Journal of Naval Science and Engineering
2011, Vol. 7, No.1, pp. 1-21

SHIP HULL GIRDER VIBRATION

Hakan UÇAR

Turkish Naval Academy
Naval Science and Engineering Institute
Tuzla, Istanbul, Turkiye
hkucar@gmail.com

Abstract

Vibration has always been an important subject of great interest to shipbuilders and marine engineers, because of its adverse effects both on the ship's structure and on the comfort of the crew. With the increase in complexity of vibration, the problem of avoiding vibration seems to be getting more rather than less difficult. Vibration is also a problem which is more amenable to mathematical analysis than are many of those connected with ships. Since many possible sources of vibration may exist on board, we are still far from the complete solution of all problems in ship hull vibration.

In warships, the addition of sensitive equipment such as radar and sonar has given a request for reducing the vibration to an absolute minimum in order to allow such devices to operate effectively. With the increasing importance of vibration in warships for reasons of defense and offence, more time is being devoted to the subject by the navies of the world. The dangers from acoustic and pressure operated offensive weapons have also focused attention on hull vibration in general and on the noise emitted by hulls, appendages and propellers, which in many cases is associated with some form of hull vibration. As a result, the hull girder vibration is an important problem in all maritime countries.

GEMİ TEKNE TİTREŞİMİ

Özetçe

Titreşim, gemi yapısı ve personel konforu üzerinde yaratabileceği olumsuz etkileri nedeniyle gemi inşa mühendisleri için önemli bir ilgi alanı olmuştur. Titreşimin kompleksliğinin artmasıyla titreşimden kaçınma problemi gittikçe daha da zor hale gelmektedir ve gemi tekne titreşimi gemilerle ilişkili birçok alandan daha çok analitik ve nümerik analizlere tabidir. Ayrıca gemi bünyesinde birçok olası titreşim kaynakları olması nedeniyle gemi tekne titreşimindeki tüm problemlerin çözümünün tamamında hala uzağız.

Harp gemilerinde radar ve sonar gibi hassas donanımların olması ve bu cihazların efektif olarak çalışması için titreşimin minimuma indirgenmesi ihtiyacı doğmuştur. Dolayısıyla, harp gemilerinde titreşim öneminin artmasıyla birlikte dünya donanmaları tarafından bu konuya daha fazla zaman ayrılmaya başlanmıştır. Ayrıca, akustik ve basınç tahrikli silahların yarattığı tehlikeler, tekne titreşimi, pervane ve diğer donanımların yaydığı yapısal titreşim kaynaklı gürültü üzerine daha fazla odaklanmasına neden olmuştur. Sonuç olarak, tüm denizci ülkelerde tekne titreşimi önemli bir problemdir.

Keywords: Vibration, Ship hull, Timoshenko Beam Theory
Anahtar Kelimeler: Titreşim, Gemi teknesi, Timoshenko Teorisi

1. INTRODUCTION

The vibration experienced on board ships can be divided into two classes. In the first type, the whole hull girder is thrown into a state of vibration at certain revolutions of the main engines, the auxiliary machinery, the propeller and the sea. In this case, the movement of the hull can be clearly seen by sighting along the length of the ship and it can reach an amplitude of as much as an inch at the bow and stern. This kind of vibration depends on the revolutions at which it occurs in relation to those required to be used in long without loosening rivets. Such vibration, affecting the whole structure, is known as *synchronous* or *resonant* vibration.

In the second type, isolated parts of the ship or certain fittings such as a mast or a plate panel, are set into a state of vibration which can be very annoying to crew but not be very dangerous to the ship. However it may be the most important vibration in warships for preventing the proper use of navigational instruments, radar and sonar devices, gun directors and similar equipment. Such vibration is usually termed *local* vibration.

Once a ship is built, it is impossible to eliminate such resonant vibration by adding material to the hull with a view to strengthening it. So the source of the disturbing forces should be taken into consideration. Some of these disturbing forces are purely mechanical and can either be eliminated or reduced to unimportant dimensions, but others are in part of hydrodynamic origin and cannot be completely avoided [1].

2. HULL GIRDER VIBRATION

Structural vibrations occur when ships are subjected to periodic or time-varying loads. If the frequencies of the disturbing forces are close to one of the natural frequencies of the ship, the permissible vibration levels may be exceeded. This high vibration may occur in the following places.

