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Using Lagrangian Relaxation Method for Asset Management Problems

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Abstract: This paper addresses an asset management problem in the context of the wind energy industry. Asset management decisions (including operation and maintenance, retrofitting and purchasing) for assets reached their end-of-life are explicitly examined in a linear programming model over a planning horizon. Unfortunately, almost all important generic classes of integer programming problems are NP-hard and many of these problems are large-size. Therefore, in order to solve practical integer programming problems we may need to use problem specific algorithms which can exploit some special structures of the problem at hand. We propose a solution approach based on a Lagrangian relaxation and the subgradient method for a large size parallel asset management problem, which originally solved by using mixed integer linear programming (MILP). The decomposition approach considers the relaxation of different sets of constraints, including the budget and energy constraints. The computational results show that the incorporation of Langrangian relaxation significantly improves the duality gap and solution time of a case study from wind turbine (WT) sector.

Using Lagrangian Relaxation Method for Asset Management Problems

Anahtar Kelimeler
Rüzgâr türbünü,
Karışık tam sayılı doğrusal programlama,
Langrangian gevşemesi

Öz: Bu makale, rüzgâr enerji sektöründe kullanım ömrünü doldurmuş rüzgar türbinlerinin varlık yönetimini ele almaktadır. Kullanım ömrü dolmuş varlıklar için mevcut olan varlık yönetimi kararları (işletme ve bakım, güçlendirme ve satın alma dahil) bir planlama ufku üzerinden doğrusal bir programlama modelinde açıkça incelenir. Ne yazık ki, tamsayı programlama problemlerinin neredeyse hepsi NP-Zor (NP-Hard)'dur ve bu problemlerin çoğu büyük boyuttadır. Bu nedenle, pratik tamsayı programlama problemlerini çözmek için, problemin bazı özel yapılarından faydalanabilecek probleme özgü algoritmalar kullanmak gerekebilir. Bu çalışmada, başlangıçta karma tamsayı doğrusal programlama (MILP) kullanılarak çözülen büyük boyutlu bir paralel varlık yönetimi problemi için Lagrange gevşetmesi ve alt gradyan yöntemine dayanan bir çözüm yaklaşımı önerilmiştir. Bütçe ve enerji kısıtları gevşetilerek çözüm elde edilmeye çalışılmıştır. Vaka çalışması olarak Rüzgar türbinlerinin kullanıldığı model sonuçları incelendiğinde, bu yöntem sayesinde, dual aralığı ve çözüm süresinin önemli ölçüde azaldığı gözlenmiştir.

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1. Introduction

As assets are utilized overtime, operating and maintenance costs increases. Therefore, an asset manager has to deal with the tradeoff between the lower operating and maintenance (O&M) costs of newer fleets and their higher initial capital costs as well as the tradeoff between conventional and new efficient technologies. Due to the nature of the problem, asset management by itself (i.e. keeping an asset with preventive maintenance or replacing with a new one) is quite similar to a parallel replacement problem. The traditional approach to solving asset management problems is using economic service life approach, dynamic programing or mixed integer linear programming (MILP). In the literature, there are limited number of studies completed in the field of asset management problems. Therefore, in this section, most of the current studies are up to year 2018.These studies are summarized below.

