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A NOVEL NONLINEAR FREQUENCY MODULATED CHIRP SIGNAL FOR SYNTHETIC APERTURE RADAR AND SONAR IMAGING

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Abstract

In order to maintain average power levels as well as high range resolution, phase-coded signals are used in radar and sonar signal processing. One of the most commonly used phase-coded signals is the linear frequency modulated chirp waveform. Various nonlinear frequency-modulated chirps are offered as alternatives to linear frequency modulated chirp. In this paper a new nonlinear frequency modulated chirp waveform is proposed. Properties like bandwidths, point spread functions and Fourier transforms are given for the proposed chirp signal. Synthetic imagery for spotlight imaging geometry is reconstructed by using the polar format and Stolt format processing techniques using the linear frequency-modulated (LFM) and proposed nonlinear frequency modulated chirp. Comparisons are presented, and it is shown that proposed waveform can improve the sonar image resolution.

YAPAY AÇIKLIKLI RADAR VE SONAR GÖRÜNTÜLEME İÇİN YENİ BİR LİNEER OLMAYAN SIKLIK DEĞİŞİMLİ SİNYAL

Özetçe

Radar ve sonar sinyal işlemede ortalama güç seviyeleriyle birlikte yüksek çözünürlüğü sağlayabilmek için evre-kodlu sinyaller kullanılmaktadır. En sıkça kullanılan sinyallarden biri de lineer sıklık değişimli (linear frequency modulated-LFM) sinyaldir. Lineer sıklık değişimli sinyale alternatif olarak bazı lineer olmayan sıklık değişimli sinyaller de ortaya atılmıştır. Bu çalışmada yeni bir lineer olmayan sıklık değişimli sinyal önerilmiştir. Ortaya atılan sinyalin; bant aralığı, nokta dağılım fonksiyonu ve Fourier dönüşümü gibi özellikleri sunulmuştur. Lineer sıklık değişimli sinyal ve önerilen lineer olmayan sıklık değişimli sinyal kullanılarak kutupsal format ve Stolt format işleme teknikleriyle sentetik *görüntüler oluşturulmuş ve ortaya atılan sinyalin sonar görüntü çözünürlüğünü geliştirdiği gösterilmiştir.*

Keywords: *synthetic aperture radar and sonar (SAR&SAS), signal processing, phase-coded signals, nonlinear frequency modulated chirps.*

Anahtar Kelimeler: *Yapay açıklıklı radar ve sonar (SAR&SAS), sinyal işleme, evre kodlu sinyaller, lineer olmayan sıklık değişimli sinyal.*

1. INTRODUCTION

Frequency modulated chirp waveforms are one of the most widely used phase-coded signal types in the radar and sonar signal processing. The most commonly used frequency modulated chirp waveform is linear frequency modulated (LFM) waveform. However, there are some nonlinear frequency-modulated chirp waveforms offered by the researchers in the past. A review of the several proposed nonlinear frequency modulated waveforms can be seen in [1]. In this study a new frequency modulated waveform is proposed. The properties like the bandwidth, Fourier transforms and point spread functions (psf) of the proposed chirp waveform is presented and compared with those of LFM. Using the same bandwidth as those of LFM chirps, it is shown that proposed waveform can improve sonar image resolution. Also by using the polar format processing (PFP) and Stolt format processing (SFP) techniques synthetic imagery are reconstructed and it is shown that proposed waveform can improve the image resolution.

2. PROPERTIES OF THE PROPOSED CHIRP WAVEFORM

2.1. Waveforms

The most commonly used chirp waveform in radar and sonar signal processing is linear frequency modulated (LFM) chirps. While different versions are present in the literature, the most commonly used form of LFM chirp is

$$p(t)^{LFM} = \exp\left(j\phi(t)^{LFM} \right) = \exp\left(j\alpha t^2 \right) \tag{1}$$

where j is the imaginary number, α is the slew (chirp) rate and t is time. Properties of the LFM chirps are well known and can be seen in [2, 3, 4, 5]. We propose a new nonlinear frequency modulated chirp waveform as

$$p(t) = \exp\left(j\phi(t) \right) = \exp\left(j\eta t^{1.75} \right) \quad (2)$$

where η is the slew (chirp) rate. This chirp waveform has several advantages over LFM chirp as it is discussed in the coming sections. A plot of this chirp can be seen in Figure 1 below for $\eta = 100$.