1. The hull girder
2. The stern and the superstructures
3. Transverse frames, plate panels and plate elements
4. The propeller shaft
5. The main engine

The most relevant global vibration modes are depicted in Figure 1. The two-noded vertical vibration mode has normally the lowest natural frequency. Typically, the vibration modes shown in the Figure 1, correspond to natural frequencies in the range of 0.6-6 Hz. [2]

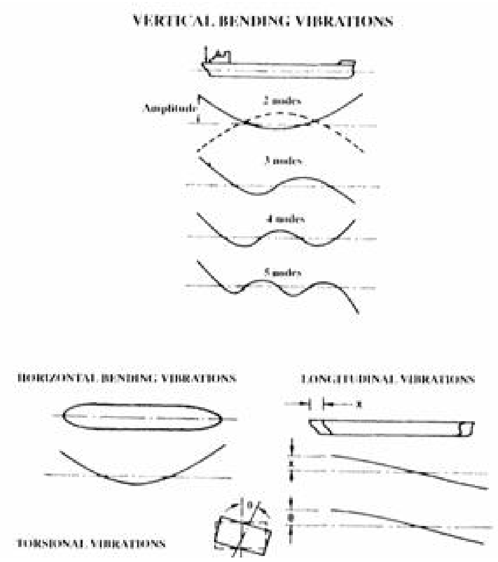


Figure 1. Beam vibration modes for a ship's hull

Simple beam models with good accuracy can often be used in order to determine the lowest natural frequencies of the hull girder. Timoshenko beam theory can be used for determination of the natural frequencies for continuous systems.

The following data must be known in order to determine the global vibrations of the hull girder.

1. Time-varying loads on the hull girder
2. The distribution of stiffness and mass of the hull girder
3. Structural and hydrodynamic damping

The vibration level is determined as a solution to a forced vibration problem. An efficient method is modal superposition where the solution is expressed as a linear combination of relevant natural vibration modes.

3. TIMOSHENKO BEAM THEORY

Timoshenko's theory of beams constitutes an improvement over the Euler-Bernoulli theory, in that it incorporates shear and rotational inertia effects. The plane cross-sections remain orthogonal to the neutral axis of the beam is replaced by the assumption that the angle between the neutral axis and the normal of the cross section is proportional to the shear force.

This modification of the Bernoulli-Euler beam theory is needed for calculation of the higher hull girder modes, where the distance between the nodes cannot be considered to be large in relation to the cross-sectional dimensions of the hull girder. [3]

Consider a beam with length *L*, modulus of elasticity *E*, shear modulus *G*, the mass per unit length *m*, the moment of inertia *I(x)*, the cross-sectional area *A(x)* and the mass moment of inertia $mr^2(x)$

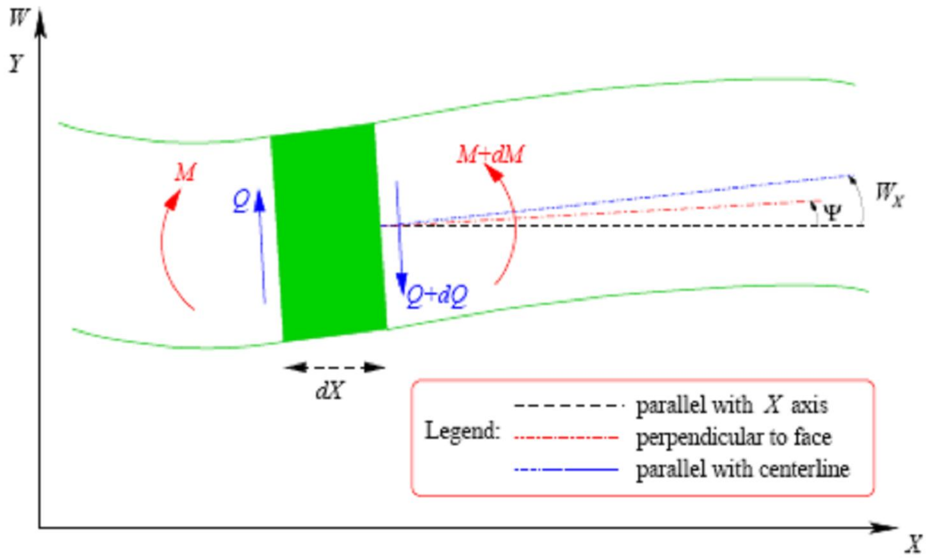


Figure 2. Timoshenko beam element [3]

Let ψ be the angle which the cross-section of the beam forms with the y-axis, when only bending is considered, then $\frac{\partial w}{\partial x} = \psi$ due to the assumption in the Bernoulli beam theory. Here W is the transverse displacement of the neutral line at a distance X from the left end of the beam at time T. Due to the effect of shear, the original rectangular element changes its shape to somewhat like a parallelogram with its sides slightly curved. The shear angle υ (or loss of slope) is now equal to the slope of bending ψ less slope of centerline W_X in the form

$$\upsilon = \psi - W_X \tag{1}$$

and the shear force Q is against the internal shear loading in the form

$$Q = -kAG\,\upsilon = -kAG(\psi - W_X) \tag{2}$$

Similarly, the bending moment M is against the internal elastic inertia in the form

$$M = -EI\,\psi_x = -EI\frac{\partial w}{\partial x} \tag{3}$$

The difference between the Euler-Bernoulli beam theory and Timoshenko beam theory can be summarized as follows [2].

