Risk Measures and Risk Capital Allocation

Uğur Karabey

Hacettepe University
Department of Actuarial Sciences
06800-Beytepe, Ankara, Türkiye
ukarabey@hacettepe.edu.tr

Abstract

The fundamental problem of the portfolio/risk management is the measurement and the allocation of risk. Various risk measures provide a solution to the former problem. However, in the recent decade's risk measures have been criticised dramatically and a new concept so called 'coherent risk measures' have been arisen. In the meantime allocation distributes the diversification benefits among the constituents of the portfolio. We show that risk allocation can provide a better risk management if one pays the necessary attention for the choice of the risk measure and the allocation method.

Keywords: risk capital allocation, risk measures, Black-Scholes model

Özet

Risk Ölçümleri ve Risk Sermayesi Dağıtımı

Riskin ölçülmesi ve dağıtılması portföy/risk yönetiminde karsılasılan temel sorunlardır. Birçok risk ölçümü risklerin ölçülmesi için çözümler sunar. Fakat son yıllarda risk ölçümleri önemli ölçüde elestirilmis ve ortaya tutarlı risk ölçümleri olarak adlandırılan yeni bir yaklasım çıkmıstır. Aynı zamanda risk dağıtımı çesitlendirmeden kaynaklanan faydaların portföyü olusturan birimlere dağıtımını yapar. Bu çalısmada risk ölçüm ve risk dağıtım yöntemlerini doğru seçen yöneticilerin daha iyi bir risk yönetimi sağlayabileceğini gösterdik.

Anahtar sözcükler: risk ölçümleri, risk dağıtımı, Black-Scholes modeli

1. Introduction

Financial institutions such as banks, firms or insurance companies (which we refer by the term portfolios) have to hold cash reserves as a cushion against unforeseen losses. This amount is called risk capital and regulators determine this capital by a measure of risk. Therefore, we can define the risk capital as the minimum amount of cash that managers have to add to their portfolios for their risk to be acceptable to the regulator.

Many risk measures exist in the literature, however lately main interest shifted to coherent risk measures that is defined by some axioms; positive homogeneity, monotonicity, sub-additivity and translation invariance [2]. We will analyse these axioms in the following sections.

On the other hand, risk capital allocation is required for management decision support, performance measurement, profitability assessment and building optimal risk-return portfolios [11]. The risk capital allocation in financial institutions has already been discussed by several authors [20, 15, 24, 26, 27, 8, 21, 10, 29, 19, 7]. Another important problem related with the allocation is the coherency. The concept of coherent allocation of risk capital has been introduced by [10] which defines some set of properties to be fulfilled by an allocation method where the calculation of risk capital is based on a coherent risk measure.

The purpose of this paper is: (i) to explore various risk measures and allocation methods; (ii) to discuss coherency of these measures and allocation methods; (iii) to simulate a stock portfolio and to discuss effects of different combinations of risk measures and allocation methods on allocations.

2. Measures of Risk

Given a probability space (Ω, F, P), we will consider the vector space $L^p(\Omega, F, P)$, or just $L^p(P)$, for $1 \leq p \leq \infty$. We treat $L^p(P)$'s elements as random variables. We have $\| X \|_p = (E_P | X |^p)^{1/p}$. A risk measure, ρ, is a mapping from a set of random variables $L^p(P)$, $1 \leq p \leq \infty$ to the real line R, i.e.

$$\rho : L^p(P) \to R$$

$$X \to \rho(X)$$

Precisely, an unacceptable risk can be transformed into an acceptable risk by adding other instruments to the position and the cost of this instrument (minimum cash) measures the risk of the position. Alternatively risk measures can be obtained from acceptance sets. An acceptance set $A \subseteq L^p(P)$ is a set of all 'acceptable' risks. This set is determined by regulators or investment managers of a company. For instance, an acceptance set could be all positions with a profit. For more details, see [2].

2.1. Value at Risk

In the 1980's, many financial institutions tried to find models to measure risk. As firms became complex, it was very important to measure the financial risk which they are exposed to. In 1994, JP Morgan constructed a measure system that was based on standard portfolio theory using estimates of the standard deviations and correlations between the losses to different instruments. It's system: Value at Risk (VaR); measures the maximum potential loss of a given portfolio over a prescribed holding period at a given confidence level α where $\alpha \in (0,1)$ and it can be described as,

$$VaR_\alpha(X) = - \inf\{x \in R : P(X \leq x) > \alpha\}. \tag{2.1}$$

VaR also can be seen as a negative α quantile of the distribution function of X. The rapid rise of VaR was due to its certain characteristics:

• VaR provides a common measure of risk across different positions and risk factors.