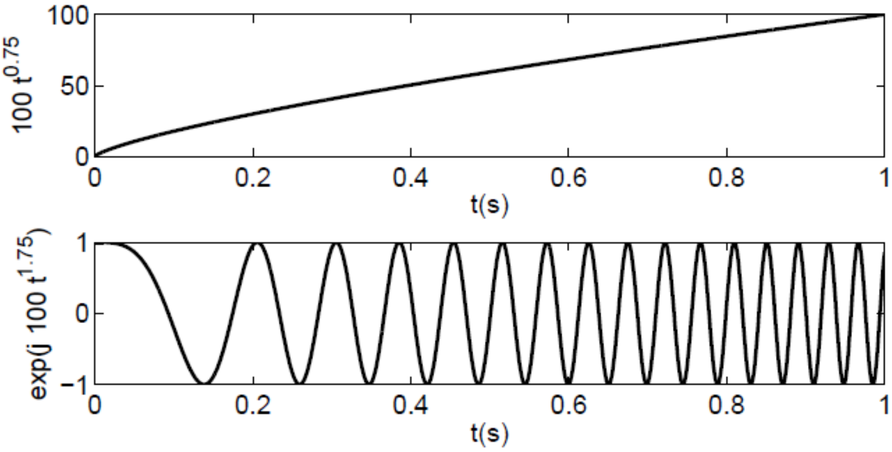


Figure 1. Plot of proposed nonlinear frequency modulated chirp for $\eta = 100$; a) Frequency modulating function b) chirp waveform.

2.2. Bandwidths

For a pulse transmitted in the time interval of $\left[0, T_p\right]$ it is straightforward to calculate the bandwidth as of the proposed chirp waveform as

$$B = \phi'(T_p) - \phi'(0) = 1.75\eta T_p^{0.75} \quad (3)$$

As discussed in [2], using a shorter pulse for this waveform the same bandwidth of the LFM pulse can be achieved. Therefore the proposed chirp waveform is more efficient since less power would be required to produce a shorter pulse.

2.3. Fourier Transforms

The Fourier transform of the proposed chirp can be calculated as

$$P(\omega) = \int_{-\infty}^{\infty} p(t) \exp\left(-j\omega t\right) dt = \int_{-\infty}^{\infty} \exp\left(j\eta t^{1.75} - j\omega t \right) dt \quad (4)$$

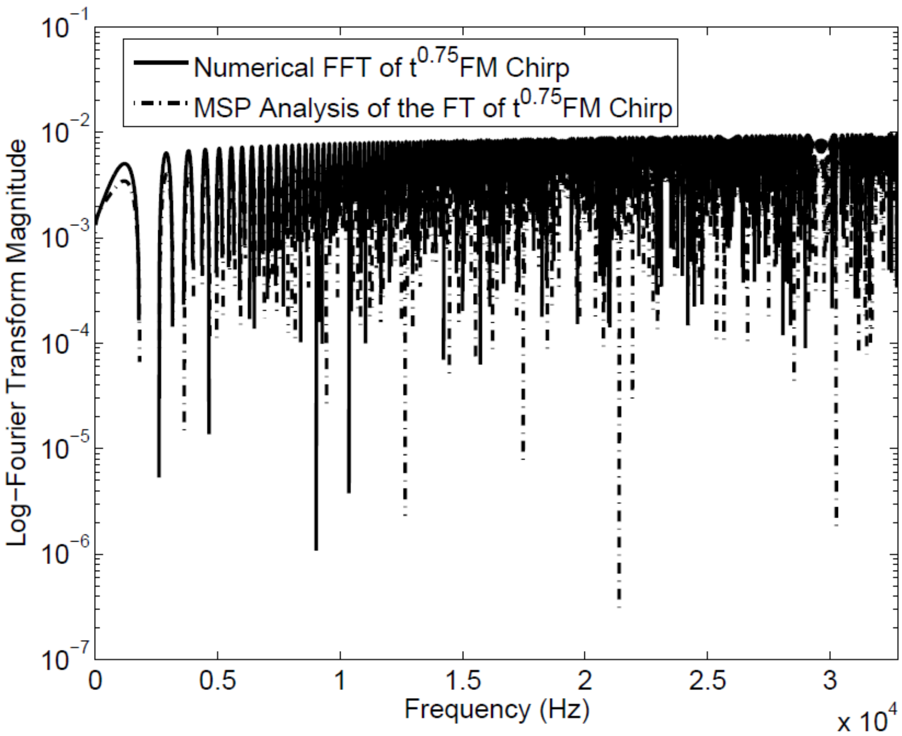


Figure 2. Comparison of the Fourier transforms of the proposed chirp waveform obtained by MSP and numerical techniques.