Timoshenko Beam Theory	Euler-Bernoulli Beam Theory
$M = EI\frac{d\psi}{dx}$	$M = EI\frac{d^2w}{dx^2}$
$\frac{dw}{dx} - \psi(x) = -\frac{V(x)}{\kappa^2 GA}$	$\psi = \frac{dw}{dx}$

We equate the transverse force and rotary inertia of the element to form the following four simultaneous pdes.

$$M + EI\,\psi_x = 0 \tag{4a}$$
$$Q + kAG(\psi - W_x) = 0 \tag{4b}$$
$$M_X - Q + \rho I \psi_{TT} = 0 \tag{4c}$$
$$Q_X - \rho A W_{TT} = 0 \tag{4d}$$

Further, Equations (4a) and (4c) involve rotational motion while Equations (4b) and (4d) involve transverse motion of the element. Eliminating M and Q from (4) yields two simultaneous PDEs in W and ψ :

$$\rho A W_{TT} + (kAG(\psi - W_x))_x = 0 \tag{5a}$$
$$\rho I_{TT} - (EI\, \psi_x)_x + kAG(\psi - W_x) = 0 \tag{5b}$$

Equation (5a) is an equilibrium of translational force per unit length against the internal shear force gradient while Equation (5b) is an equilibrium of rotational torque per unit length equating to the gradient of internal bending moment against the internal shear force. This form is convenient for finding the normal modes and frequency of free vibration and the solution is in the form of (W, ψ). [3]

In the case of a uniform beam, ψ can be eliminated from the above two equations to form a single equation.

$$\frac{EI}{\rho A} W_{XXXX} - \frac{I}{A}\left(\frac{E}{kG} + 1\right) W_{XXTT} + \frac{\rho I}{kGA} W_{TTTT} + W_{TT} = 0 \tag{6}$$

This equation has four terms in the unit of force per unit mass or acceleration. They are the terms involving *bending moment, shear force, rotational motion* and *translational motion* respectively. When the shear and rotational terms are small and disregarded, the equation will be that of the Euler-Bernoulli beam.

The standard homogeneous boundary conditions for this system of equations are as follows.

Hinged type :$W = 0$, $M = EI\,\psi_x = 0$
Clamped type :$W = 0$, $\psi = 0$;
Free type :$Q = kAG(\psi - W_x) = 0$, $M = EI\,\psi_x = 0$

The solutions to the system of equations with a sst of homogeneous boundary conditions will have this form;

$$\begin{aligned} W(x,t) &= u(x)\sin(\omega t + \phi) \\ \psi(x,t) &= \alpha(x)\sin(\omega t + \phi) \end{aligned} \tag{7}$$

This solution can be inserted in Eq.(5) in order to determine the ordinary differential equations. For example, the natural frequency of a uniform, homogeneous, simply supported beam is determined as follows.

$$\omega_n^2 = \frac{\left\{1+\left(\frac{n\pi}{L}\right)^2\left(\frac{EI}{kGA}+r^2\right)\right\} \pm \sqrt{\left\{1+\left(\frac{n\pi}{L}\right)^2\left(\frac{EI}{kGA}+r^2\right)\right\}^2 - 4r^2\frac{m}{kGA}\left(\frac{EI}{m}\right)\left(\frac{n\pi}{L}\right)^4}}{2r^2\frac{m}{kGA}} \tag{8}$$

including the solution n=0.

It is seen that, for each value of *n*, two different ω_n^2 are obtained. The vibration modes therefore can be sketched as in Figure 3.

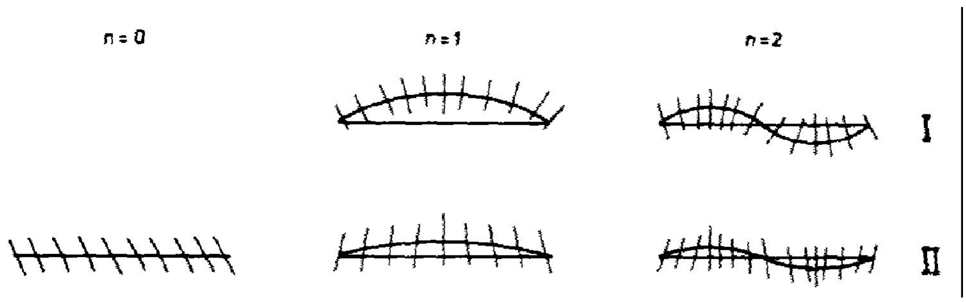


Figure 3. Lowest natural vibration modes for a uniform, homogeneous Timoshenko beam, simply supported at the ends [2].