• VaR enables us to aggregate the risks of positions taking account of the ways in which risk factors correlate with each other.

• VaR is probabilistic and gives useful information on the probabilities associated with specified loss amounts.

• VaR is expressed in simple and understandable way, namely, 'lost money' [12].

In 1996, the Basel Committee approved the use of VaR for calculating capital requirements for banks and VaR has become the most widely used risk measure, heretofore. However, VaR is deficient for many reasons. VaR is capable of measuring the worst loss but it fails to address how large this loss can be, if the α probability events occur. VaR can not consider tail losses beyond the selected quantile, therefore, it is risky to rely VaR completely. Another important problem with VaR is nonconvexity. So, it is mostly impossible to find unique global minimum in optimization problems. The most important problem of VaR is, it can penalize the diversification in portfolios instead of rewarding it. For these reasons, a number of consistent risk measures have been introduced in the literature.

2.2. Coherent Measures of Risk

Shortcomings of VaR led many researchers to seek alternative risk measures. In 1997 Artzner et al. criticized

VaR and introduced a more theoretical approach to risk measurement in their study [1]. In their context, they define some properties that a good risk measure should satisfy. However, VaR does not belong to these -so called- coherent risk measures. The theory of coherent risk measures relies on the idea that an appropriate risk measure is consistent with economic intuition and finance theory.

A risk measure $\rho : L^p(P) \rightarrow R$, for $1 \leq p \leq \infty$ is called a *'coherent risk measure'* on $L^p(P)$ if it satisfies the following properties.

Subadditivity: For all $X_1, X_2 \in L^p(P)$

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$$

This property ensures that the combination of two positions reduces the risk; therefore it represents the 'diversification' effect.

Positive Homogeneity: For all $X \in L^p(P)$ and for all real numbers $\lambda \geq 0$ we have

$$\rho(\lambda X) = \lambda \rho(X)$$

Positive Homogeneity ensures that the risk of a position depends linearly on the size of the position.

Monotonicity: For all $X_1, X_2 \in L^p(P)$

$$\rho(X_1) \geq \rho(X_2), \text{ if } X_1 \leq X_2 \text{ a.s.}$$

Monotonicity represents that a position X_2 with a higher value than a position X_1 has a lower risk.

Translation Invariance: For all $X \in L^p(P)$ and for all real numbers a
$$\rho(X + a) = \rho(X) - a$$

Translation Invariance property states that adding cash to a position reduces the risk by the same amount. For more details, see [2,18]. Note that we assume interest rates are zero; therefore there is no discounting factor in definitions.

VaR is not a coherent risk measure due to the missing subadditivity property. This property expresses the fact that a portfolio made of sub-portfolios will risk an amount which is at most the sum of the separate amounts risked by its sub-portfolios. This is the most important character of a risk measure. For a sub-additive measure, portfolio diversification always lead to risk reduction, while for measures which violate this property, diversification may produce an increase in the risk.

2.3. The Expected Shortfall

$$ES_\alpha(X) = -E[X \mid X \leq -VaR_\alpha(X)] \tag{2.2}$$

is called expected shortfall (ES) at level α (α usually close to 1) which is defined as an average of VaRs of X at level α and higher. VaR is only the minimal loss in the 'bad' cases which happens with the probability α, whereas expected shortfall measures the average loss in these 'bad' cases. ES is very familiar to actuaries, it is also known as the Conditional Tail Expectation (CTE) or Tail VaR. In the mean time, it has been variously labelled as Expected Tail Loss, Tail Conditional Expectation, Conditional VaR, Tail Conditional VaR and Worst Conditional Expectation in financial risk management area. There is no consistency of terminology in either literature. In the case of continuous loss distribution, all of these measures give the same result but in the discrete case they can differ.

Expected shortfall belongs to the family of coherent risk measures and it provides better approach for risk management. It is more sensitive to the shape of the loss distribution and it counts tail of the loss distribution completely. For more details, see [18].

2.4. Further Measures of Risk

We now define some particular risk measures which are commonly used in the literature.