Using the method of stationary phases (MSP) as discussed in [2], one can evaluate the integral given in (4)

$$P(\omega) \approx 1.99\frac{\omega^{1/6}}{\eta^{2/3}}e^{\left[j\left(\frac{\pi}{4}-0.20\frac{\omega^{7/3}}{\eta^{4/3}}\right)\right]} \tag{5}$$

Details of this derivation can be seen in [2]. On the Figure 2 above, a comparison of the Fourier transforms of the proposed chirp obtained by the MSP and obtained numerically is presented.

2.4. Point Spread Functions

For $\omega \in [\omega_c - \omega_o, \omega_c + \omega_o]$ where ω_c is the carrier frequency and $\omega_o = 2\pi f_o$ is the half bandwidth, the point spread function of the proposed chirp waveform can be calculated [2, 5] as

$$psf(t) = I.F.T\left[\left|P(\omega)\right|^2\right] = \frac{1}{2\pi}\int_{\omega_c-\omega_o}^{\omega_c+\omega_o}\left|P(\omega)\right|^2 \exp\left(j\omega t\right)d\omega \qquad (6)$$

where I.F.T denotes the inverse Fourier Transform operation. Integral in the equation (6) is calculated numerically and compared with the psf of the LFM chirp waveform in Figure 3 below. As given in [6], sidelobe reduction of the psf can be achieved to any level desired. However this operation requires the usage of an envelope function which causes the power of the signal to decrease and therefore signal-to-noise ratio (SNR) decreases. This is an undesired effect. Therefore it is crucial to decrease the sidelobe level without altering the SNR. In the Figure 3 below, it is possible to see that-3 dB mainlobe width of the proposed chirp waveform and the LFM chirp waveforms are of negligible difference [2]. The reading of the peak sidelobe values approximately lead to -13.25 dB for LFM chirp and -14.30 dB for the proposed chirp waveform [2]. Therefore a reduction of 1.05 dB can be achieved using the proposed chirp without having an adverse effect either on the -3 dB mainlobe width or the SNR. Therefore image quality can be improved [2].

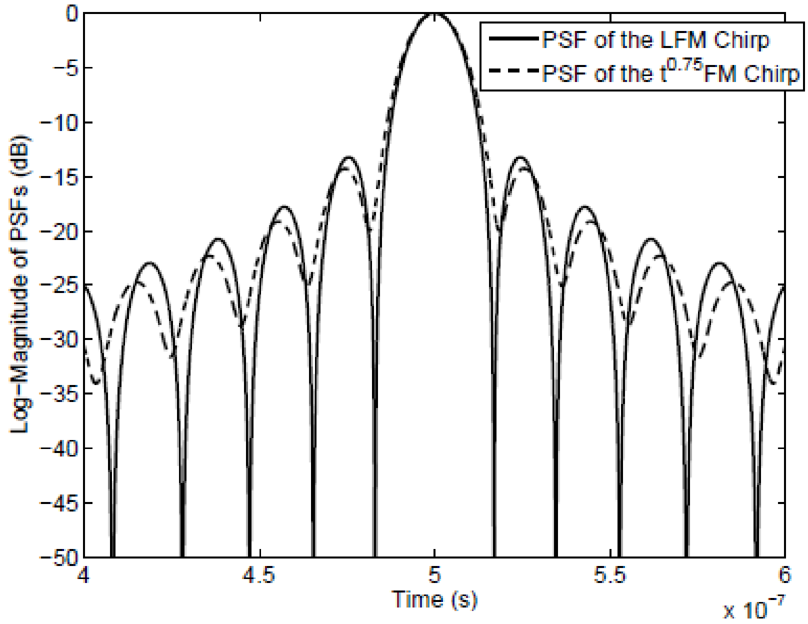


Figure 3. Comparison of the point spread functions of the proposed chirp waveform and LFM chirp waveform.