Jones et al. (1991) analyzed the parallel machine replacement problem using dynamic programming and assuming that a fixed charge was incurred in each period of replacement, thus proving that cluster of the same-age assets would either be kept or replaced (no-splitting rules) as a group in any period if demand was constant [1]. Hartman (2000) examined the impact of fluctuating demand and budgeting consideration of the parallel replacement problem under economies of scale, where the purchase of an asset is subject to a fixed charge, regardless of the order size, and a capital budgeting constraint in each period. He recognizes that problem can get difficult when the number of assets is large because of the large state-space [2]. Karabakal et al. (1994) addressed a different parallel replacement problem with capital budgeting constraints solved by a branch-and-bound algorithm [3]. Karabakal et al. (2000) provide a heuristic multiplier adjustment method for solving large, realistically sized problems [4]. Hartman and Dearden (1999) presented integer programming solutions that allow decision makers to determine minimum cost-replacement strategies with variable utilization schedules and categorize assets based on age and cumulative utilization [5]. Hartman and Clarke (2002) integrated a production planning problem with a parallel replacement problem, and illustrated how their integer programming model accurately predicts costs production decisions and may alter replacement decisions [6]. Hartman (2004) examined asset replacement decisions, based on age and cumulative utilization, under variable periodic utilization [7]. Laksuwong et al. (2014) presented a new approach for the parallel fleet replacement problem. In this study, the aim was to determine an optimal replacement schedule for a fleet of vehicles that results in minimal total cost of owning and operating the fleet [8]. Seif and Rabbani (2014) assessed life cycle costing (LCC) of equipment based on the failure rates of machine components. The LCC was assessed, mathematically modeled, and incorporated to the parallel machine replacement problem with capacity expansion consideration [9]. Chen (1998) study solution algorithms for the parallel replacement problem under economy of scale [10]. Abhishek (2000), examined three or more assets and higher number of discrete utilization levels per asset under stochastic demand by different algorithm including exhaustive search dynamic programing, demand approximation, relaxed integrated approximation and decision approximation. Based on the results, it was concluded that decision approximation algorithm provided the best result[11]. Ching-Jung and Kuo-Rui (2016), proposed to use Lagrangian relaxation heuristic that can be an effective solution method for the capacitated p hub median location problem with multiple capacity levels [12]. Hartman (2000) studied the parallel asset replacement problem under economies of scale and non-decreasing demand with branch-and-cut. He showed that parallel replacement problems (PRP) are NP-hard [13].

Mainly two options, either keeping asset with preventive maintenance or purchase a new asset has been considered for PRP. Yet, there can be another alternative, which is retrofitting, for asset management in the field of wind energy. There exists little literature regarding the PRP with retrofitting as an improvement options in an optimization model. Therefore, there is a need for such an asset management model that provides the strategic operation and management plan to decision makers under budget and energy constraints. To close this gap, Cinar et al. (14) prosed a MILP to determine the optimal replacement scheduling incorporating retrofitting options into the model under a limited budget [12]. As this proposed parallel replacement problem with retrofitting (PRP-R) considered as NP-hard, to reduce the solution time for a large problems, we propose an approximate solution approach based on Lagrangian relaxation and the subgradient method as an alternative to obtain approximate solutions. Coupled with a Lagrangian relaxation, this should provide us with feasible solutions and estimates of the potential error. Our main objective have been to analyze the potential of this approach for a difficult asset management problem and, hence, we have applied the wind farm. To the best of our knowledge, this is the first study that Lagrangian relaxation algorithm developed for PRP-R. Therefore, the proposed model can be considered as the main research contribution. Our computational results show that the incorporation of Langrangian relaxation significantly improves the integrality gap and reduces the solution time of a case study problem of the WTs asset management.

Brief summary of the Lagrangian relaxation method is given in Section 2. A description of the problem is outlined in Section 3 and the mathematical model, which takes form of a MILP, is then presented in Section 3.1. Section 3.2 briefly describe the sub-gradient algorithm used for the model. Section 4 presents a case study that validates the model. Finally, Section 5 presents conclusions and suggestions for future research.

2. Langrangian Relaxation for Proposed Model

The power of commercial MILP solvers has improved greatly over the last ten to fifteen years. This is due partly to much faster Linear Programing (LP) solvers, which allow a much quicker processing of the nodes in the Branch-and-Bound tree. In addition, increased use of logical processing and heuristic tools for tightening models and for finding solutions help the process. Despite the fact that some of the combinatorial optimization problems are still very difficult to solve using Branch-and-Bound alone. This seems to be the case for problems containing different components that are only loosely connected together by constraints.

In our original problem, when we increase the size of the problem, we observe that computational time to solve the problem increased exponentially. Therefore, in order to solve the problem faster, we develop and implement Lagrangian relaxation algorithm to pick a set of complicating constraints, which, if relaxed, is much easier to solve than the original problem.

