sierpinski space with the topology $\tau = \{\emptyset, \{a\}, X\}$. Let $f : (X, \tau) \to (X, \tau)$ be defined by: $f(a) = b$ and $f(b) = a$. It can be easily observed that f is contra gb-continuous. But f is not gb-continuous, since $\{a\}$ is open and its preimage $\{b\}$ is not gb-open.*

Example 3.11. *The identity function on the real line with the usual topology is continuous* [23, *Example* 2] *and hence gb-continuous. The inverse image of $(0, 1)$ is not gb-closed and the function is not contra gb-continuous.*

Definition 3.12. *A subset A of a space (X, τ) is called*

(a) a generalized semiclosed set (briefly gs-closed) [8] *if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open;*

(b) an $\alpha-$generalized closed set (briefly αg-closed) [25] *if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open;*

(c) a generalized pre-closed set (briefly gp-closed) [26] *if* $pcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open;*

(d) a generalized β*-closed set (briefly g*β*-closed)* [12] *if* $\beta cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open.*

Definition 3.13. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called contra* αg*-continuous* [23] *(resp. contra gs-continuous* [17]*, contra gp-continuous, contra g*β*-continuous) if the preimage of every open subset of* Y *is* αg*-closed (resp. gs-closed, gp-closed, g*β*-closed) in* X*.*

We obtain the following diagram by using Definition 2.1, 2.3, 3.1, 3.12 and 3.13.

contra continuous
$\downarrow$
contra αg-continuous
$\swarrow$ $\searrow$
contra gs-continuous   contra gp-continuous
$\searrow$ $\swarrow$
contra gb-continuous
$\downarrow$
contra $g\beta$-continuous

However, the converses are not true in general as shown by the following examples.

Example 3.14. *Let* $X = \{a, b, c\}, \tau = \{\emptyset, \{a\}, X\}$ *and* $\sigma = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$*. Then the identity function* $f : (X, \tau) \to (X, \sigma)$ *is contra* αg*–continuous but not contra continuous.*

Example 3.15. *Let* $X = \{a, b\}$ *with the indiscrete topology* τ *and* $\sigma = \{\emptyset, \{a\}, X\}$*. Then the identity function* $f : (X, \tau) \to (X, \sigma)$ *is contra gb-continuous but not contra gs-continuous, since* $A = \{a\} \in \sigma$ *but* A *is not gs-closed in* (X, τ)*.*

Example 3.16. *Let* $X = \{a, b, c, d\}$ *and* $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, b\}, \{a, b, c\}, \{b, c, d\}, X\}$*. Define a function* $f : (X, \tau) \to (X, \tau)$ *as follows:* $f(a) = b, f(b) = a, f(c) = d$ *and* $f(d) = c$*. Then* f *is contra gs-continuous. However,* f *is not contra* αg*-continuous, since* $\{c, d\}$ *is a closed set of* (X, τ) *and* $f^{-1}(\{c, d\}) = \{c, d\}$ *is not* αg*-open.*

Example 3.17. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ *and* $Y = \{1, 2\}$ *be the Sierpinski space with the topology* $\sigma = \{\emptyset, \{1\}, Y\}$*. Let* $f : (X, \tau) \to (Y, \sigma)$ *be defined by:* $f(a) = 1$ *and* $f(b) = f(c) = 2$*. Then* f *is contra gb-continuous but not contra gp-continuous.*

Theorem 3.18. *If a function* $f : X \to Y$ *is contra gb-continuous and* Y *is regular, then* f *is gb-continuous.*

Proof. Let x be an arbitrary point of X and V be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set G in Y containing $f(x)$ such that $cl(G) \subset V$. Since f is contra gb-continuous, so by Theorem 3.7 there exists $U \in gbO(X, x)$ such that $f(U) \subset cl(G)$. Then $f(U) \subset cl(G) \subset V$. Hence, f is gb-continuous. □

Definition 3.19. *A space (X, τ) is said to be:*
(a) gb-space if every gb-open set of X is open in X,
(b) locally gb-indiscrete if every gb-open set of X is closed in X.

The following two results follow immediately from Definition 3.19.

Theorem 3.20. *If a function $f : X \to Y$ is contra gb-continuous and X is gb-space, then f is contra continuous.*

Proof. Let $V \in O(Y)$. Then $f^{-1}(V)$ is gb-closed in X. Since X is gb-space, $f^{-1}(V)$ is closed in X. Thus, f is contra continuous. □

Theorem 3.21. *Let X be locally gb-indiscrete. If a function $f : X \to Y$ is contra gb-continuous, then it is continuous.*

Proof. Let $V \in O(Y)$. Then $f^{-1}(V)$ is gb-closed in X. Since X is locally gb-indiscrete space, $f^{-1}(V)$ is open in X. Thus, f is continuous. □

Recall that for a function $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by G_f.

