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Number Line Estimations, Place Value Understanding and Mathematics Achievement*

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Abstract

Current study investigated the relationships among mathematics achievement, place value understanding and number line estimations. We used a curriculum-based math achievement test (MAT, Mathematics Achievement Test), a place value test (PVT), and a mental number line estimation test (MNL) as the data collection tools. A total of 355 fourth graders participated in the study. They were recruited from schools located in middle-low socioeconomic areas of central Anatolia. Correlations were statistically significant among all these tests scores (MAT, PVT, MNL). The highest correlation was obtained between PVT and MAT scores. PVT have also stronger correlations with MNL (0-1000). The average scores of MAT and MNL tests were correlated inversely as expected. The correlations between mathematics achievement scores and MNL (0-1000) was highest. The three MNL tests alltogether have explained 40% of the variance in MAT. Similar to PVT, MNL (0-1000) alone accounted for 36% of the variance in MAT. Meanwhile, MNL (0-1000) (large numbers) contributed MAT more. PVT explained 70% of the variance in MAT. In all tests, PVT contributed more to MAT. Generally, boys did better in all MNL tests but only the result of MNL (0-1000) was significant favoring boys. We can conclude, based on these results that both approximate number acuity and place value understanding contribute to the general math achievement as well as each other. While the place value concept seems to be a representational tool that enhence the acuity of approximate numbers, the relative magnitudes of numbers support place value understanding conceptually.

Keywords: Number line, number sense, place value understanding, math achievement.

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Sayı Doğrusunda Tahmin, Basamak Değeri Anlayışı ve Matematik Başarısı*

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Öz

Bu çalışmada, sayı doğrusu tahmin becerisi, basamak değeri anlayışı ve matematik başarısı arasındaki ilişki araştırılmıştır. Veri toplama aracı olarak müfredata dayalı sayı görevlerini içeren matematik başarı testi (MBT) ve basamak değeri testi (BD) ile zihinsel sayı doğrusu tahmin testi (ZST) kullanılmıştır. Çalışmaya toplam 355 dördüncü sınıf öğrencisi katılmıştır. Okullar, İç Anadolu'da yer alan bir ilde düşük-orta sosyoekonomik düzeye sahip bölgede bulunan okullardan seçilmiştir. Tüm test puanları (MBT, BD, ZST) arasında istatistiksel olarak anlamlı korelasyon bulunmuştur. En yüksek korelasyon BD ve MBT puanları arasındadır. BD kavrayışı, ZST (0-1000) ile daha güçlü korelasyona sahiptir. MBT ve ZST testlerinin ortalama puanları arasındaki korelasyon beklendiği gibi ters orantılıdır. Matematik başarı puanları ile ZST (0-1000) arasındaki korelasyon yüksek olarak elde edilmiştir. Her üç ZST testi, MBT'deki varyansın %40'ını açıklamaktadır. BD'ye benzer şekilde, MNL (0-1000) tek başına MBT'deki varyansın %36'sını oluşturmuştur. Öte yandan MNL (0-1000), MBT'deki başarıya daha fazla katkıda bulunmaktadır. BD'deki başarı, MBT'deki varyansın %70'ini açıklamaktadır. Tüm testlerde BD kavrayışı, MBT'deki başarıya daha fazla katkıda bulunmuştur. Genel anlamda çocuklar tüm ZST testlerinde başarılı olmuşlardır ancak sadece ZST (0-1000)'deki tahmin keskinliğinde erkek öğrenciler daha başarılıdırlar. Bu sonuçlara dayanarak hem yaklaşık sayı keskinliği hem de basamak değeri anlayışının, genel matematik başarısına ve birbirine katkıda bulunduğu sonucuna varılabilir. Basamak değeri kavramı, yaklaşık sayıların keskinliğini artıran temsili bir araç gibi görünse de, sayıların göreceli büyüklükleri basamak değeri anlayışını kavramsal olarak da desteklemektedir.

Anahtar Sözcükler: Sayı hissi, zihinsel sayı doğrusu, matematik başarısı, basamak değeri.

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Introduction

Two basic stones of mathematics are number and shape. In other words, at the core, mathematics is mainly about establishing relationships between spatial structures and quantities (Skagerlund, 2016). In fact, at the early stages, doing/learning mathematics is considered to be a quantification process. The quantities might either be discrete like five marbels, three elephants etc. or continuous like time, distance, area, mass etc. According to triple coding theory (Dehaene, 1992) we represent these quantities in three formats: analog (●●●), symbolic (3), and verbal (three). Before developing symbolic notations we use analog representattions for numbers either discrete of continuous. In other words, what considered to be core knowledge of number, which comes with birth (Spelke & Kinzler, 2007), becomes mental representations of quantities by interacting with these external representations. Any countable object can be used to represent discrete quantities. Analog representations of a continuous quantity on the other hand could be any distance on a continuous object like a stick or number line pictorially.

