Journal of Naval Science and Engineering
2011, Vol.7, No.2, pp. 30-45

AXIAL HEAT CONDUCTION IN LAMINAR DUCT FLOWS

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Abstract

In this study, a numerical model for axial heat conduction effects in laminar circular tube flows has been developed using the finite difference method. The calculations have been done using different Peclet numbers in the range of 0.5 to 100. Constant heat flux and constant wall temperature boundary conditions have been used. The validity of the numerical model has been checked by some data in the literature. The results were presented for temperature variation, Nusselt number, and energy absorbed before the heated section. The axial conduction effects may be important for micro tubes, which are to be used in micro heat exchangers.

LAMİNAR KANAL AKIŞLARINDA EKSENEL ISI İLETİMİ

Özetçe

Laminar silindirik kanal akışlarında eksenel ısı iletimi etkileri için sonlu farklar yöntemi kullanılarak sayısal bir model geliştirildi. Hesaplamalar 0.5-100 Peclet sayıları aralığında yapıldı. Sabit ısı akısı ve sabit yüzey sıcaklığı sınır koşulları kullanıldı. Sayısal yöntemin geçerliliği literatürdeki bazı verilerle kontrol edildi. Sıcaklık dağılımı, Nusselt sayısı ve ısıtılan yüzeyden önce absorbe edilen enerji için sonuçlar ortaya kondu. Eksenel ısı iletim etkileri mikro ısı değiştiricilerde kullanılan mikro tüpler için önemli olabilir.

Keywords : *Axial Heat Conduction, Laminar Duct Flows***.**
Anahtar Kelimeler : *Eksenel Isı İletimi, Laminar Kanal Akışları.*

1. INTRODUCTION

The Graetz problem is known as the heat transfer problem of laminar fluid flow in ducts. It has many applications and has been studied extensively since Graetz (1885). The classical Graetz problem considers the forced convection heat transfer of the fluids flowing in ducts while neglecting axial conduction in the fluid.[1]

This study covers the Peclet number range from those of liquid metals where axial conduction may not be neglected to oil flows where the axial conduction effects is not important on the temperature distribution inside the laminar duct flows. Nowadays developing micro machines, the axial conduction effects may be important in ducts of the micro heat exchangers.

The axial conduction effects of laminar flows in circular ducts for constant wall temperature and constant heat flux have been studied in this study. Finite difference method has been used to solve the governing equations [2,3]. The validity of the results has been compared with some existing results in the literature. The results were presented for bulk mean temperature variation, Nusselt number behavior, and energy absorbed before the heated section.

2. DEFINITION OF THE PROBLEM

2.1. The Geometry of the Problem

Consider a circular tube where a laminar fully developed flow is inside (Figure1). The flow is heated from x=0 by two different mechanisms. In the first case, it is heated by the wall at constant temperature. In the second case, it is heated by constant heat flux. The upstream wall is insulated.