4. TIME-VARYING LOADS ON THE HULL GIRDER

The most common source for the generation of hull vibrations is propeller-induced forces. Formerly, the main engines were also a considerable source of vibration problems, but better balancing of the movable parts in the large diesel engines has reduced significantly the magnitude of unbalanced vibratory forces and moments. Wave-induced forces may also cause hull girder vibrations.

4.1. Propeller Induced Forces

When the propeller of the ship rotates in the inhomogeneous wake field, periodic forces will arise in the stern. These hydrodynamic forces will act partly on the propeller and be transferred to the hull girder via the bearings of the propeller axis and on the plating of the stern, as shown in Figure 4. It is very difficult to calculate these forces by theoretical methods because of the complicated hydrodynamic flow conditions around the propeller. Therefore, it is often necessary to use model experiments and empirical formulas [2].

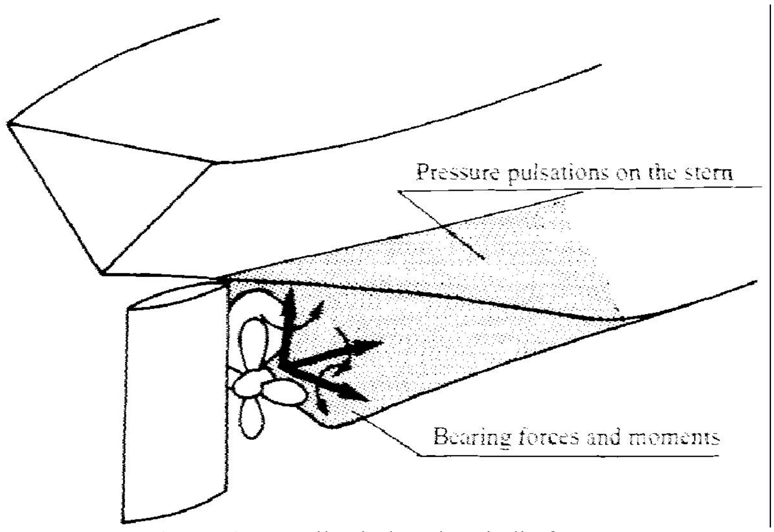


Figure 4. Propeller-induced periodic forces

The magnitude of the periodic forces and moments can be determined by calculating the hydrodynamic lift *L* on each propeller blade. The lift is a function of the position of the blade, given by the angle θ relative to a vertical position of the propeller blade, as shown in Figure 5.

For each blade, the lift L_j can be divided into two force components. The blade thrust $T_j(\theta)$ and the resistance $P_j(\theta)$, having effect in respectively the direction of the propeller axis and perpendicularly to the axis of the propeller blade. T_j and P_j can be expanded in Fourier series

$$T_j(\theta) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\theta$$
$$P_j(\theta) = \frac{1}{2}b_0 + \sum_{n=1}^{\infty} b_n \cos n\theta \tag{9}$$

As a consequence, the resulting load components on the propeller axis at the propeller can be determined by adding up the loads T_j and P_j from the total of Z similar propeller blades [2].

Propeller thrust	$T(\theta) = \sum_{j=1}^{Z} T_j(\theta_j)$
Propeller moment	$Q(\theta) = r\sum_{j=1}^{Z} P_j(\theta_j)$
Vertical Force	$F_V(\theta) = \sum_{j=1}^{Z} P_j(\theta_j)\sin\theta_j$
Vertical Bending Moment	$M_V(\theta) = r\sum_{j=1}^{Z} T_j(\theta_j)\cos\theta_j$
Horizontal Force	$F_H(\theta) = \sum_{j=1}^{Z} P_j(\theta_j)\cos\theta_j$
Horizontal Bending Moment	$M_H(\theta) = r\sum_{j=1}^{Z} T_j(\theta_j)\sin\theta_j$

where $$\theta_j = \theta + \frac{2\pi}{Z}(j-1) \tag{10}$$

It is seen from the results that all load components are periodic with the period $2\pi/Z$, because the same propeller configuration occurs each time a new blade gets in the same position as the preceding blade. If the propeller axis rotates with the constant frequency Ω then $\theta = \Omega t$ and *blade frequency* is $Z\Omega$.