The Standard Deviation

$$\rho_{sd}(X) = \sigma(X) = \sqrt{Var(X)} \qquad (2.3)$$

where Var(X) denotes the variance. The standard deviation is a widely used measure of the variability or dispersion. It shows how much variation there is from the mean. A lower standard deviation indicates that the data points tend to be very close to the mean, whereas a higher standard deviation indicates that the data are spread out over a large range of values. This measure is not coherent because it is not translation invariant. Also, it penalizes not only the risk of a return below the mean but also the risk of a return above the mean. For more details, see [18].

A risk measure based on standard deviation

$$\rho_{sd,a}(X) = -E[X] + a\,\sigma(X),\ a > 0 \qquad (2.4)$$

where E[X] denotes the expected value and $\sigma(X)$ the standard deviation. This measure includes a risk load that is proportional to the standard deviation of the risk. This risk measure is translation invariant, subadditive and positively homogeneous. However, it is not monotonic for a properly chosen X, hence, is not coherent [6]. We refer this measure by the term MSD, hereafter.

A risk measure based on one-sided moments

$$\rho_{p,a}(X) = -E[X] + a\,\sigma_p^-(X),\ a > 0 \qquad (2.5)$$

where X^- defined as $\max\{-X, 0\}$, $\sigma_p^-(X) = \left\| (X - E[X])^- \right\|_p$ for $1 \le p \le \infty$. It is well known fact that most of the loss distributions are skewed. Therefore, if one understands risk as an asymmetrical concept related to outcomes below the mean (or target level), standard deviation is inadequate as both positive and negative deviations from the mean increase risk. Considering this, one needs to measure the downside risk which is measured by semi-standard deviation. This measure is coherent if $0 \le a \le 1$, as shown by [13]. We refer this measure by the term MSSD, hereafter.

3. Allocation of Risk Capital

Once a manager has determine the risk capital of the portfolio (or company), she must then allocate it back to each risk component in the portfolio. The allocation of risk capital has its own challenges due to the nature of dependence structures of combined risks. There are many motives behind the risk capital allocation. Firstly, by comparing different losses on capital for each component, it is often possible to answer if a component is worth to keep or not. Secondly, since capital is defined as a risk measure of whole company, one can assess the riskiness of each component's position by splitting this capital, and compare one to another. Another motive is, allocation provides a useful device for assessment of performance of managers, which can be linked to their compensations. Last but not least, insurers may want to use the allocation in pricing [22, 30].

Consider now that a portfolio has *n* sub-portfolios where N={1,2,...,*n*} is the set of all sub-portfolios. Each sub-portfolio's loss is represented by X_i, $i \in N$ then, aggregate loss of the portfolio can be described as $\sum_{i=1}^{n} X_i = X$

In the literature, many researchers have proposed a set of axioms that any desirable allocation method is expected to satisfy [10, 16, 17]. The following Axioms are adapted from [10].

Let *D* be the set of risk capital allocation problems: pairs (N,ρ) composed of a set of *n* lines and a coherent risk measure ρ. Allocated capital for line *i* is denoted by a_i. An allocation is a functional $\Pi: D \to R^n$ that maps each allocation problem, (N,ρ), into a unique allocation:

$$\begin{bmatrix} \Pi_1(N,\rho) \\ . \\ . \\ . \\ \Pi_n(N,\rho) \end{bmatrix} = \begin{bmatrix} a_1 \\ . \\ . \\ . \\ a_n \end{bmatrix} \tag{3.0}$$

3.1. Properties for a coherent Allocation

An allocation Π is said to be coherent, if for every allocation problem (N,ρ) satisfies the following properties.

Full Allocation: The allocated capitals add up to the total capital.

$$\rho(X) = \sum_{i=1}^{n} a_i$$

No Undercut: The risk of any subset M of the total risk N is always lower than the sum of stand-alone risks of that subset.

$$\forall\, M \subseteq N,\ \sum_{i \in M} a_i \le \rho(\sum_{i \in M} X_i)$$

Symmetry: For any subset $M \subseteq N \setminus \{i, j\}$, if sub-portfolios i and j make the same contribution to the risk capital of subset M , then $a_i = a_j$. This property ensures that a sub-portfolio's allocation depends only on its contribution to risk within the portfolio.

Riskless Allocation: Assume that last portfolio (line) is riskless with the initial price 1 and strictly positive price r in any state of nature at time T. Therefore, $X_n = \alpha r$ and

$a_n = \rho(X_n) = \rho(\alpha r) = -\alpha$

According to this axiom, a riskless portfolio should be allocated exactly its risk measure which can be negative. It is easy to see that this axiom is related to the translation invariance axiom of coherent risk measures.