Lagrangean relaxation was developed in the early 1970's with the pioneering work of Held and Karp on the travelling salesman problem. [15]. Lagrangian relaxation is a relaxation and decomposition method to solve mathematical programming problems. The main idea of Lagrangian relaxation is to separate the constraints as easy constraints and the hard constraints, and eliminate the hard constraints from the constraint set of the mathematical programming model [16]. Lagrangian relaxation is a technique commonly used to relax these complicating constraints in order to have an easier problem to optimize [17]. The method of Lagrangian relaxation lifts these complicating constraints and makes use of the special structure to solve the relaxed problem. This produces a lower bound for a minimization problem. There are two general techniques are available: sub-gradient optimization and multiplier adjustment methods. With Lagrangian relaxation, a good approximate solution or the best (tight) bound can be obtained by subgradient optimization methods where a subgradient vector is obtained by minimizing the relaxed primal problem and the dual variables are updated iteratively along the direction of this subgradient vector [18]. Lagrangian relaxation with subgradient optimization method is provided in literature by several authors [17].

Considering PRP-R being NP-hard, in this study, we use the Lagrangian relaxation with subgradient optimization method for our problem. We provide insight into solving large-scale problems with the use of Langrangian relaxation method for the problem developed in previous research [14]. Considering the motivations above, the objective of this work is to extend the PRP-R model that we develop to aid the replacement and retrofitting decision-making in wind farms. The work focuses on the economic and financial aspects of the replacement problem, trying to identify the optimal replacement, retrofitting or recycling timing and the consequences of alternative decisions.

3. Solving Original Parallel Replacement Problem-Retrofitting (PRP-R) Mathematical Model by Langrangian Relaxation

The model used in this study is originally proposed by Cinar et al. (2018) [14]. In this model, a MILP is developed to determine the optimal replacement policy for the WT industry. The proposed MILP model consists of formulating the total cost, including the cost for operating, retrofitting and replacing WTs. As PRP-R model defined as NP-Hard, we propose to solve the original model by using Langrangian relaxation to reduce the solution time.

3.1. Model Details

In the original MILP model, we select the constraint (3) and (4) as complicating constraints and modify the objective function by adding two difficult constraints to the objective function and solving the problem with subgradient optimization. Constraint 3 and 4 are dualized into original objective function. We represent the notation and model formulation with a network illustration, shown in Figure 1 [14].