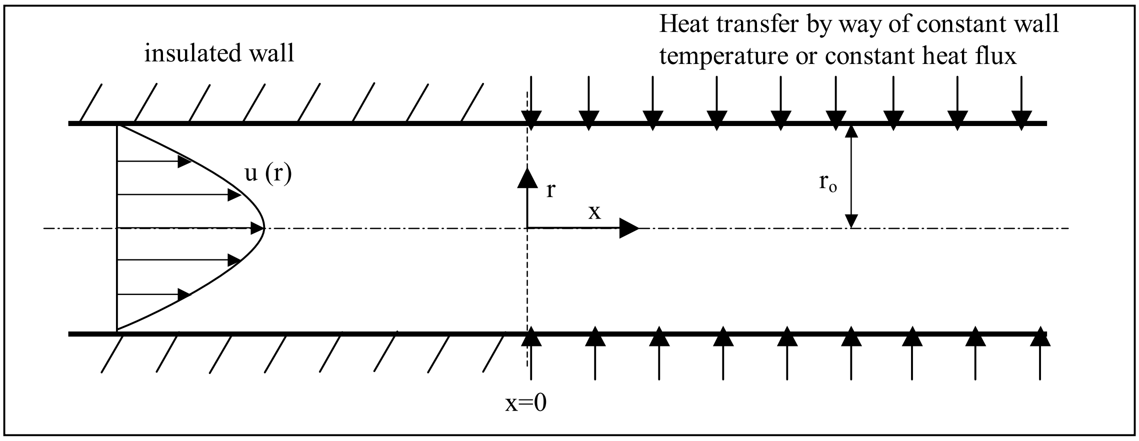

Figure 1. Laminar fully developed flow inside the geometry of the problem

2.2. The Important Parameters

The Reynolds Number for the duct flows is defined as

$$Re_D = \frac{VD}{\upsilon} \tag{1}$$

where V is mean velocity, D is diameter, υ is kinematic viscosity of the fluid. For laminar duct flows, $Re_D \leq 2300$.

The Prandtl number is defined as

$$Pr = \frac{\upsilon}{\alpha} \tag{2}$$

where α is thermal diffusivity.

The Prandtl number is:

Pr<<1 for the liquid metals

$Pr \cong 1$ for the gases

Pr>>1 for the oils.

The value of the Prandtl number strongly affects the relative growth of the velocity and thermal boundary layers. If the Prandtl number is about 1, the velocity profile and the temperature profile develop together and at the same rate.

Peclet number is defined as

$$Pe = Re \,.\, Pr \tag{3}$$

and it is a measure of the quantity of the axial heat conduction effects in the fluid. The axial conduction is assumed negligible for Pe>10 and the axial conduction term can be assumed small in the governing equation for this case. But the axial conduction effects can be significant when the Peclet number is smaller. The purpose of this numerical study is to show the importance of the axial conduction effects and the heat absorption upto the entrance from where the heating starts when the Peclet number is small.

The Nusselt number is defined as

$$Nu = \frac{hD}{k} \tag{4}$$

where k is the thermal conductivity of the fluid and h is the convection coefficient.

The velocity profile in the laminar fully developed flow for the circular tube is:

$$u(r) = 2V\left[1-\left(\frac{r}{r_o}\right)^2\right] \tag{5}$$

where r is measured from the centerline and r_o is tube radius.

The heat transfer for a duct flow can be expressed using Newton's law of cooling,

$$q'' = h(T_w - T_m) \tag{6}$$

where h is the convection heat transfer coefficient, T_w is the wall temperature and T_m is the mean temperature of the fluid, where T_m is

$$T_m = \frac{\int_{Area} \rho u c_v T dA}{\dot{m} c_v} \tag{7}$$

For the constant c_v and incompressible flow through the circular tube, T_m is

$$T_m = \frac{2}{V r_o^2} \int_0^{r_o} uTrdr \qquad [2] \tag{8}$$

3. EQUATIONS

3.1. Constant Wall Temperature Case

3.1.1 The Governing Equation

For circular duct flow, the viscous energy equations in cylindrical coordinates is

$$u\rho \frac{di}{dx} + v_r \rho \frac{di}{dr} - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r} \right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial r} \right) \right] = 0 \tag{9}$$

where i is enthalpy $di = cdT + \frac{dP}{\rho}$ [3].For constant k, Symmetric temperature distribution $\left(\frac{\partial T}{\partial \phi} = 0 \right)$, no radial velocity $(v_r = 0)$, equation becomes

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial x^2}=\frac{u}{\alpha}\frac{\partial T}{\partial x} \tag{10}$$

The equation can be nondimensionalized by

$$\theta=\frac{T_o-T}{T_o-T_e},\ r^+=\frac{r}{r_o},\ u^+=\frac{u}{V},\ x^+=\frac{x/r_o}{\mathrm{Re}\,\mathrm{Pr}},\ u^+=2(1-r^{+^2})$$

where T_o is wall temperature, T_e is fluid entrance temperature.