3.2. Methods of Allocation

There are various allocation methods available in the literature. Theoretical and practical aspects of different allocation methods have been analyzed in a number of papers [9, 10, 13, 23, 27, 28, 29].

3.2.1. Proportional Allocation

Proportional allocation is a naive allocation method which is given by

$$a_i^{P,\rho} = \frac{\rho(u_i X_i)}{\sum_{j \in N} \rho(u_j X_j)} \rho(X(u))$$

(3.1)

where diversification effect is distributed proportionally to the risks. This method simply calculates stand-alone risk measures for each risk and then allocates the total risk capital in proportion to separate risk measures. This approach guarantees the full allocation principle. However, it is not coherent as an allocation method and it ignores the stochastic dependencies between the risks.

3.2.2. Variance-Covariance Allocation

This allocation method, proposed by [23], is given by

$$a_i^{V-C,\rho} = \frac{Cov(u_i X_i, X(u))}{Var(X(u))} \rho(X(u))$$

(3.2)

where $Cov(u_i X_i, X(u))$ is the covariance between the individual risks $u_i X_i$ (sub-portfolios) and aggregate risk (portfolio) X(u), Var(X(u)) is the variance of the portfolio X(u). This method focuses on how individual business units contribute to the standard deviation of the portfolio. It's clear that sum of these individual covariance's is equal to the variance of the portfolio. Therefore, the full allocation principle is satisfied. This relation can be given by

$$Var(X(u)) = Var\left(\sum_i u_i X_i\right) = \sum_i u_i Cov\left(X_i, \sum_j u_j X_j\right) = \sum_i \sum_j u_i u_j Cov(X_i, X_j)$$

(3.3)

3.2.3. Merton-Perold Allocation

This method is based on the option pricing model of the firm. In this approach, the value of the policyholders' claim on the firm is equal to the present value of losses minus the value of the 'insolvency put option'. The insolvency put option is the expected loss due to the possibility of default of the firm. Simply for one period model the firm issues policies at time 0 and claim payments occur at time 1. If assets exceed liabilities at time 1, the firm pays the losses and the equity owners receive the difference between assets and liabilities. However, if liabilities exceed assets, the insurer defaults and the policyholders receive the assets. Therefore the payoff of this option at time 1 is L − max(L − A, 0), where L is losses and A is assets and max(L − A, 0) is the payoff on the insolvency put option [8]. Merton-Perold approach is an incremental capital allocation which focus on what happens to the insolvency put option if all sub-portfolios are added or removed from the firm. Then the allocated capital for line i is given by

$$a_i^{S,\rho} = \sum_{S \subseteq N} \frac{(|s|-1)! \, (n-|s|)!}{n!} (\rho(S) - \rho(S \setminus \{i\}))$$

(3.4)

An important characteristic of this allocation method is that the incremental amounts do not add up to total risk capital. Hence, it is not coherent.

3.2.4. Game Theoretical Allocation

Game-theoretic methods provide a suitable framework for cost allocation problems [4, 5, 25]. Shapley method is an example for these methods. It describes how coalitions can be formed in a way that none of the players benefits more as a stand-alone than as a group. Aumann-Shapley method is another example for that kind of methods and it allows for fractional units and requires less computation compared to the Shapley method.

A coalitional game (N, ρ) consists of

• a finite set of *n* players, N,

• a cost function, ρ, which associates a real number, ρ(S), to any subset (coalition) S, of N .

Then each player wants to minimise her cost and her strategies consist of agreeing or not to take part in coalitions. The main question in coalitional games is the allocation of the cost, ρ(N), between all players and this question is formalized by the concept of value.

A value is a functional Π which maps each game (N, ρ) into a unique allocation as in equation (3.0).

The core of a game
Given the subadditivity of ρ, the players of a game have an incentive to take part in the coalition, since by doing this they minimise the total cost when compared to the their stand-alone costs. Any player does not take part in the coalition if her allocated cost is higher than her stand-alone cost. The set of allocations that do not allow any player to be apart from coalition is called the core. *The core* of a coalitional game, (N, ρ), is the set of allocations $a \in R^n$ for which $\sum_{i \in S} a_i \leq \rho(S)$ for all coalitions S⊆N.