Symbolic representations of numbers have notational procedures learned in school. For numbers from zero to nine we have one symbol for each number, called numerals (0, 1, 2, 3... 9). When the numbers get larger than nine however, we use the same numbers for different quantities in accordance with the place of the symbols. For example, when we write 24 in a base-ten structure, 2 is no more two but "two tens" because it is in the place of tenths. The positional or the place value understanding of our number system is central to developing number sense and is also the basis for aritmetic operations on symbolic representations of numbers (Nataraj & Thomas, 2007). Students develop meaning for place value when they understand the positional notation on numerals. Place-value knowledge is particularly important in understanding regrouping, gaining number sense, and making mental computations (Burns & Tank, 1988).

Another representation of number is the number line. Numbers can be placed on number lines according to their magnitudes. Mapping symbolic numbers on a number line, on the other hand, requires an understanding of the Hindu-Arabic Number System. The distances between the numbers on the number line are assumed to be linearly equidistant (Moeller, Pixner, Kaufmann, & Nuerk, 2009); for instance, the interval between 10 and 100 must be 10 times as large as the interval between 1 and 10.

Number is also represented mentally. Emprical evidence shows that mental magnitude of a number is not exact but approximate. The approximate number system (ANS) is not unique to human being. Brannon, Jordan, and Jones (2010) claimed that we share this system phylogenetically with some other species, such as monkeys, pigeons and rats. Evolutionally, we can rationalize the emergence of the ANS across species because it may enable animals to perceive and represent quantities in the immediate environment. This skill is vital and very important for survival of the animals, for example, in foraging and hunting. They require the apprehension first, and then perhaps discrimination, of one or more sets of vital objects in the immediate surroundings (Skagerlund, 2016). Similarly, ANS is even more important for human being. It may facilitate children's early understanding of numbers such as assigning values to countable quantities, and later the acquisition of other number related concepts (Chu, vanMarle, & Geary, 2015). In fact, accumulating evidence persistently shows that the ANS acuity and later mathematics performance are strongly related (Skagerlund, 2016).

Symbolically representing large numbers requires place value notations. In understanding multi-digit numbers and arithmetic, for example, an understanding of the place-value system is necessary as well as an understanding of the numerical magnitudes (Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011). Children who have good place-value understanding make more accurate number-line estimations than their peers (Bicknell & Young-Loveridge, 2015). These findings may have implications for mathematics education practice in making connections between number and space in further mathematical topics since they are not limited to numbers only. For instance, the ability to place numbers on a number line has been found to be strongly related to their understanding of proportional reasoning as well as mathematical achievement in general (Rouder & Geary, 2014). However, there are still vigorous debates about the contribution of the approximate number system

(ANS) acuity to learning these basic symbolic competencies and more broadly to mathematics achievement (van Marle, Chu, Li, & Geary, 2014).

Chu and colleagues' (2015) claimed that rather than ANS acuity, children's understanding of the cardinal value of number words is a more suitable symbolic knowledge. Chu et al. (2015), used both competencies to simultaneously predict mathematics achievement in order to find the relative importance of the symbolic knowledge and ANS. They found that children's cardinal knowledge is stronger and enduring for predicting mathematics achievement, while the contribution of ANS disappears. They concluded that this result does not imply that ANS acuity is not related to children's achievement in mathematics, since they found that it already is, but this relation appears to be mediated by conceptual understanding of number symbols and the relations among them (Chu et al., 2015). If so, then young children who can easily differentiate the relative quantities of object collections will more easily realize that we use different number symbols for different quantities. The importance of the ANS for early mathematics achievement may decrease once children achieve this insight and establish the meaning of number symbols (Chu et al., 2015). Still however, it might be possible that both symbolic understanding of number and ANS acuity contribute to mathematics achievement independently.

Assigning meaning to number symbols, for example learning the cardinal value of number symbols, may be a critical early link for young children because the mental magnitudes of numbers represented by the ANS may correspond to the cardinal value (though approximate) of countable quantities (Gallistel & Gelman, 2005). There are two measures of ANS accuity: Weber fraction and task accuracy. The relation between ANS acuity and achievement in mathematics was fully mediated by children's success on the symbolic quantity tasks, with knowledge of cardinal value appearing as a significantly important mediator. The patterns of available findings suggest that ANS acuity facilitates the early use of symbols for quantities and that this early use indirectly influences mathematics achievement through this knowledge (van Marle et al., 2014). The findings by Lyons and Beilock (2011) similarly showed that the adults' ANS acuity and their ability to order the magnitudes of numbers presented in Arabic numerals are strongly related. This suggests that some features of the ANS may lead, in part, the ordinal ability (Gallistel & Gelman, 1992).