The governing equation becomes

$$\left(1-r^{+^2}\right)\frac{\partial\theta}{\partial x^+}=\frac{\partial^2\theta}{\partial r^{+^2}}+\frac{1}{r^+}\frac{\partial\theta}{\partial r^+}+\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial x^{+^2}} \tag{11}$$

3.1.2 Boundary Conditions

The boundary conditions for this case are:

at $r^+=0$, $\frac{\partial\theta}{\partial r^+}=0$ (symmetry boundary condition)

at $r^+=1$, $\begin{cases}\theta=0 & for\quad x^+\geq 0\\ \frac{\partial\theta}{\partial r^+}=0 & for\quad x^+<0\end{cases}$

as $x^+\to-\infty$, $T\to T_e$, $\theta\to 1$

as $x^+\to+\infty$, $T\to T_o$, $\theta\to 0$

3.1.3 The Bulk Mean Temperature

The mean temperature at a cross-sectional area of duct is :

$$T_{m(x)}=\frac{2}{Vr_o^2}\int_0^{r_o}u_{(x,r)}T_{(x,r)}rdr \tag{12}$$

Let's define the bulk mean temperature, $\theta_m = \dfrac{(T_m - T_o)}{(T_e - T_o)}$

Then the bulk mean temperature becomes:

$$\theta_{m(x^+)} = 4\int_0^1 \left(1 - r^{+2}\right)\theta_{(x^+,r^+)} r^+ dr^+ \qquad (13)$$

3.1.4 The Nusselt Number

The Nusselt number is :

$$Nu = \frac{hd}{k} \qquad (14)$$

When the convection equation $q_o^{"} = h(T_o - T_m)$ and the conduction equation $q_o^{"} = -k\left.\dfrac{\partial\theta}{\partial r}\right|_{r=r_o}$ are plugged in Nusselt number, the Nusselt number becomes:

$$Nu = -\frac{2}{\theta_m}\left.\frac{\partial\theta}{\partial r^+}\right|_{r^+=1} \qquad (15)$$

3.1.5 Heat Absorption Upto the Entrance

The energy absorbed upto the entrance is :

$$E_{absorbed} = 2\pi\rho c_p \int_{-\infty}^{0}\int_{0}^{r_o} (t - t_e) r dr dx \qquad (16)$$

when nondimensionalized with the non-dimensional parameters:

$$\frac{E_{absorbed}}{(t_o - t_e)\rho c_p r_o^3} = -2\pi Pe \int_{-\infty}^{0}\int_{0}^{1} (\theta - 1) r^+ dr^+ dx^+ \qquad (17)$$

3.2. Constant Heat Flux Case

3.2.1 The Governing Equation

The governing equation is :

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial x^2}=\frac{u}{\alpha}\frac{\partial T}{\partial x}$$

The equation can be nondimensionalized by

$$\theta=\frac{t_e-t}{q^{"}d/k},\ r^+=\frac{r}{r_o},\ u^+=\frac{u}{V},\ x^+=\frac{x/r_o}{\mathrm{Re}\,\mathrm{Pr}},\ u^+=2(1-r^{+^2})$$

The governing equation becomes

$$\left(1-r^{+^2}\right)\frac{\partial\theta}{\partial x^+}=\frac{\partial^2\theta}{\partial r^{+^2}}+\frac{1}{r^+}\frac{\partial\theta}{\partial r^+}+\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial x^{+^2}} \qquad (18)$$

3.2.2 Boundary Conditions

The boundary conditions are:

at $r^+=0$, $\frac{\partial\theta}{\partial r^+}=0$ (symmetry boundary condition)

at $r^+=1$, $\begin{cases}\frac{\partial\theta}{\partial r^+}=-\frac{1}{2} & for \quad x^+\geq 0\\ \frac{\partial\theta}{\partial r^+}=0 & for \quad x^+<0\end{cases}$

as $x^+\to-\infty$, $\quad t\to t_e$, $\quad \theta\to 0$

as $x^+\to+\infty$, $\quad \theta\to -2x^+ +\left[\frac{7}{48}+\frac{r^{+^4}}{8}-\frac{r^{+^2}}{2}\right]$ [4]