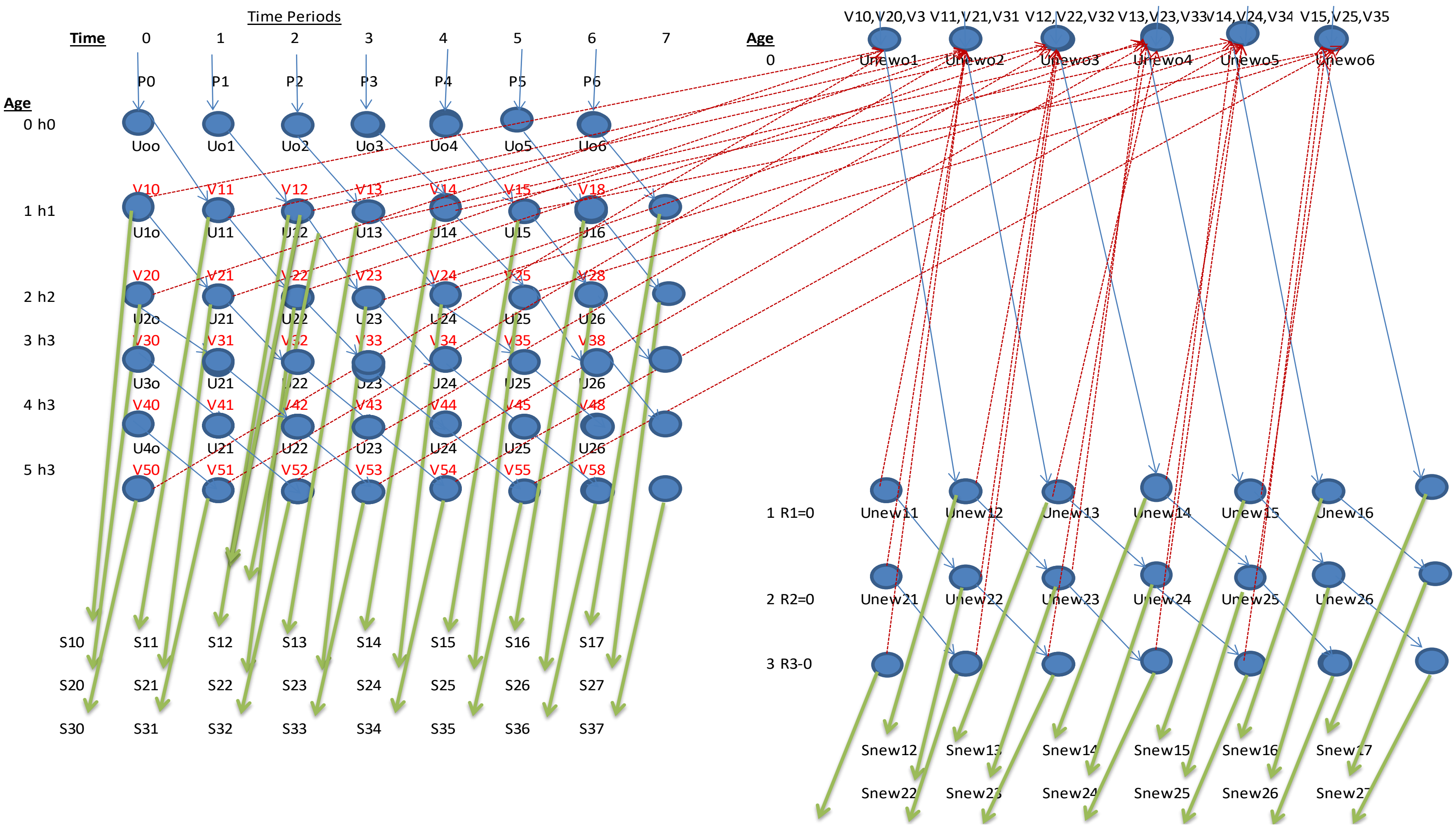

Figure 1. Representation of RRR as a network with flow representing purchase (P), retrofitting (V, VN), utilization (U), salvage (S), retrofitted WT salvaged (SNEW), retrofitted WT utilized (UNEW) variables, and initial WT (h, R), initial retrofitted WT (R) with N=3 and T=7. [124]

The model for the proposed problem has the following sets, indexes, parameters, cost, and decision variables.

i:	asset age, $i \in I = \{1,\ldots,\|I\|\}$
t:	time period, $t \in T = \{1,\ldots,\|T\|\}$

Parameters:

B_t:	budget at time period t (\$)
D_t:	energy demand at time period t (kW)
F_t:	fixed cost of retrofit at the end of year t (\$)
EO_{it}:	energy production of an old asset at age i at time period t (kW)
EN_t:	energy production of a new asset at time period t (kW)
ER_t:	energy production of a retrofitted asset at time period t (kW) ($EN_t \geq ER_t$)
E_t:	unit cost of electricity at time period t (\$/Kw)
h_i:	number of existing assets at age i (initial cluster size)
r_i:	number of retrofitted assets at age i at time period zero

Costs:

OP_i:	operation and maintenance costs for i year old asset receiving preventive maintenance (\$)
OR_i:	operation and maintenance costs for i year old asset receiving retrofit (\$)
SA_{it}:	salvage revenue (negative cost) from selling i year old assets at time period t (\$)
SR_{it}:	salvage revenue (negative cost) from selling i year old retrofitted asset at time period t (\$)
θ_t:	cost of purchasing a new asset at the end of year t (\$)
dr:	inflation rate

Decision Variables:

U_{it}:	number of i-year-old assets that received preventive maintenance at end of year t
UN_{it}:	number of i-year-old assets that received retrofitting and utilized at end of year t
V_{it}:	number of i-year-old assets that received retrofit at end of year t
VN_{it}:	number of i-year-old retrofitted assets that received another retrofit at end of year t
P_t:	number of assets purchased at end of year t (new asset)
S_{it}:	number i-year-old assets salvaged at end of year t
SN_{it}:	number i-year-old retrofitted asset salvaged at end of year t for retrofitting