There is abundance of literature on preschool children's emerging number words knowledge and Arabic number symbols and their knowledge of counting, ordinality, cardinality and arithmetic. Very few of them, however, has related preschoolers' knowledge in these areas to ANS acuity or concurrent or later mathematics achievement, aside from the above cited studies (van Marle et al., 2014). In an earlier study by Huntley-Fenner and Cannon (2000), it has been found that preschoolers' knowledge of the cardinal value of number words was not correlated with their ANS acuity. In other words, children's numerical judgments seem to be mediated by an analog system and that there is no relation between this mechanisms and verbal counting ability. It may also indicate that ordering numerical magnitude via the ANS mechanism is a preverbal ability.

Current findings supports the claim that efficiency in ANS may support young children's preliminary understanding of the meaning of basic number words and Arabic numerals, but may not be directly related to achievement in mathematics besides these basic relations (van Marle et al., 2014). In other words, the finding that children's number words knowledge, Hindu-Arabic numerals and their meaning fully mediated the relation between ANS acuity and achievement in mathematics is in line with Lyons and Beilock (2011) claim; specifically, symbolic quantitative knowledge mediates the relation between ANS acuity and mathematics achievement.

In sum, the Arabic numerals knowledge, cardinal value, number words, and their relative sizes were critical preschool competencies that accounts for achievement in mathematics and subsidized to the relation between mathematics achievement and ANS acuity (van Marle et al., 2014). Children's intuitive number sense may have a positive effect on their achievement in mathematics by facilitating the learning of Hindu-Arabic numerals, verbal numbers and their understanding of the meaning of these symbols, however it is not clear whether and how this early number sense extends to better mathematics achievement in subsequent years.

Namkung and Fuchs (2016) discussed their findings in terms of differences between whole-number and fraction learning and between arithmetic and number line learning. They concluded that number line estimation competence and whole-number and fraction calculation may share a set of cognitive resources but, given the different developmental processes, they also may require distinct cognitive abilities (Namkung & Fuchs, 2016). The ability of ordering numbers symbolicly seem to fully mediate the relation between ANS acuity and more complex mathematical skills. This may suggest that ordering numbers symbolicly may serve as a stepping stone from approximately representating numbers to competence in mathematics. If these are the case then we can use ANS acuity and then symbolic number-ordering ability as reliable markers of mathematics performance during schooling towards more complex math skills in designing math-education curricula (Lyons & Beilock, 2011).

If the acquisition of symbolic numerical order is linked to ANS, then more efficient assessment of ordinal relations in symbolic numbers should be related to greater ANS acuity (Lyons & Beilock, 2011). Furthermore, this view proposes that the approximate number system; ANS (i.e., an intuitive sense of approximate quantity) should be a fundamental aspect of any symbolic number—that is, as claimed by Dehaene (2008) there should be considerable intersection between symbolic and nonsymbolic numerical processes.

Based on the discussions above, it seems that approximate number acuity as measured with number line estimation tasks is a relatively more intuitive construct than other mathematical measures such as cardinality, place value and overall achievement. However, more evidences are needed to support this hypothesis. The purpose of this study was to investigate the relationships among the number line estimations, place value understanding and overall mathematics achievement. The findings of this study may shed light on paving the road towards designing curricula for better mathematical learning at the elementary school level.

Method

This study used a relational survey design. The relationships between students' number line estimations, place value understanding and mathematics achievement scores were investigated.

Participants

The sample of the study consisted of 355 fourth graders, selected from a school that accommodates a wide range of socio-cultural backgrounds to achieve maximum diversity within our sample including a wide range of socio-economic strata (Buyukozturk, Cakmak, Akgun, Karadeniz, & Demirel, 2008). There were approximately equal numbers of boys (n=184) and girls (n=171) in the study.

Data Collection Tools

Participants were administered three different tests: a curriculum-based Math Achievement Tests (MAT), a mental number line estimation test (MNL), and place value test (PVT).

Mathematics Achievement Test (MAT): Mathematics Achievement Tests was developed by Fidan (2013) for grades 1-4 based on the number domain of the Turkish national education mathematics curriculum. It includes numbers, counting, number patterns, four arithmetic operations, and fractions. KR-20 coefficients of the test were .96 for the grade four students. The administration of the test took one class hour.