The Shapley value
The Shapley value was introduced by [25] as a method for each player to expect a benefit from playing a game and ever since has received interest. The Shapley value for the game (N, ρ) can be given by,

$$a_i^{S,\rho} = \sum_{S \subseteq N} \frac{(|s|-1)! \, (n-|s|)!}{n!} (\rho(S) - \rho(S \setminus \{i\}))$$

(3.5)

where *s* is the number of players in coalition S and *n* is the total number of players. Note that for any game with n players there are $2^n - 1$ nonempty possible coalitions and the calculation of the Shapley Value may become harder if n gets bigger. The Shapley Value can be seen as expected marginal benefit added by each player. For more details, see [25].

The Aumann-Shapley value
[5] extended the concept of Shapley value to non-atomic games. Here non-atomic means divisible players/portfolios. The Aumann-Shapley value can be given by

$$a_i^{A\&S,\rho} = u_i \int_0^1 \frac{\partial}{\partial u_i} \rho(tX(u)) dt$$

(3.6)

for a positively homogeneous risk measure ρ this simplifies to

$$a_i^{A\&S,\rho} = u_i \frac{\partial \rho(X(u))}{\partial u_i}$$

(3.7)

where the payoff $X(u) = \sum_{i=1}^n u_i X_i$ of a portfolio $u = (u_i)_{1 \leq i \leq n} \in R^n$ consists of sub-portfolios with payoffs Xi. In the case of a positively homogeneous, convex and differentiable cost function the core of such a game consists one element: the gradient of the cost function due to normed weights of players [3]. Therefore, in the case of subadditive and positively homogeneous risk measure which is differentiable at a portfolio u ∈ R^n, the gradient is the 'unique fair' per unit allocation. For more details, see [10].

Euler Principle: Consider a function ρ : L^p(P) → R, if ρ is positively homogeneous and differentiable at u ∈ R^n, then we have

$$\rho(X(u)) = \sum_{i=1}^{n} u_i \frac{\partial \rho(X(u))}{\partial u_i}$$
(3.8)

Under Euler principle the capital allocated to the sub-portfolio X_i of X is the derivative of associated risk measure ρ at X in direction of X_i. For a positive homogeneous and differentiable risk measure the Aumann-Shapley method coincides with the Euler method. Game theoretical methods and the Euler method are coherent allocation methods. The Euler principle is the most-preferred allocation method in the literature due to its coherent properties, for more details see [18]. Note also that in this study all used risk measures are positive homogeneous and differentiable, therefore Euler method represents the game theoretical methods.

For the calculation of allocations under Euler principle we need partial derivatives of risk measures with respect to asset weights. Detailed information about partial derivatives of risk measures can be found in [18].

4. Scenario and a Simulation Study

In order to show how theory works, we used a stock portfolio of five companies and data covers the period from February 2003 to February 2012. We assume that given total wealth is £1,000,000 which is equally weighted to sub-portfolios. We consider a one-period framework; therefore between time 0 and T no trading is possible. Note also that we study 1 year time horizon, thus T =1. We assume 'risk' to be given by a random variable X representing a cash flow at time T. Precisely, the corresponding loss variable at time T can be defined by X = X(T) − X(0). Individual stock returns are modelled by geometric Brownian motions (GBM), i.e. price processes $X_i(i = 1,...,5)$ with

$$X_i(t) = X_i(0) \exp\left((\mu_i - \sigma_i^2 / 2)t + \sigma_i \sqrt{t} Z\right), \ t \geq 0$$
(4.0)

where μ_i is the drift, σ_i is the diffusion coefficient, $X_i(0)$ is the price of the i-th asset at time 0 and Z is a standard normally distributed random variable.

With stochastic simulation we compute N market scenarios and by using these realizations we can compute estimates using the empirical distribution given by the simulation output. For simulation we need a discretization. For more details about discretization, see [14]. Note that used time-step is 1/12 (prices are monthly) for discretization. Estimated geometric Brownian motion parameters are given in Table 1. Estimated covariance matrix of hypothetical historical increments of underlying Brownian motions is given in Table 2. For computation of risk measures and risk capital allocations we simulated N=100,000 realizations of the random vector X = (X_1, ..., X_5).

Allocation proportions of stock portfolio are given in Table 3. Allocations based on different allocation methods and different risk measures indicate that allocations show significant differences. We prefer the combination of expected shortfall and the Euler method due to their coherent properties. By assuming this combination results fair allocations, we can see how other risk measures and allocation methods perform.