The most important components in relation to generation of the hull vibrations are the terms which vary with the blade frequency. If only these terms are kept, the result is as follows;

$$\begin{aligned}
T_1 &= Za_z \cos Z\Omega t \\
Q_1 &= rZb_z \cos Z\Omega t \\
F_{V1} &= \frac{1}{2} Z(b_{Z-1} - b_{Z+1}) \sin Z\Omega t \\
M_{V1} &= \frac{r}{2} Z(a_{Z-1} - a_{Z+1}) \cos Z\Omega t \\
F_{H1} &= \frac{1}{2} Z(b_{Z-1} - b_{Z+1}) \cos Z\Omega t \\
M_{H1} &= \frac{r}{2} Z(a_{Z-1} - a_{Z+1}) \sin Z\Omega t
\end{aligned} \qquad (11)$$

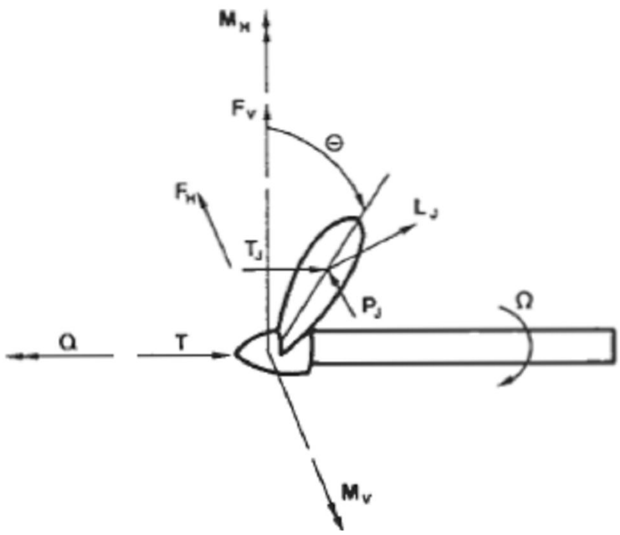


Figure 5. Resulting forces and moments on the propeller [2]

For conventional ships, the size of the time-varying loads T_1, Q_1 and the others are of the order of magnitude of 5-20 % of respectively the mean propeller thrust and moment.

The significance of the time-varying loads on the propeller is mainly that they may cause too large vibrations of the propeller axis. Their contribution to the generation of hull girder vibrations is normally smaller than the contribution from the pulsating hydrodynamic forces induced on the stern as a consequence of the inhomogeneous wake field. If the propeller cavitates, this effect strongly enhances the latter load but does not increase the forces on the propeller.

4.2. Unbalanced Forces from Diesel Engine

A schematic cross-section of a cylinder in a diesel engine and forces are shown in Figure 6. It is seen from the figure that the vertical motion x of the piston can be written

$$x = (r + \ell) - (r\cos\theta + \ell\cos\varphi) \tag{12}$$

where r is the radius of the crank motion and ℓ is the length of the connecting rod. From the definition of the angles, it is seen that $\ell\sin\varphi = r\sin\theta$ and $\theta = \omega t$ where ω is the frequency of revolutions of the engine. If φ is eliminated, the result is

$$x(t) = r(1-\cos\theta) + \ell\left[1 - \sqrt{1-\left(\frac{r}{\ell}\right)^2 \sin^2\theta}\,\right] = r\left(1 - \cos\omega t + \frac{1}{2}\frac{r}{\ell}\sin^2\omega t\right) \tag{13}$$

If the above expression is differentiated twice with respect to time, the acceleration and therefore the resulting D'Alembert force F_1 is obtained.

$$F_1 = m_1\ddot{x}(t) = m_1 r\omega^2\left(cos\omega t + \frac{r}{\lambda}cos2\omega t\right) \quad \text{(positively upwards)} \quad (14)$$

The centrifugal force F_2 as a result of the circular motion of the crank must be added to the force F_1. The resolved centrifugal forces in the vertical (F_{2V}) and the horizontal (F_{2H}) direction are as follows;

$$F_{2V} = m_2 r\omega^2 \cos\omega t$$
$$F_{2H} = m_2 r\omega^2 \sin\omega t \quad (15)$$

where m_2 is the part which follows the motion of the crank shaft.

The resulting mass forces are F_1+F_{2V} in the vertical direction and F_{2H} in the horizontal direction. In order to balance these forces and moments for the engine as a whole, the phase shift between the ignition for the single cylinders can be chosen in an appropriate way and rotating masses can be added to the crankshaft. The engine manufacturers provide very accurate balanced engines today by using various correction procedures.