Mental Number Line Test (MNL): There were three subtests in MNL, MNL1 (0-10), MNL 2(0-100), and MNL 3(0-1000). MNL 1 and 2 were originally developed by (Olkun & Sari, 2016) and consisted of number placement tasks on 0-10, and 0-100 number lines. In addition to the number line test developed by Sari & Olkun (2018), 0-1000 number line was included in this study (MNL 3). A typical number line is a horizontal or vertical line with zero on the left end, and 10, 100 or 1000 on the other end. Students are requested to place the numbers shown one at a time on the number line by drawing a hash mark on the number line (see Figure 1). No timing was recorded for this test. Only the absolute values of the difference between the estimation and to be estimated numbers were recorded in number to position tasks. There were a total of 58 items in MNL tests, 18 items in MNL 1, 20 items in MNL 2 and 20 items in MNL 3. There were 2 practice items before each of the actual test.

Place Value Test (PVT): It was developed by Sari & Olkun (2018) by considering the relevant acquisitions regarding the concept of place value in the 4th grade primary school mathematics curriculum, and was composed of 14 questions in total. The KR-20 reliability coefficients of the test were .84. The administration of the test took one class hour.

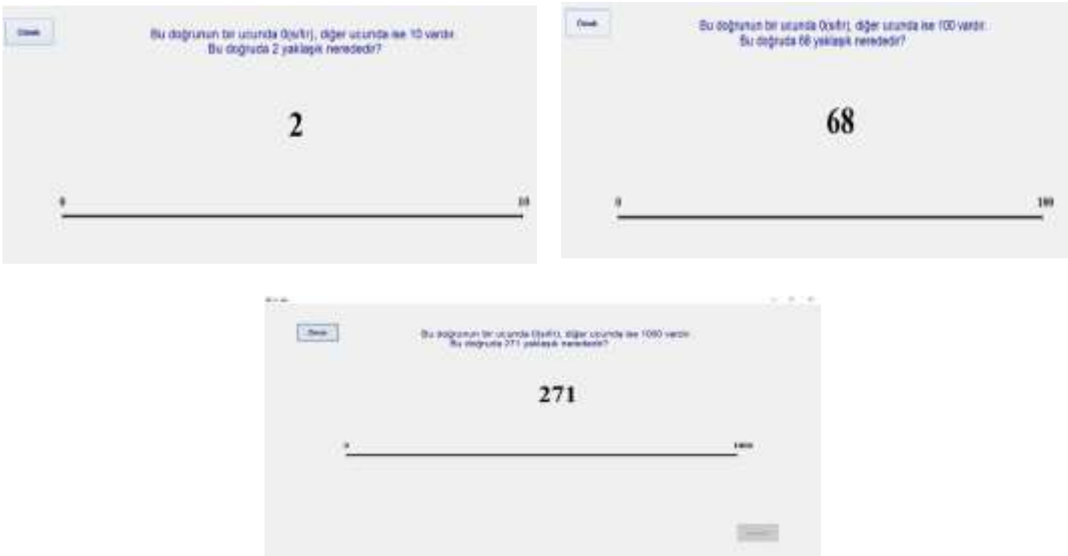

Figure 1. *Sample items from the MNL 1, MNL 2 and MNL 3*

Analysis

The row scores (i.e. the number of correct answers) were used for the tests, MAT and PVT. The total absolute error (TAE) scores were calculated for the MNL 1, 2, and 3 tests by using the formula "I Estimations – to be estimated number)/scale I" as suggested by Siegler and Booth (2004). Multiple regression analyses (enter method) were carried out in order to determine the explanatory power of the place value understanding and mathematics skills on number line estimation accuracies. Correlations among the test scores were also calculated before the regression analysis.

Students were divided into two groups based on their TAE scores. Students who are at the bottom 27% on all of the three MNL tests were placed into low performer group. Students who are at the top 27% were placed on the high performer group. These two groups were compared in terms of their PVT and MAT scores with Independent-Samples T-tests. We also compared male and female students' scores on MAT, PVT and MNL test through Independent-Samples T-tests.

Results

Before analyzing the explanatory power of the tests used in this study on number line estimations we calculated the correlations among the tests. Results are depicted in Table 1.

Table 1
Correlations among the tests used in the study

Tests	MAT	MNL1	MNL2	MNL3
PVT	.836**	-.519**	-.517**	-.599**
MAT	-	-.496**	-.544**	-.597**
MNL1		-	.576**	.565**
MNL2			-	.824**

** Correlation is significant at the .01 level. PVT: Place Value Test, MAT: Mathematics Achievement Test, MNL1: Mental Number Line 0-10, MNL2: Mental Number Line 0-100, MNL3: Mental Number Line 0-1000.