The objective function, shown as equation (1), includes the operation and maintenance cost, purchase cost of a new asset, and fixed cost of retrofitting the main components (i.e., structure, motors, gearbox, blades, generator, etc.) of an existing asset and minimizes the sum of purchasing, maintaining, operating, and salvaging over the period of analysis, i.e., from time zero (present) to the end of year t. Constraints (2) and (3) provide that there is enough energy production to satisfy periodic demands at time zero and after time zero, respectively. Constraint (4) shows that purchase costs, operation and maintenance costs, and fixed costs cannot exceed the yearly budget. Constraints (5) to (10) are referred to as flow conservation constraints. The age of any asset in use will increase by one year after each time period (5). Constraint (6) ensures that the conservation of assets, i.e., the initial asset (not 0-age ones) must be used, sold, or retrofitted. At the end of the last time period, there will be no asset in use for any age or type of asset, i.e., all assets will be sold. Constraint (7) is the initial boundary condition for an asset at age 0 at time period t is equal to a newly purchased plus retrofitted asset at period $t-1$ (previous period). Constraints (8) and (9) are flow balance constraints for the last period of an existing asset, which should be either retrofitted or salvaged from the previous period. Constraint (10) is a flow balance constraint for the number of assets utilized during year zero, which must be equal to the sum of existing assets plus purchased assets. The decision variables associated with purchasing, utilization, retrofitting, and salvaging decisions must be integer positive numbers, as shown in expression (11). To be able to determine if retrofitting is profitable as a WT asset management strategy, we modified the objective function based on total profit, which includes total electricity production price minus total operating, purchasing, and retrofitting costs.

$$Minimize\ z = \sum_{t\in T\setminus\{|T|\}} \sum_{i\in I\setminus\{|I|\}} OP_{it} * U_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}} \sum_{i\in I\setminus\{|I|\}} OR_{it} * UN_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}} \Theta_t * P_t * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}} \sum_{i\in I} F_{it} * V_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}} \sum_{i\in I} F_{it} * VN_{it} * (1+dr)^{-t} - \sum_{t\in T} \sum_{i\in I\setminus\{0\}} SA_{ti} * S_{it} * (1+dr)^{-t} - \sum_{t\in T} \sum_{i\in I\setminus\{0\}} SR_{ti} * SN_{it} * (1+dr)^{-t} \quad (1)$$

Subject to:

$$\sum_{i\in I\setminus\{|I|\}} U_{i(to)} * EO_i + P_{(t0)} * EN_{(i0)} \geq D_{(t0)} \quad (2)$$

$$P_{t+1} * EN_{(i)} + \sum_{i\in I} V_{(i+1)t} * ER_{(i)} + \sum_{i\in I} VN_{(i+1)t} * ER_{(i)} \sum_{i\in I} U_{i(t+1)} * EO_i + \sum_{i\in I} UN_{i(t+1)} * EN_i \geq D_{(t+1)} \qquad t \in T\setminus\{|T|\} \quad (3)$$

$$\sum_{i\in I\setminus\{|I|\}} U_{it} * OP_{it} + \sum_{i\in I\setminus\{|I|\}} UN_{it} * OR_{it} + \sum_{i\in I\setminus\{0\}} V_{it} * F_{it} + \sum_{i\in I\setminus\{0\}} VN_{it} * F_{it} + \sum_{t\in T\setminus\{|T|\}} P_t * \Theta_t - \sum_{i\in I\setminus\{0\}} S_{it} * SA_{ti} - \sum_{i\in I\setminus\{0\}} SN_{it} * SR_{ti} \leq B_t \qquad t \in T\setminus\{|T|\} \quad (4)$$

$$U_{(i-1)(t-1)} = U_{it} + V_{it} + S_{it} \qquad i \in I\setminus\{0\}, \quad t \in T\setminus\{0\} \quad (5a)$$