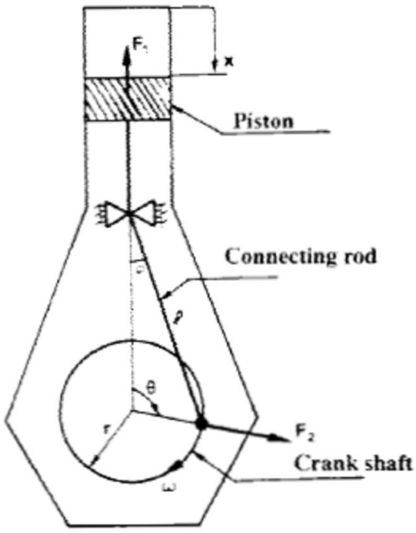


Figure 6. Schematic cross-section of a cylinder [2]

4.3. Wave Induced Loads

The wave-induced load per unit length along the hull girder (the x-axis) can be written as a sum of the harmonic components.

$$q(x,t) = \sum_{j=1}^{n} a_j \Phi_q(x,\omega_j)\cos\left(\omega_{e,j} t + \hat{\varepsilon}_j(x) + \varepsilon_{qj}\right) \tag{16}$$

where a_j is the wave amplitude for the wave component which has the frequency ω_j and where $\Phi_q(x,\omega_j)$ is the amplitude of the function, defined as the amplitude of the load in the position x=x.

The linear response of the ship becomes statistically normally distributed with a mean value of zero and a variance equal to the sum of the variances for the response calculated for each load components. The response of the ship for each individual component can thus be considered separately without accounting for the stochastic phase angle ε_{qj}, which does not enter into the variance.

Wave-induced vibrations of the hull girder only occur in relatively rare cases. The reason is that the wave amplitude is normally negligible small for frequencies of encounter $\omega_{e,j}$ of the order of the lowest natural frequency of the hull girder [4].

5. STIFFNESS DISTRIBUTION OF THE HULL GIRDER

It is necessary to know the stiffness and mass distribution of the hull girder in order to determine the natural frequencies and natural vibration modes of the hull girder.

The relevant stiffness parameters for vertical and horizontal vibration modes are the *bending stiffness* ($EI_y(x)$ for the vertical vibrations and $EI_z(x)$ for horizontal vibrations) and the *shear stiffness* ($k_zGA(x)$ and $k_yGA(x)$).

The effectiveness of longitudinal elements which do not extend along the whole length of the ship is important. The hatched areas, as shown in Figure 7, are ineffective to the bending stiffness of the hull girder. A reasonable value for the angle θ is 15^o.

It is also important to define the shear stiffness, kGA. While the cross-sectional area *A* is easy to calculate, the calculation of the dimensionless constant *k* depends on some assumptions which can approximately the real three-dimensional deformation pattern with relevant beam deformation measures. Therefore, several calculation methods for the constant *k* are found. The most consistent procedure for determination of k has been given by Cowper (1966). The reduction of the three-dimensional elasticity theory to a beam theory given there is relatively complicated and here only the result for a cross-section built up of thin-walled elements is presented, assuming the same modulus of elasticity *E* throughout. [2]

$$k = \frac{2(1+\upsilon)I_y}{\frac{\upsilon}{2}(I_z - I_y) + A\int_{\ell} \psi\tau_0 h ds} \quad (17)$$

where I_y and I_z : the moments of inertia about respectively the y- and the z-axis

τ_0 : the unit shear stress distribution
υ : Poisson ratio

The function ψ is given by;

$$\psi = 2(1+\upsilon)I_y\tau_0(s) + \frac{\upsilon}{2}\left((z^2 - y^2)\cos\varphi + 2yz\sin\varphi\right) \quad (18)$$

where φ is the angle between the plate element at s=s and the z-axis.

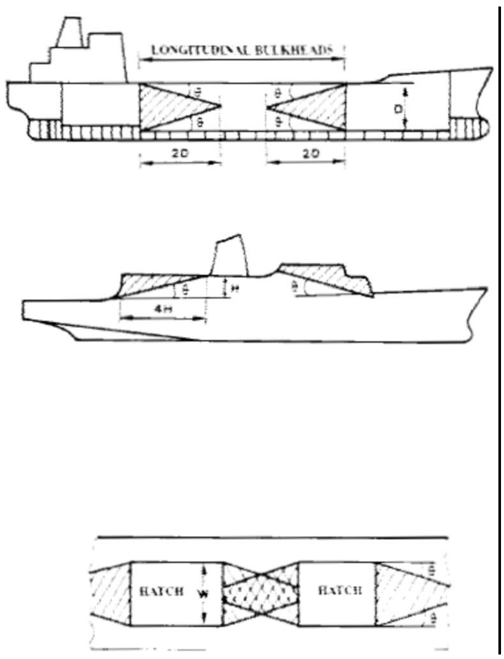


Figure 7. Efficiency of longitudinal elements [2]

From the equation above, it is understood that if the shear stress distribution τ_0 is known, the shear coefficient k can be determined. For realistic hull cross-sections the shear area kA will be of the order of magnitude of 50-90 % of the projected area.