3.2.3 The Bulk Mean Temperature

The mean temperature at a position is :

$$T_{m(x)} = \frac{2}{Vr_o^2} \int_0^{r_o} u_{(x,r)} T_{(x,r)} r dr$$

Let's define the bulk mean temperature, $\theta_m = \frac{(T_e - T_m)}{q_o^{"} d / k}$

Then the bulk mean temperature becomes:

$$\theta_{m(x^+)} = 4 \int_0^1 \left(1 - r^{+2}\right) \theta_{(x^+, r^+)} r^+ dr^+ \tag{19}$$

3.2.4 The Nusselt Number

The Nusselt number is :

$$Nu = \frac{hd}{k}$$

When the convection equation $q_o^{"} = h\left(T_{x^+, r^+ = 1} - T_m\right)$ and the non-dimensional bulk mean temperature are plugged in Nusselt number, the Nusselt number becomes:

$$Nu = \frac{1}{\theta_m - \theta_{x^+, r^+ = 1}} \tag{20}$$

3.2.5 Heat Absorption Upto the Entrance

The energy absorbed upto the entrance is :

$$E_{absorbed} = 2\pi \rho c_p \int_{-\infty}^{0} \int_0^{r_o} (t - t_e) r dr dx$$

when nondimensionalized with the non-dimensional parameters:

$$\frac{E_{absorbed}}{q_o^{"} \rho c_p r_o^4 / k} = -4\pi Pe \int_{-\infty}^{0} \int_0^1 \theta r^+ dr^+ dx^+ \tag{21}$$

4. NUMERICAL RESULTS

4.1. Bulk Mean Temperature vs. x^+

The bulk mean temperature was plotted vs. x^+ for both constant wall temperature case and constant heat flux case. The results were calculated and plotted for Peclet numbers 0.5, 1, 2, 3, 5, 10, 50, and 100. The validity of Figure 2 has been checked by comparison with Ref. 6.

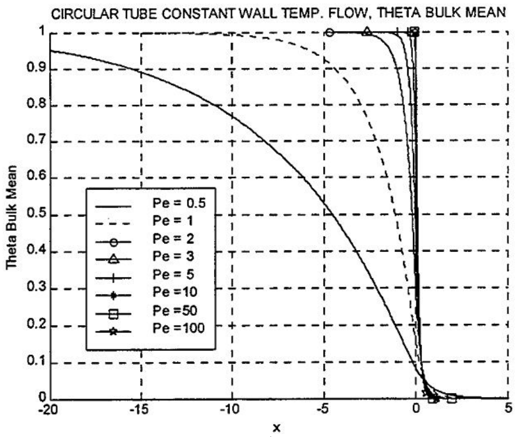

Figure 2.a : Bulk Mean Temperature vs. x^+ at Constant Wall Temperature Case

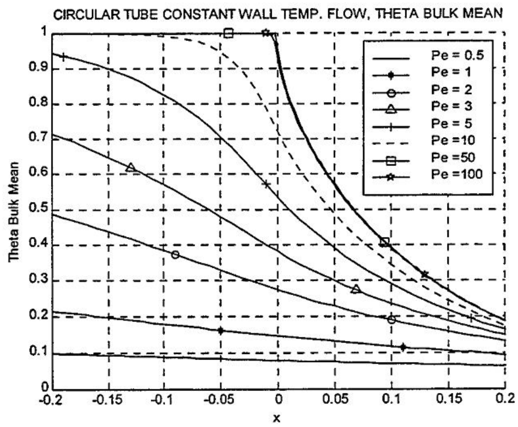

Figure 2.b : Enlarged view of Figure 2.a

The Bulk Mean Temperature vs. x^+ for constant wall temperature and constant heat flux cases are seen on Figures 2 and 3. As it is seen from the graphs, when Peclet number gets smaller, the axial conduction effects becomes very important and the temperature of the fluid upto the entrance increases. As the Peclet number increases, the axial conduction effects decrease. When the Peclet number is 50 and 100, the bulk mean temperature at x^+=0 is almost equal to the entrance temperature.