2.5. Other Possible Uses of the Proposed Waveform

One of the most successful waveforms in improving the image resolution is the hybrid waveform that includes tangent function part and the LFM part, also known as tan-LFM waveform [6]. This hybrid waveform is given by

$$p_{hybrid_1}(t) = \exp\left(j\phi_{hybrid_1}\ t \right) \tag{7}$$

where

$$\phi_{hybrid_1}(t) = \pi B\left[-\frac{2T_p\beta}{\gamma\tan(\gamma)} \ln\left|\cos(t\gamma/T_p)\right| + \frac{(1-\beta)t^2}{T_p} \right] \tag{8}$$

and

$$\phi'_{hybrid_1}(t) = \pi B\left[\frac{2\beta\tan(t\gamma/T_p)}{\tan(\gamma)} + \frac{2(1-\beta)t}{T_p} \right] \tag{9}$$

In a similar manner using the proposed nonlinear chirp waveform we propose a new hybrid chirp waveform given in the form of

$$p_{hybrid_2}(t) = \exp\left(j\phi_{hybrid\ 2}\ t \right) \tag{10}$$

where

$$\phi_{hybrid\ 2}(t) = \pi B \left[-\frac{2T_p\beta}{\gamma \tan(\gamma)} \ln\left|\cos(t\gamma/T_p)\right| + \frac{8(1-\beta)t^{1.75}}{7T_p^{0.75}} \right] \tag{11}$$

and

$$\phi'_{hybrid\ 1}(t) = \pi B \left[\frac{2\beta \tan(t\gamma/T_p)}{\tan(\gamma)} + \frac{2(1-\beta)t^{0.75}}{T_p} \right] \tag{12}$$

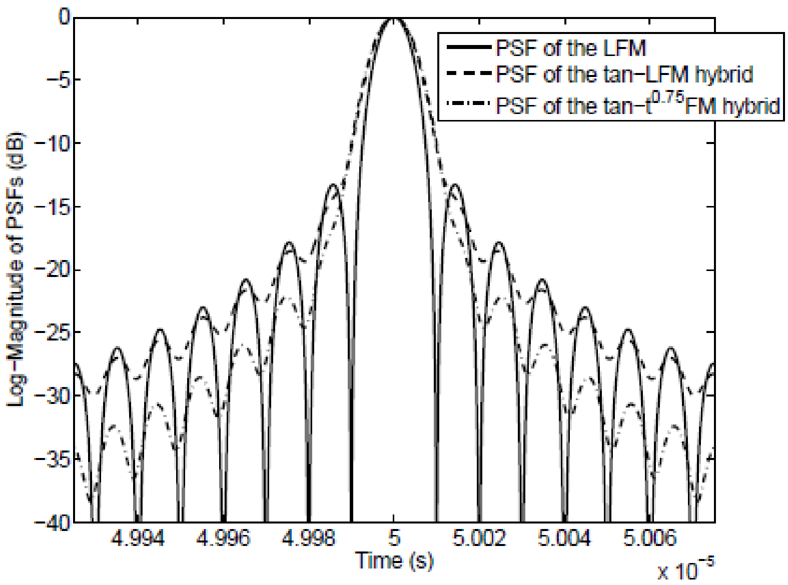


Figure 4. Comparison of point spread functions of the proposed hybrid chirp waveform and tan-LFM chirp waveform.

In Figure 4 above, a comparison of the point spread functions for the tan-LFM and hybrid waveform proposed is presented for a bandwidth of 100 MHz. For the given parameters the point spread functions of the LFM, tan-LFM and the proposed hybrid chirp waveforms are compared. This comparison suggests that although there is a slight increase in -3 dB mainlobe width, peak sibelobe values approximately lead to -13.25 dB for LFM, -18.52 dB for the tan-LFM and -22.14 dB for the proposed hybrid chirps. Therefore, a reduction of 5.27 dB can be achieved using the tan-LFM instead of LFM. A further reduction of 3.62 dB can be obtained by using proposed hybrid waveform instead of using tan-LFM [2]. Therefore, image quality can be improved using the proposed nonlinear chirp in the hybrid form as well without having an adverse affect on SNR so that image resolution can be improved.

3. REVIEW OF THE IMAGE RECONSTRUCTION TECHNIQUES

In order to show the advantages of using the proposed chirp waveform rather than using the LFM chirp waveform, the spotlight SAS/SAR imaging in considered and synthetic imagery are reconstructed. A detailed discussion of spotlight SAR imaging can be seen in some manuscripts such as [4, 5, 7]. Two common methods of spotlight image reconstruction are summarized here. These two methods are the polar format processing (PFP) and the Stolt format processing (SFP). Matched filtering for PFP and SFP are performed in the wavenumber domain thus wavenumber mappings are introduced to be used in the inverse 2D Fourier transform which is utilized in the reconstruction of the target scattering function.