Table 1: Parameters of Stock Return Distributions

Parameters	BP Ltd	GSK Ltd	PRU Ltd	TOMK Plc	TSCO Plc
			Portfolio - Risk Sources		
Drift μ	0.093	0.012	0.112	0.016	0.151
diffusion σ	0.197	0.172	0.356	0.319	0.207

Table 2: Covariance matrix of hypothetical historical increments
of underlying Brownian motions.

	BP Ltd	GSK Ltd	PRU Ltd	TOMK Plc	TSCO Plc
			Covariance Matrix		
BP Ltd	1.0000	0.1884	0.2279	0.1447	0.1753
GSK Ltd	0.1884	1.0000	0.2996	0.2189	0.4457
PRU Ltd	0.2279	0.2996	1.0000	0.5480	0.4292
TOMK Plc	0.1447	0.2189	0.5480	1.0000	0.3034
TSCO Plc	0.1753	0.4457	0.4292	0.3034	1.0000

PRU Ltd. and TOMK Plc. turn out the most risky portfolios whereas BP Ltd. and TSCO Plc. seem less risky. Proportional method draws some inconsistencies compared to the other methods. BP Ltd. and TSCO Plc. have negative allocations under MSSD risk measure for other methods. However, proportional method has positive allocations for these portfolios. Allocations under this method are also dramatically different from allocations under the fair combination.

Variance-Covariance method gives equal allocations with the combination of standard deviation and Euler method because this method is a fair allocation method for standard deviation (derivative of the standard deviation equivalent to the Variance-Covariance method).

Allocation differences between expected shortfall and VaR are also noteworthy. By considering the fact that VaR is much sensitive to the confidence level, we can expect that these differences become more considerable for higher confidence levels.

Table 3: Allocation Proportions of Sub-Portfolios at 95% Confidence Level

Risk Measures	BP Ltd	GSK Ltd	PRU Ltd	TOMK Plc	TSCO Plc
		Euler Method			
Value at Risk	6.51	15.32	34.71	32.83	10.63
Exp. Shortfall	**9.25**	**14.84**	**33.04**	**30.88**	**11.99**
S. Deviation	10.34	10.12	36.46	25.77	17.31
MSSD	-14.36	27.38	45.03	67.87	-25.92
MSD	-0.34	15.61	42.37	42.50	-0.14
		Proportional Method			
Value at Risk	14.57	16.37	27.80	28.56	12.69
Exp. Shortfall	15.14	16.30	27.15	27.30	14.11
S. Deviation	15.73	12.66	29.80	24.09	17.71
MSSD	9.16	22.43	28.58	41.08	-1.25
MSD	12.08	16.76	30.29	32.63	8.23
		Merton-Perold Method			
Value at Risk	3.63	16.01	37.67	35.05	7.64
Exp. Shortfall	6.32	15.32	35.19	32.48	10.68
S. Deviation	9.31	10.22	37.36	25.56	17.55
MSSD	-36.35	37.70	57.09	95.12	-53.56
MSD	-5.84	17.59	46.07	47.49	-5.32
		Shapley Method			
Value at Risk	8.45	15.91	33.37	32.78	9.49
Exp. Shortfall	10.21	15.54	31.82	30.61	11.81
S. Deviation	11.96	10.93	34.50	25.12	17.50
MSSD	-10.39	29.23	40.80	66.23	-25.87
MSD	2.52	17.02	38.92	41.35	0.19
		Variance-Covariance Method			
All Risk Measures	10.34	10.12	36.46	25.77	17.31

5. Conclusions

In this paper we have explored different risk measures and risk capital allocation methods. We have discussed their properties and have compared them by a simulation study. A more comprehensive simulation study which considers alternative risk models, examination of combinations of different risk models, risk measures and allocation methods have to be postponed to future research.

We used the Euler method as a fair unique allocation method based on the results of game theory and risk adjusted performance management. Hence, comparisons are done between the Euler and other allocation methods. We found that not only choice of risk measure is important for efficient risk management but also choice of allocation method really matters. Managers can provide better risk management by giving the necessary attention to the choice of risk measures and allocation methods.

References

[1] P. Artzner, F. Delbaen, J.M. Eber, and D. Heath, 1997, Thinking coherently. RISK, 10:68-71.