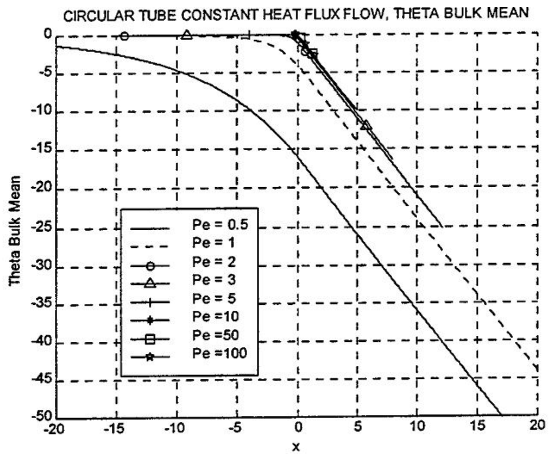

Figure 3.a. : Bulk Mean Temperature vs. x^+ at Constant Heat Flux Case

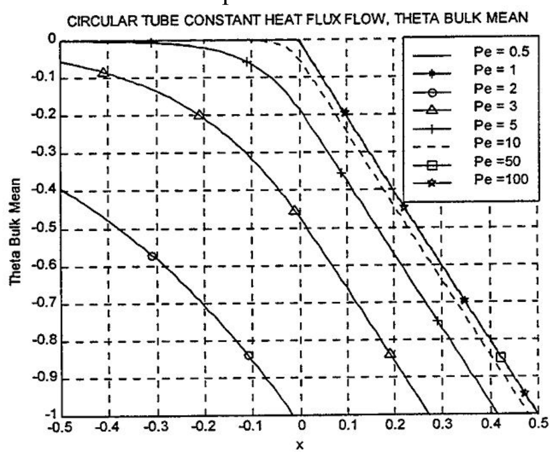

Figure 3.b : Enlarged view of Figure 3.a

4.2. Nusselt Number vs. x^+

The Nusselt number vs. x^+ was plotted for both constant wall temperature and constant heat flux cases. The results were calculated and plotted for Peclet numbers 0.5, 1, 2, 3, 5, 10, 50, and 100 to see how Peclet number affects the nusselt number distribution in the flow. For laminar fully developed conditions and for large Peclet numbers, Nusselt number goes to 3.66 for constant wall temperature. This case is plotted in Figure 4. The numerical results where Nusselt number goes to as x^+ gets larger is seen in Table 1.

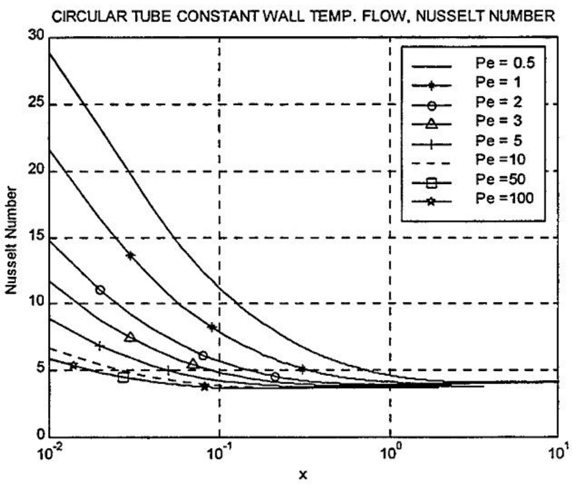

Figure 4 : The Nusselt Number vs. x^+ at Constant Wall Temperature Case

Pe	*0.5*	*1*	*2*	*3*	*5*	*10*	*50*	*100*
Nu. no	4.0971	4.0276	3.9224	3.8506	3.767	3.6948	3.6586	3.6572

Table 1 : The Nusselt Numbers for Various Peclet Numbers as x^+ goes to infinity for Constant Wall Temp Case

Nusselt number goes to 4.36 as x^+ gets larger for constant heat flux case. As it is seen on Figure 5, Nusselt number goes to 4.36 independent of Peclet number.