The importance of the shear stiffness kGA compared with the bending stiffness EI grows with the number of nodes in the natural vibration mode. For the two-noded vertical natural vibration mode, the bending stiffness is normally dominant; but the shear stiffness contributes to the deformation in all other vertical and horizontal natural vibration modes. While both the magnitude of the bending and the shear stiffnesses are of importance to the natural vibration modes, their variation along the hull girder will often be of less importance. Therefore, it is usually enough to calculate these stiffnesses for a few cross-sections along the hull girder and use interpolation between these values. Figure 8 shows the three lowest natural vibration modes corresponding to horizontal bending-torsion for a container ship

Figure 8. The three lowest natural vibration modes corresponding to horizontal bending-torsion models for a container ship [Pedersen (1983)]

6. MASS DISTRIBUTION OF THE HULL GIRDER

When the hull girder vibrates, the surrounding water will be forced to follow the motions of the ship. The motion of the water will be the same as the motion of the hull when it is close to the hull. At a larger distance from the hull, the amplitude of the water will quickly decrease while the frequency remains unchanged.

The relevant mass date used in the calculation of the natural vibrations of the hull must therefore contain both the mass distribution of the hull girder, including the mass of the cargo, and a contribution which reflects the associated motion of the water. [2]

The determination of the mass distribution $m_s(x)$ of the hull girder can be made from the knowledge of the steel weight of the ship and the equipment weight. In addition to the mass m_s per unit length, the associated mass radii of gyration $r_y(x)$ and $r_z(x)$ for vibrations in respectively the horizontal and the vertical plane should be determined. The added mass of water per unit length $m_w(x)$ by a vertical motion of a hull section can be written as

$$m_w(x,n) = J_n C_m(x) \rho A(x) \tag{19}$$

where ρ : the density of the water

C_m : the dimensionless coefficient depending on the shape of cross-section

$A(x)$: the submerged area of the section

J_n : the three-dimensional reduction factor for the three-dimensional flow around the hull girder where n is the number of nodes in the vibration mode.

J factor is determined by two different methods, Townsin and Kumai. Figure 9 shows a comparison of the two methods.

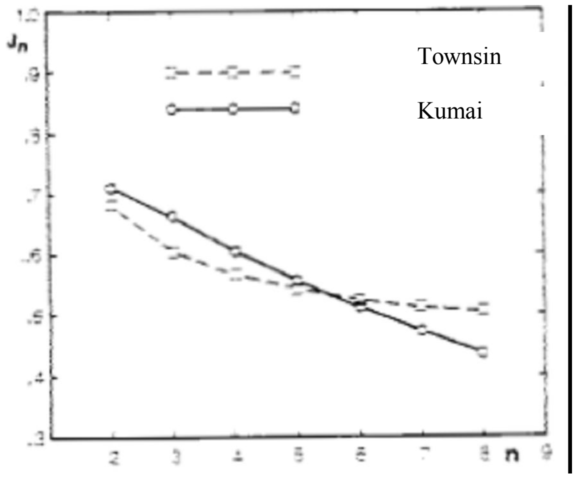


Figure 9. Three-dimensional vertical reduction factor J_n for a 340,000 dwt tanker [2]

It is seen from Figure 9 that at the two-noded vibration mode the J factor represents a reduction of 30 % of the two-dimensional mass of water. As the added mass of water is of the same order of magnitude as the mass of the ship, this reduction has a considerably influence on the natural frequency.

7. DAMPING

A classical mass-spring-damper system is illustrated in Figure 10 which exhibits a vibrating concentrated mass *m*, held by a spring with the stiffness *k* and a viscous damper with the damping *b*.