$$UN_{(i-1)(t-1)} = UN_{it} + VN_{it} + SN_{it} \qquad i \in I\setminus\{0\}, \quad t \in T\setminus\{0\} \quad (5b)$$

$$h_i = U_{i0} + V_{i0} + S_{i0} \qquad i \in I\setminus\{0\} \quad (6a)$$

$$r_{i0} = 0 \qquad i \in I\setminus\{0\} \quad (6b)$$

$$U_{(0)(t+1)} = P_{t+1} + \sum_{i\in I} V_{it} \qquad i \in I\setminus\{0\}, \quad t \in T\setminus\{|T|\} \quad (7a)$$

$$UN_{(0)(t+1)} = \sum_{i\in I} V_{it} + \sum_{i\in I} VN_{it} \qquad i = \{|I|\}, \quad t \in T\setminus\{|T|\} \quad (7b)$$

$$U_{(I-1)t} = S_{I(t+1)} + V_{I(t+1)} \qquad i = \{|I|\}, \quad t \in T\setminus\{|T|\} \quad (8a)$$

$$UN_{(|I|-1)t} = SN_{|I|(t+1)} \qquad i = \{|I|\}, \quad t \in T\setminus\{|T|\} \quad (8b)$$

$$U_{i(|T|-1)} = S_{(i+1)|T|} \qquad i \in I\setminus\{|I|\} \quad (9a)$$

$$UN_{i(|T|-1)} = SN_{(i+1)|T|} \qquad i \in I\setminus\{|I|\}, \quad t = \{|T|\} \quad (9b)$$

$$U_{00} = P_0 + h_0 \quad (10)$$

$$U_{it},\ S_{it}, SN_{it}, V_{it}, VN_{it}, UN_{it,}, P_t \in Z+ \qquad i \in I, t \in T \quad (11)$$

3.2. Model Details

In the PRP-R, when the size of the problem is increased, it can be seen that the computational time to solve it increases exponentially. In order to solve the problem faster, a Lagrangian relaxation algorithm is proposed. This algorithm penalizes a set of complicating constraints to the objective function, which results in a problem easier to solve than the original problem, and a subgradient optimization method is used to update the penalty.
In other words, the Lagrangian relaxation method lifts these complicating constraints and makes use of the special structure to solve the relaxed problem. This produces a lower bound for a minimization problem. There are two general techniques to implement a Lagrangian relaxation algorithm: sub-gradient optimization and multiplier adjustment methods [19]. Several authors, including Fisher (2004) and Lemar´echal (2001) use Lagrangian relaxation with the subgradient optimization method [15, 20]. A multiplier adjustment method is an iterative method that generates a series of monotonically increasing lower bounds. In general, a multiplier adjustment method requires less iteration than the subgradient method per iteration. On the other hand, it cannot guarantee lower bounds that are a good as those produced by the subgradient method [21]. As, subgradient method has always appeared to give good lower bounds which is often close to the optimal integer solution, this method is usually preferred.

In the problem here, constraints (3) and (4) (production and budget constraints, which are inequalities) are selected as complicating constraints and the objective function is modified by penalizing these constraints to the objective function and solving the problem using the subgradient optimization technique. The resulting objective function is as follows:

$$Minimize\ \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I\setminus\{|I|\}} OMPN_{it} * U_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I\setminus\{|I|\}} OMLR_{it} * UN_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}} PUR_t * P_t * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I} FRET_{it} * V_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I} FRET_{it} * VN_{it} * (1+dr)^{-t} - \sum_{t\in T}\sum_{i\in I\{0\}} SAL_{ti} * S_{it} * (1+dr)^{-t} - \sum_{t\in T}\sum_{i\in I\setminus\{0\}} SAL_{ti} * SN_{it} * (1+dr)^{-t} + \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I\setminus\{|I|\}} \Phi_{it} * (EnDem - P_{t+1} * ENNEW_{(i0)} + \sum_{i\in I} V_{(i+1)t} * ENRET_{(i0)} + \sum_{i\in I} VN_{(i+1)t} * ENRET_{(i0)} \sum_{i\in I} U_{i(t+1)} * ENOLD_i + \sum_{i\in I} UN_{i(t+1)} * ENNEW_i) + \sum_{t\in T\setminus\{|T|\}}\sum_{i\in I\setminus\{|I|\}} \delta_{it} * (\sum_{i\in I\setminus\{|I|\}} U_{it} * OMPN_{it} + \sum_{i\in I\setminus\{|I|\}} UN_{it} * OMLR_{it} + \sum_{i\in I\setminus\{0\}} V_{it} * FRET_{it} + \sum_{i\in I\setminus\{0\}} VN_{it} * FRET_{it} + \sum_{t\in T\setminus\{|T|\}} P_t * PUR_t - \sum_{i\in I} S_{it} * SAL - \sum_{i\in I} SN_{it} * SAL_{ti} - Budget) \quad (12)$$