Definition 3.22. *The graph G_f of a function $f : X \to Y$ is said to be contra gb-closed if for each $(x, y) \in (X \times Y) - G_f$ there exists $U \in gbO(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G_f = \emptyset$.*

Lemma 3.23. *The graph G_f of a function $f : X \to Y$ is contra gb-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G_f$ there exist $U \in gbO(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \emptyset$.*

Theorem 3.24. *If a function $f : X \to Y$ is contra gb-continuous and Y is Urysohn, then G_f is contra gb-closed in the product space $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G_f$. Then $y \neq f(x)$ and there exist open sets H_1, H_2 such that $f(x) \in H_1$, $y \in H_2$ and $cl(H_1) \cap cl(H_2) = \emptyset$. From hypothesis, there exists $V \in gbO(X, x)$ such that $f(V) \subset cl(H_1)$. Therefore, we obtain $f(V) \cap cl(H_2) = \emptyset$. This shows that G_f is contra gb-closed. □

Theorem 3.25. *If $f : X \to Y$ is gb-continuous and Y is T_1, then G_f is contra gb-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G_f$. Then $y \neq f(x)$ and there exist open set V of Y such that $f(x) \in V$ and $y \notin V$. Since f is gb-continuous, there exists $U \in gbO(X, x)$ such that $f(U) \subseteq V$. Therefore, we obtain $f(U) \cap (Y - V) = \emptyset$ and $(Y - V) \in C(Y, y)$. This shows that G_f is contra gb-closed in $X \times Y$. □

Definition 3.26. [16] *A space X is said to be strongly S-closed if every closed cover of X has a finite subcover.*

Theorem 3.27. *If (X, τ_{gb}) is a topological space and $f : X \to Y$ has a contra gb-closed graph, then the inverse image of a strongly S-closed set A of Y is gb-closed in X.*

Proof. Assume that A is a strongly S-closed set of Y and $x \notin f^{-1}(A)$. For each $a \in A$, $(x, a) \notin G_f$. By Lemma 3.23 there exist $U_a \in gbO(X, x)$ and $V_a \in C(Y, a)$ such that $f(U_a) \cap V_a = \emptyset$. Then $\{A \cap V_a : a \in A\}$ is a closed cover of the subspace A, since A is strongly S-closed, then there exists a finite subset $A_0 \subset A$ such that $A \subset \cup\{V_a : a \in A_0\}$. Set $U = \cap\{U_a : a \in A_0\}$, but (X, τ_{gb}) is a topological space, then $U \in gbO(X, x)$ and $f(U) \cap A \subset f(U_a) \cap [\cup\{V_a : a \in A_0\}] = \emptyset$. Therefore, $U \cap f^{-1}(A) = \emptyset$ and hence $x \notin gbcl(f^{-1}(A))$. This show that $f^{-1}(A)$ is gb-closed. □

Theorem 3.28. *Let Y be a strongly S-closed space. If (X, τ_{gb}) is a topological space and $f : X \to Y$ has a contra gb-closed graph, then f is contra gb-continuous.*

Proof. Suppose that Y is strongly S-closed and G_f is contra gb-closed. First we show that an open set of Y is strongly S-closed. Let U be an open set of Y and $\{V_i : i \in I\}$ be a cover of U by closed sets V_i of U. For each $i \in I$, there exists a closed set K_i of X such that $V_i = K_i \cap U$. Then the family $\{K_i : i \in I\} \cup (Y - U)$ is a closed cover of Y. Since Y is strongly S-closed, there exists a finite subset $I_0 \subset I$ such that $Y = \cup\{K_i : i \in I_0\} \cup (Y - U)$. Therefore, we obtain $U = \cup\{V_i : i \in I_0\}$. This shows that U is strongly S-closed. By Theorem 3.27, $f^{-1}(U)$ is gb-closed in X for every open U in Y. Therefore, f is contra gb-continuous. □

Theorem 3.29. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra gb-continuous, then f is contra gb-continuous.*

Proof. Let U be an open set in Y, then $X \times U$ is an open set in $X \times Y$. Since g is contra gb-continuous. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is an gb-closed in X. Thus, f is contra gb-continuous. □

Theorem 3.30. *$f : X \to Y$ is contra gb-continuous, $g : X \to Y$ contra continuous, and Y is Urysohn, then $E = \{x \in X : f(x) = g(x)\}$ is gb-closed in X.*