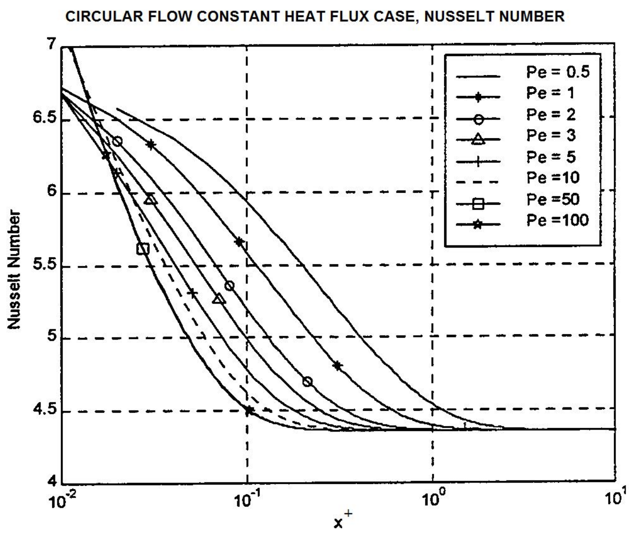

Figure 5 : The Nusselt Number vs. x^+ at Constant Heat Flux Case

4.3. Heat Absorption upto the Entrance

Since bulk mean temperature upto the entrance increases, it shows that heat absorption by the fluid upto the entrance increases by decreasing Peclet number. The absorbed heat for constant wall temperature and constant heat flux cases were plotted vs. Peclet number in Figure 6 and Figure 7.

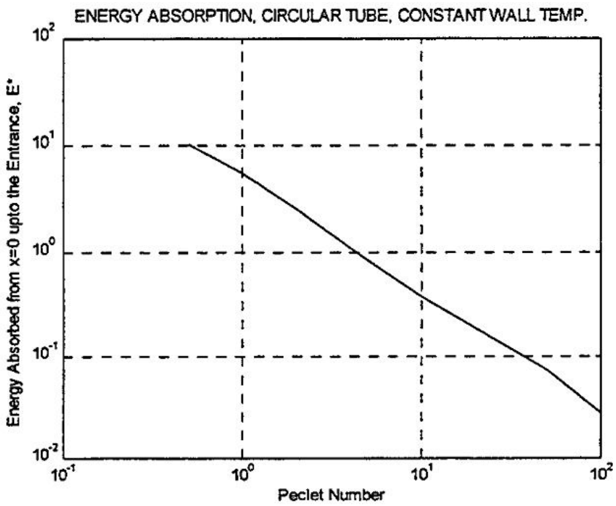

Figure 6 : The Absorbed Energy upto the Entrance for Constant Wall Temp Case

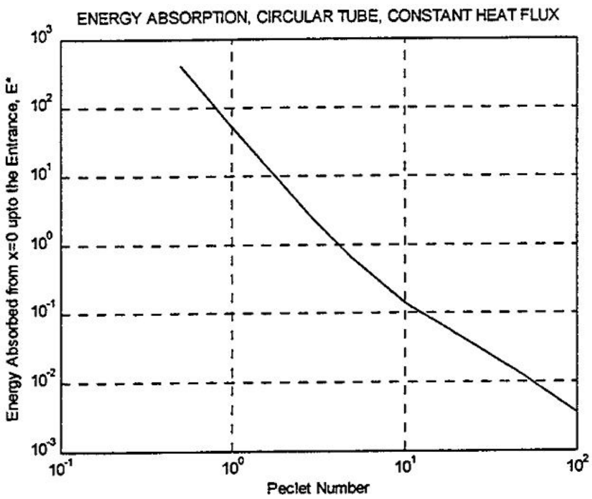

Figure 7 : The Absorbed Energy upto the Entrance for Constant Heat Flux Case

5. CONCLUSION

Axial conduction effects in duct flows are important and have to be considered for the cases of Pe<10. The bulk mean temperature and Nusselt number variations vs. downstream distance were plotted as seen in the results. It is seen obviously seen the increase in the temperature against the flow direction from x=0 as the Peclet number decreases. Heat absorption by the fluid in the insulated region was also quantified by using the nondimensional parameter E^*. The heat absorption increases logarithmically as the Peclet number decreases, as seen in the heat absorption graphs.

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