3.1. Review of the Polar Format Processing

The PFP reconstruction of the target scattering function can be performed by utilizing the following steps. Considering a recorded SAS/SAR signal

$$s(t,u)=\int\int f(x,y)\,p\left[t-\frac{2\sqrt{(X_c-x)^2+(Y_c+u-y)^2}}{c}\right]dxdy \qquad (13)$$

where (X_c, Y_c) are the coordinates of spot center. Taking fast-time Fourier transform one can obtain

$$S(\omega,u)=P(\omega)\int\int f(x,y)\exp\left[-j2k_w\sqrt{(X_c-x)^2+(Y_c+u-y)^2}\right]dxdy \qquad (14)$$

where $k_w=\omega/c$ is the electromagnetic wavenumber. Applying matched filtering, denoting matched-filtered $S(\omega,u)$ as the $S(\omega,u)^M$ and performing plane wave approximation as given in [2, 4, 5], one can obtain

$$F(k_x,k_y)\approx S(\omega,u)^M\exp\left[-j2k_w\sqrt{X_c{}^2+(Y_c+u)^2}\right] \qquad (15)$$

where k_x,k_y are the PFP wavenumber mappings are given by

$$\begin{aligned} k_x &= 2k_w X_c/\sqrt{X_c{}^2+(Y_c+u)^2} \\ k_y &= 2k_w\left(Y_c+u\right)/\sqrt{X_c{}^2+(Y_c+u)^2} \end{aligned} \qquad (16)$$

By taking the inverse 2D Fourier transform of (15) one can reconstruct the target scattering function $f(x,y)$ which completes the formulation of the PFP reconstruction. A detailed discussion of spotlight SAR/SAS imaging can be seen in some books such as [4, 5, 7].

3.2. Review of the Stolt Format Processing

The SFP, or sometimes referred as digital reconstruction via spatial frequency interpolation, is performed in the aperture coordinate system. Considering a recorded signal of the form

$$s(t,u)=\int\int f(x,y)\,p\left[t-\frac{2\sqrt{x^2+(y-u)^2}}{c}\right]dxdy \qquad (17)$$

Taking the Fourier transform in the fast time domain one obtains

$$S(\omega,u)=P(\omega)\int\int f(x,y)\exp\left[-j2k_w\sqrt{x^2+(y-u)^2}\right]dxdy \qquad (18)$$

Taking the Fourier transform in the slow-time domain using method of stationary phases as given in the [2] one obtains

$$S(\omega,k_u)=P(\omega)\int\int f(x,y)\exp\left[-j\sqrt{4k_\omega^2-k_u^2}\,x-jk_u y\right]dxdy \qquad (19)$$

where

$$\begin{aligned} k_x &= \sqrt{4k_\omega^2-k_u^2} \\ k_y &= k_u \end{aligned} \qquad (20)$$

can be recognized as Stolt wavenumber mappings as first suggested in [8]. Applying matched filtering in wavenumber space one obtains

$$FF(k_x, k_y) = S(\omega, k_u)^M \tag{21}$$

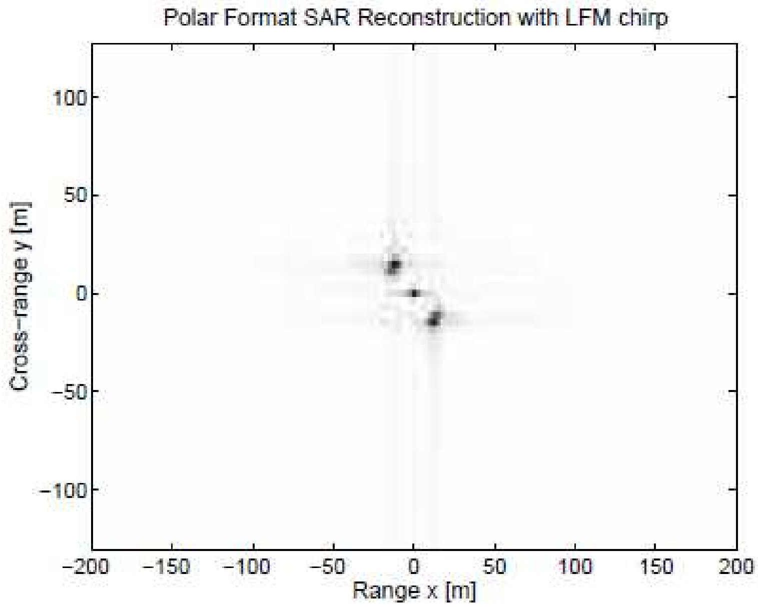
where $S(\omega, k_u)^M$ denotes the matched filtered spectrum in 2D dimensions. Then applying an inverse 2D Fourier transform to (21), one can obtain the target scattering function. A more detailed discussion of SFP can be seen in [4, 7].