[2] P. Artzner, F. Delbaen, J.M. Eber, and D. Heath, 1999, Coherent measures of risk, Mathematical Finance, 9-3: 203-228.

[3] J-P. Aubin, 1979, Mathematical Methods of Game and Economic Theory, North-Holland Publishing Co., Amsterdam.

[4] J-P. Aubin, 1981, Cooperative fuzzy games, Mathematics of Operations and Research, 6-1:1-13.

[5] R. J. Aumann and L. S. Shapley, 1974, Values of Non-Atomic Games, Princeton University Press, Princeton.

[6] A. Buch and G. Dorfleitner, 2008, Coherent risk measure, coherent capital allocations and the gradient allocation principle, Insurance: Mathematics and Economics, 42:235-242.

[7] A. Buch, G. Dorfleitner, and M. Wimmer, 2009, Rethinking risk capital allocation in a rorac framework.

[8] J.D. Cummins, 2000, Allocation of capital in the insurance industry, Risk Management & Insurance Review, 3:7-27.

[9] F. Delbaen and E.. Ossische, 2000, Technische Hochschule. Coherent risk measures on general probability spaces, ETH Zurich.

[10] M. Denault, 2001, Coherent allocation of risk capital, Journal of Risk, 4-1:7-21.

[11] J. Dhaene, A. Tsanakas, E. A. Valdez, and S. Vanduffel, 2010, Optimal capital allocation principles, Journal of Risk and Insurance, Forthcoming.

[12] K. Dowd and D. Blake, 2006, Discussion paper pi-0603 after var: The theory, estimation and insurance applications of quantile-based risk measures, Technical report, The Pensions Institute.

[13] T. Fischer, 2003, Risk capital allocation by coherent risk measures based on one-sided moments. Insurance: Mathematics and Economics, 32-1:135-146.

[14] T. Fischer and A. Roehrl, 2003, Risk and performance optimization for portfolios of bonds and stocks, Proceedings of the International AFIR Colloquium

[15] K.A. Froot and J.C. Stein, 1998, Risk management, capital budgeting, and capital structure policy for financial institutions: in integrated approach. Journal of Financial Economics, 47:55-82.

[16] O. Hesselager and U. Anderson, 2002, Risk sharing and capital allocation, Technical report, Tryg Insurance, Ballerup, Denmark.

[17] M. Kalkbrener, 2005, An axiomatic approach to capital allocation. Mathematical Finance, 15-3:425-437.

[18] U. Karabey, 2012, Risk Capital Allocation and Risk Quantification in Insurance Companies, PhD thesis, Heriot-Watt University.

[19] J. Kim and M. Hardy, 2008, A capital allocation based on a solvency exchange option. Insurance: Mathematics and Economics, 44-3:357-366.

[20] R. C. Merton and A. Perold, 1993, Theory of risk capital in financial firms. Journal of Applied Corporate Finance, 6:16-32.

[21] S. C. Myers and J. A. Read, 2001, Capital allocation for insurance companies. Journal of Risk and Insurance, 68:545-580.

[22] M. B. Neil, 2007, Capital allocation by percentile layer. Enterprise Risk Management Symposium, Society of Actuaries.

[23] L. Overbeck, 2000, Allocation of economic capital in loan portfolios. In Measuring Risk in Complex Stochastic Systems, W. Hardle & G. Stahl, Berlin.

[24] F. Saita, 1999, Allocation of risk capital in financial institutions. Financial Management, 28:95-111.

[25] L. Shapley, 1953, A value for n-person games. In Contribution to the Theory of Games, Princeton University Press Vol. 2.

[26] N. M. Stoughton and J. Zechner, 1999, Optimal capital allocation using raroc and eva, Technical report, University of California Irvine und University of Vienna.

[27] D. Tasche, 1999, Risk contributions and performance measurement, Technical report, Research paper, Zentrum Mathematik (SCA).

[28] D. Tasche, 2002, Expected shortfall and beyond. J. Banking Finance, 26:1519-1533.

[29] M. Urban, J. Dittrich, C. Klüppelberg and R. Stölting, 2004, Allocation of risk capital to insurance portfolios, Blatter DGVFM Deutsche Gesellschaft fur versicherungs und finanzmathematike, V. XXVI, 3:389-406.

[30] A. E. Valdez and A. Chernih, 2003, Wang's capital allocation formula for elliptically contoured distributions. Insurance: Mathematics and Economics, 33:517-532.