Subject to constraints (2), (5a), (5b), (6a), (6b), (7a), (7b), (8a), (8b), (9a), (9b), (10), and (11).

The subgradient algorithm used for this problem is shown below. Two Lagrangian relaxation multipliers based on MILP formulation are used for the original problem. First, the budget and energy constraints are relaxed; then a relaxation with the integrality property is obtained. This is useful for solving the MILP relaxation of the original problem approximately. Relaxation is obtained by dualizing the set of constraints (4) and (5) with Φ_{it} and $\delta_{it} \geq 0$, where Φ_{it} *and* δ_{it} are multipliers associated with budget and energy constraints, respectively, of period t.

$$\Phi_{it}\ \ and\ \ \delta_{it} \geq 0 \quad i \in I \text{ and } t \in T \quad (13)$$

The pseudo code for the proposed Lagrangian algorithm is presented in Table 1.

Table 1. Pseudo Code for Proposed Subgradient Optimization Algorithm for PRP-R

Algorithm Steps	Explanation for each step
Input: LLBP();	//Lagrangian lower bound
Input: Lup;	//Upper bound value for original problem
{Initialization}	
$\theta := \theta init$;	//Subgradient agility, suggested $\theta init$=2 [22]
An initial value $\Phi^0 \geq 0$, $\delta^0 \geq 0$	//Lagrangian multiplier
$Lmax = -\infty$;	//Best lower bound so far
for μ=0,1,…. {Sub-gradient iterations}	
$\gamma^t \coloneqq g(x^t)$ $\sigma^t \coloneqq g(y^t)$	//Gradient of $L(\Phi^t)$ and //Gradient of $L(\delta^t)$
$\partial_t \coloneqq \theta_t(L^* - L(\Phi^t))/\|\| \gamma^t\|\|^2$ and $\psi_t \coloneqq \theta_t(L^* - L(\delta^t))/\|\| \sigma^t\|\|^2$	//Compute step size
$\Phi^{t+1} \coloneqq \max\{0, \Phi^t + \partial_t\gamma^t\}$ and $\delta^{t^{t+1}} \coloneqq \max\{0, \delta^{t^t} + \psi_t\sigma^t\}$	//Update step size
If $\|\|\Phi^{t+1}$-$\Phi^t\|\| < \varepsilon$ and If $\|\|\delta^{t+1}$-$\delta^t\|\| < \varepsilon$ then	// Suggested $\varepsilon = 0.01$(1)

Stop
End if
If no progress in more than K iterations, then
$\theta_{t+1} := \theta_t/2$
	//Reduce agility

else
$\theta_{t+1} := \theta_t$
End if
$\mu := \mu + 1$
end for	

4. Results and Discussion

For each instance we first solved the original formulation using Branch and Bound algorithm. We then solved the Lagrangian relaxation algorithm. To further illustrate the applicability of the model and solution method, we apply our methodology on available data for the GE 1.5 MW WT, which is widely used throughout the US in wind farms. Even though, there are many types and style of WTs can exist at each wind farm; we consider only one type and one style of WTs. For implementation, we consider 50 replacement periods (years), and three scenarios with small, medium, and large size of initial inventory of assets. Summing all the cost data provided above, cost data for the experimental design is presented in Table 2.