4. RESULTS

4.1. Using Polar Format Processing Reconstruction

In this section, the comparisons based on PFP reconstruction technique are presented. The first example is for a scene with 5 scatterers where X_c = 1500m; Y_c = 0m; X_o = 100m; L = 150m with signal parameters T_p =10x10^{-6}s, f_c = 200x10^6 Hz, f_o = 15x10^6 Hz and where PFP reconstruction algorithms is used. In the Figure 5 below, the image reconstructed by PFP algorithm with an LFM chirp and the same image reconstructed with same algorithm but using the proposed chirp waveform is presented.

In the Figure 5 below, it can be realized that proposed nonlinear frequency modulated chirp waveforms results in less speckle so that light gray zones around the scatterers are decreased. This is due to reduced peak sidelobe levels of the proposed nonlinear frequency modulated chirp waveform compared to the LFM chirp waveform. However in this example the advantages of the using the proposed waveform is not very clear. This is due to the approximate wavenumber mappings of the PFP algorithm. Approximate wavenumber mappings limit the visualization of the improvement due to the waveform. The imagery reconstructed with SFP algorithm is presented in the next section for better visualization.


Polar Format SAR Reconstruction with LFM chirp
100
50
0
−50
−100
Cross−range y [m]
−200
−150
−100
−50
0
50
100
150
200
Range x [m]

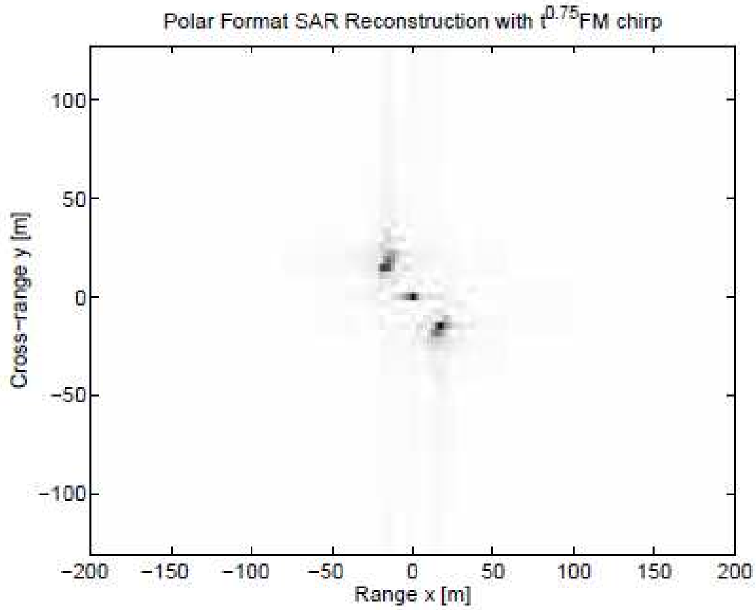

Polar Format SAR Reconstruction with t^0.75 FM chirp
100
50
0
−50
−100
Cross−range y [m]
−200
−150
−100
−50
0
50
100
150
200
Range x [m]

Figure 5. PFP reconstruction of a scene with 5 scatterers a) using the LFM chirp b) using the proposed nonlinear frequency modulated chirp.

4.2. Using Stolt Format Processing Reconstruction

In this section, the comparisons based on SFP reconstruction technique are presented. The second example is for the same scene as in the first example which has five scatterers and the same parameters are used. However, this time the SFP reconstruction algorithm is used. In the Figure 6 below, the image reconstructed by SFP algorithm with an LFM chirp and the same image reconstructed with the same algorithm but using the proposed chirp waveform is presented.