Proof. Let $x \in X - E$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exists open sets V and W such that $f(x) \in V$, $g(x) \in W$ and $cl(V) \cap cl(W) = \emptyset$. Since f is contra gb-continuous, then $f^{-1}(cl(V))$ is gb-open in X and g is contra continuous, then $g^{-1}(cl(W))$ is open in X. Let $U = f^{-1}(cl(V))$ and $G = g^{-1}(cl(W))$. Then U and G contain x. Set $A = U \cap G$ is gb-open in X. And $f(A) \cap g(A) \subset f(U) \cap g(G) \subset cl(V) \cap cl(W) = \emptyset$. Hence $f(A) \cap g(A) = \emptyset$ and $A \cap E = \emptyset$ where A is gb-open therefore $x \notin gbcl(E)$. Thus E is gb-closed in X. □

Theorem 3.31. *Let $\{X_i : i \in I\}$ be any family of topological spaces. If $f : X \to \Pi X_i$ is a contra gb-continuous function. Then $P_i o f : X \to X_i$ is contra gb-continuous for each $i \in I$, where P_i is the projection of ΠX_i onto X_i.*

Proof. We shall consider a fixed $i \in I$. Suppose U_i is an arbitrary open set in X_i. Then $P_i^{-1}(U_i)$ is open in ΠX_i. Since f is contra gb-continuous, $f^{-1}(P_i^{-1}(U_i)) = (P_i o f)^{-1}(U_i)$ is gb-closed in X. Therefore $P_i o f$ is contra gb-continuous. □

Theorem 3.32. *If $f : X \to Y$ is a contra gb-continuous function and $g : Y \to Z$ is a continuous function, then $gof : X \to Z$ is contra gb-continuous.*

Proof. Let $V \in O(Y)$. Then $g^{-1}(V)$ is open in Y. Since f is contra gb-continuous, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is gb-closed in X. Therefore, $gof : X \to Z$ is contra gb-continuous. □

Definition 3.33. *A function $f : X \to Y$ is said to be:*
(a) [21] *gb-irresolute if the preimage of a gb-open subset of Y is a gb-open subset of X,*
(b) $pre - gb$-open if image of every gb-open subset of X is gb-open.

Theorem 3.34. *Let $f : X \to Y$ be surjective gb-irresolute and $pre - gb$-open and $g : Y \to Z$ be any function. Then $gof : X \to Z$ is contra gb-continuous if and only if g is contra gb-continuous.*

Proof. The "if" part is easy to prove. To prove the "only if" part, let $gof : X \to Z$ be contra gb-continuous and let F be a closed subset of Z. Then $(gof)^{-1}(F)$ is a gb-open subset of X. That is $f^{-1}(g^{-1}(F))$ is gb-open. Since f is $pre - gb$-open $f(f^{-1}(g^{-1}(F)))$ is a gb-open subset of Y. So, $g^{-1}(F)$ is gb-open in Y. Hence g is contra gb-continuous. □

4 Applications

Definition 4.1. *A topological space X is said to be:*
(a) gb-normal if each pair of non-empty disjoint closed sets can be separated by disjoint gb-open sets,
(b) ultranormal [32] *if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.*

Theorem 4.2. *If $f : X \to Y$is a contra gb-continuous, closed injection and Y is ultranormal, then X is gb-normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of X. Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y. Since Y is ultranormal $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2, respectively. Hence $F_1 \subset f^{-1}(V_1)$, $F_2 \subset f^{-1}(V_2) \in gbO(X)$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus X is gb-normal. □

Definition 4.3. [9] *A topological space X is said to be gb-connected if X is not the union of two disjoint non-empty gb-open subsets of X.*

Theorem 4.4. *A contra gb-continuous image of a gb-connected space is connected.*

Proof. Let $f : X \to Y$ be a contra gb-continuous function of a gb-connected space X onto a topological space Y. If possible, let Y be disconnected. Let A and B form a disconnectedness of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \emptyset$. Since f is contra gb-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty gb-open sets in X. Also, $f^{-1}(A) \cap f^{-1}(B) = \emptyset$. Hence X is non-gb-connected which is a contradiction. Therefore Y is connected. □

Theorem 4.5. *Let X be gb-connected and Y be T_1. $f : X \to Y$ is a contra gb-continuous, then f is constant.*

Proof. Since Y is T_1 space, $v = \{f^{-1}(y) : y \in Y\}$ is disjoint gb-open partition of X. If $|v| \geq 2$, then X is the union of two non-empty gb-open sets. Since X is gb-connected, $|v| = 1$. Therefore, f is constant. □

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