Table 2. 1.5 MW WT data for model illustration

Parameter	Symbol	Data	Reference
Unit purchase cost	$P_{1;1}$	$1,400-$2000 per kw increasing by 2% each time period t	[23]
Unit O&M cost	$OMPN_{1;1}$ $OMLR_{1;1}$	$ 9 -$20 per MWh increasing by 10% each time period	[23]
Unit salvage value	$S_{1;1}$ $SN_{1;1}$	80% of $P_{1;1}$ and decreasing by 20% each time period t	Estimated value
Demand	EnDem	15,500,000-45,000,000 kWh per year	Estimated value
Energy production	EN	4,500,000 kWh per year for 1.5 MW WT	[24]
Budget	Budget	$250,000-$1,500,000	Estimated value
Cost of electricity	ECost	0. 02-0.05$/kW	[24]
New wind turbine cost (GE 1.5 XLE 1.5 MW)	$P_{1;1}$	$1, 400,000	[25], [26]
Gearbox 10-15% of total cost of WT	$V_{1;1}$, $VN_{1;1}$	$140,000-$210,000	[25], [26]
Generator % 5-10 of total cost of WT (including installation cost)		$70,000-$140,000;	
Blades % 10-25 of total cost of WT (including installation cost)		$140,000-$210,000;	
Tower cost %10-35 of total cost of WT (including installation cost)		$140,000-$350,000	

We solve original problem using CPLEX and alternative method for three scenarios, present results, and illustrate the solution effectiveness in Table 3. The computational experiments with Computational experiments with Lagrangian relaxation suggest that subgradient optimization is quite effective in solving large-scale problems. When the problem planning period is 100 time units, the subgradient algorithm solves the problem in approximately less than 1.5% of the CPU time that the PRP-R model requires to find the optimal solution using CPLEX. Each run results in a number of iterations, which are provided in Table 4. Lagrangian relaxation suggest that sub-gradient optimization is quite effective in solving the large scale problems.

Table 3. Run Times with Lagrangian Relaxation

Time Period	Main Problem CPU (sec)	LR CPU (sec)	Number of Iterations
25	10	0.08	3
50	70	2	7
100	4,568	60	15
100—Case a (20% operation and maintenance cost increase)	4,452	56	12
100—Case b (20% retrofitting cost increase)	4,376	49	10
100—Case c (20% increasing energy demand)	4,735	83	14

Based on the results obtained it was concluded that the Branch and Bound algorithm was not able to solve the basic formulation in a reasonable time, except in the small instance. However, a significant improvement is obtained by strengthening the formulation of the model. This benefits both the straightforward use of Branch and Bound, and the Lagrangian relaxation. Notice that the Branch and Bound leads higher CPU times compared to the Linear Relaxation approach.

This paper present Lagrangian relaxation with subgradient method for the previously designed MILP programing for an asset replacement problem. A significant number of previous studies have focused on both theoretical development and mathematical modeling of asset management problems. To the best of our knowledge this is the first study considers Lagrangian relaxation approach to solve the NP-Hard PRP-R. As a result this is a significant contribution to the field of study.

5 Conclusion

In this study, we have analyzed WTs asset management problem modelled through a combinatorial optimization formulation that can be viewed as the composition of a location problem and a fixed charge problem. Due to the difficulty of solving a given the combinatorial complexity of the subproblems, we used Lagrangian relaxation approach to solve the problem. We tested a WT asset management problem with both small and large instances.

In summary, we note that strengthening of the formulation does significantly improve the solutions process, and Lagrangian relaxation seems to be a promising approach for larger, more difficult to solve problems. Strengthening the formulation led to optimal solutions for the large and medium sized problems in reasonable CPU time. Detail testing is needed to achieve better conclusions on the applicability of Lagrangian relaxation, but this preliminary study results suggests that the approach is promising, specially if it is combined with strengthening procedures.

Further research is suggested in the analysis of other Lagrangian heuristics, such as simple versions of Ant Colony Optimization and GRASP, among other well-known heuristics and metaheuristics to compare the model solution time. In addition, to be able to test the proposed algorithm, different application (such as different type of end of life product including vehicles and electronic goods) can be studied.

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