As it can be realized in the Figure 6 below, proposed chirp waveform performs much better so that 5 scatterers in the scene is clearly identified in the second image. However in the image reconstructed by the LFM chirp waveform this identification is not possible. It is also possible to realize that speckle in the form of gray shaded zones is reduced. It is much easier to realize enhancements in the imagery reconstructed by SFP algorithm compared to imagery reconstructed by PFP algorithm for different chirp waveforms. This occurs since SFP wavenumbers are exact so that less distortion are created in the synthetic imagery. With improved radar and sonar image resolution using the proposed chirp waveform, the remote sensing quality for various purposes such as discussed in [3, 4, 5, 7, 9] can be enhanced.

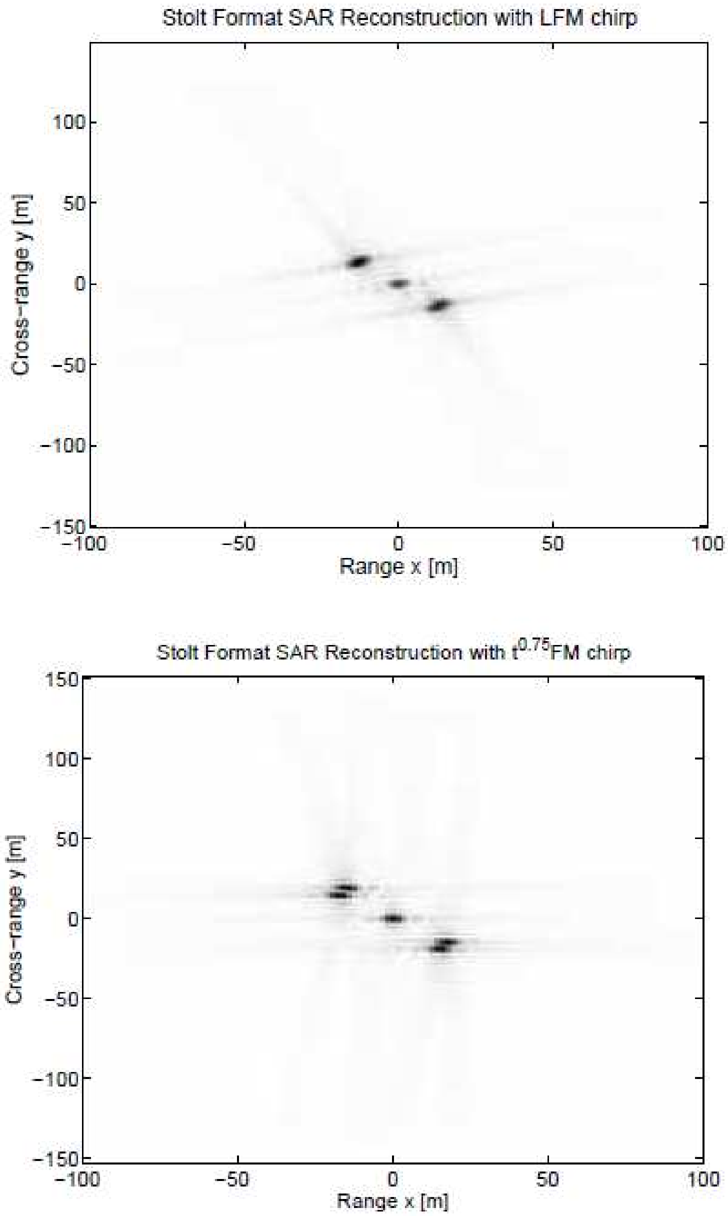


Figure 6. SFP reconstruction of a scene with 5 scatterers a) using the LFM chirp b) using the proposed nonlinear frequency modulated chirp.

5. CONCLUSION

In this paper a new nonlinear frequency modulated chirp signal is proposed as an alternative to the commonly used linear frequency-modulated (LFM) chirp signal. It is shown that for the same bandwidth, the proposed waveform has smaller peak sidelobes compared to the LFM waveform while the difference in the -3dB mainlobe width is of negligible difference. Also proposed form does not cause a reduction in the SNR, therefore can be effectively used. It is also shown that proposed waveform is more advantageous compared to LFM when used in the hybrid form similar to hybrid waveforms known as tan-LFM. By utilizing the polar format processing and Stolt format processing algorithms for the spotlight synthetic aperture sonar imaging geometry, it is shown that proposed waveform can be used to reconstruct imagery with better resolution. The proposed chirp waveform can be used both for improving the radar and sonar image resolution.

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