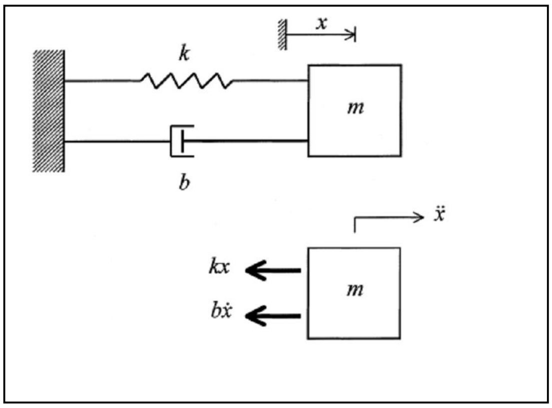


Figure 10. Natural vibrations of damped system with one degree of freedom

Equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Solution for underdamped case:

$$x = e^{-\zeta\omega_n t}[A_1 \cos\omega_d t + A_2 \sin\omega_d t]$$

Natural frequency of damped system: $\omega_d = \sqrt{1-\zeta^2}\,\omega_n$

Natural frequency of undamped system: $\omega_n = \sqrt{\frac{k}{m}}$

Damping ratio: $\zeta = \frac{b}{2\sqrt{km}} = \frac{b}{b_0}$

Logarithmic decrement : $\delta = 2\pi\xi / \sqrt{1-\xi^2}$

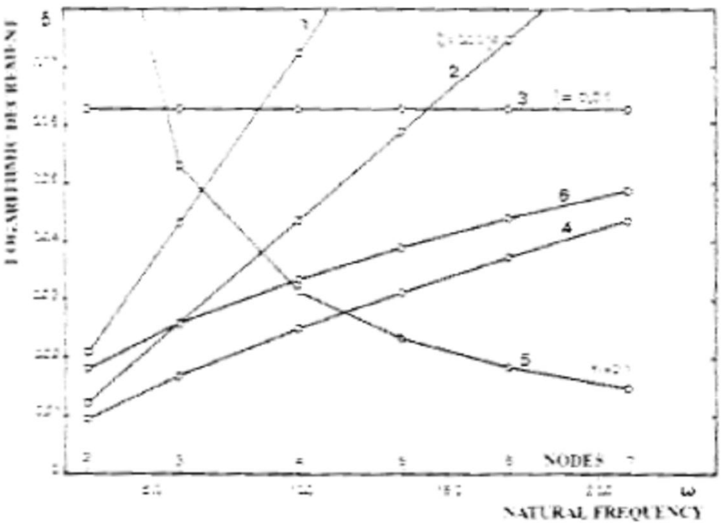
It is seen from Figure 10 that if the damping b is much smaller than the critical damping b_0, the natural frequency ω_d for the damped system will coincide with the natural frequency ω_n of the undamped system. So the damping ratio ζ is a somewhat inconvenient quantity. Therefore, the logarithmic decrement δ, defined as the natural logarithm to the relation between two successive maxima in x, is often used.

While the damping in slightly damped vibrations may be neglected in the determination of the natural frequencies of the system, the damping will have a significant influence on the vibration amplitude around the natural frequencies. For the system shown in Figure 10, *the dynamic amplification factor* Q is given by

$$Q = \frac{x_0}{F_0 / k} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}} \qquad (20)$$

The dynamic amplification factor Q determines the motion amplitude of the mass m when this is subjected to a periodic force $F_0 \cos\omega t$.

If ship hull vibrations are considered, damping will mainly be due to structural damping from hysteresis effects in the steel, especially as a consequence of welding. Damping also takes place in cargoes of grain and the like, as well as through hydrodynamic damping. However all these effects are usually so small that they can be neglected in relation to the internal structural damping in the welded steel structure in the frequency range of interest. Calculation of the magnitude of damping in hulls is not possible because the theoretical damping mechanisms cannot today be calculated for so complex structures as ships. Therefore, calculation of the forced hull vibrations must be based on the empirical formulas. A number of these formulas are shown in Figure 11.



1	$\delta_n = 0.0073\omega_n$	4	$\delta_n = 3.5 / L(\omega_n / \omega_2)^{3/4}$
2	$\delta_n = \zeta\pi\omega_n$	5	$\delta_n = \eta\pi / \omega_n$
3	$\delta_n = 2\pi\omega_n$	6	$\delta_n = 0.01065\omega_n^{1/2}$

Figure 11. Examples of published values for the logarithmic decrement δ as a function of frequency for a 340,000 dwt tanker [Jensen and Madsen (1977)]

REFERENCES

[1] F.H.Todd, “Ship Hull Vibration”, Edward Arnold London, 1961
[2] J.J.Jensen, “Load and Global Response of Ships”, Elsevier, 2001
[3] S.W.Taylor, S.C.B.Yau, “Boundary Control of a Rotating Timoshenko Beam”, Anziam, 2003
[4] J.J.Jensen, M.Dogliani, “Wave-Induced Ship Hull Vibrations in Stochastic Seaways”, Elsevier, 1996