There were statistically significant correlations among all the tests used for the study. The highest correlation was calculated between PVT and MAT scores. Also, PVT have stronger correlations with MNL3. Similarly, MNL3 was highly correlated with mathematics achievement scores (-0.597).

Table 2

Summary of regression results for MNL1, MNL2, and MNL3 with PVT

Model	Variables	R	R^2	F	β	t	p
1	MNL1 MNL2 MNL3	.637	.406	80.069	12.337	37.958	.001

a Predictors: (Constant), MNL1 (Mental Number Line 0-10), MNL2 (Mental Number Line 0-100), MNL3 (Mental Number Line 0-1000)

The results of the multiple regression analyses carried out to determine the explanatory power of the students' number line estimation skills on place value understanding are presented in Table 2. When MNL1, MNL2, and MNL3 were entered in the regression, we saw that all of the tests have significant explanatory power on place value test (PVT). As seen in Table 2, MNL1, MNL2, and MNL3 together have explained 41% of the variance in PVT. MNL3 alone accounted for 36% of the variance in the PVT.

Another linear regression analysis was performed to see if MN1, MNL2, and MNL3 explains any variance in MAT (see Table 3).

Table 3

Summary of regression results for MNL1, MNL2, and MNL3 with MAT

Model	Variables	R	R^2	F	β	t	p
1	MNL1 MNL2 MNL3	.629	.396	76.627	14.384	34.911	.001

a Predictors: (Constant), MNL1 (Mental Number Line 0-10), MNL2 (Mental Number Line 0-100), MNL3 (Mental Number Line 0-1000)

Similarly, when MNL1, MNL2, and MNL3 were entered in the regression we saw that all of the tests have significant explanatory power on mathematics achievement test (MAT). Results showed that MNL1, MNL2, and MNL3 together have explained 40% of the variance in MAT. As in PVT, MNL3 alone accounted for 36% of the variance in the MAT. On the other hand, MNL 0-1000 (large numbers) more contributed to mathematics achievement.

The result of the independent samples t-test analysis carried out to determine whether there was a significant difference between the students' PVT and MAT scores in the Low and High performer groups formed by bottom and top %27 based on the Total Absolute Errors obtained from MNL1, MNL2 and MNL3 is shown in Table 4.

Tablo 4

Comparison of the Lower and Upper groups PVT and MAT scores

Variables	Groups	N	Mean	Sd	df	t	p
PVT	Sub-group	42	3.30	2.56	79	-14.822	.001*
	Upper group	39	11.20	2.20			
MAT	Sub-group	42	3.66	2.66	79	-14.999	.001*
	Upper group	39	12.87	2.85			

*p< .01

According to Table 4, in terms of TAE scores in MNL1, MNL2 and MNL3 there is a significant difference in the PVT [$t_{(79)} = -14.822$, $p < .001$] and MAT achievements of the students [$t_{(79)} = -14.999$, $p < .01$] in Low and High performer groups on behalf of High performer group.

In order to see if PVT scores explain any variance in MAT, a linear regression analysis was run. Results showed that PVT (R= .836, R^2= .698) explained 70% of the variance ($F_{(1-353)}$= 817.53, p<.01) in MAT. On the other hand, PVT contributed more to mathematics achievement.

We also investigated whether any gender differences existed. Gender analysis showed that there were no statistically significant differences between boys and girls in PVT, MAT, MNL1 and MNL2. However as seen in Table 5, there are gender differences in the MNL3 favoring boys (p<.05). Boys consistently did better in estimating the relative magnitude of numbers on external number lines, but only the results of MNL3 was significant.

Table 5
Gender differences

	N	PVT Mean	MAT Mean	MNL1 TAE	MNL2 TAE	MNL3 TAE
Boys	184	7.95	8.73	29.44	199.6	2533.8
Girls	171	7.85	9.27	31.16	218.3	2944.7
	p	(.789)	(.263)	(.368)	(.237)	(.014)*

*p<.05, PVT: Place Value Test, MAT: Mathematics Achievement Test, MNL1: Mental Number Line 0-10, MNL2: Mental Number Line 0-100, MNL3: Mental Number Line 0-1000

Discussion, Conclusion and Recommendations

If we summarize the basic findings of the current research, we see that both PVT and MAT have stronger correlations with MNL3 (0-1000). MNL1 (0-10), MNL2 (0-100), and MNL3 (0-1000) together have explained 40% of the variance in MAT. As in PVT MNL (0-1000) alone accounted for 36% of the variance in MAT. PVT also explained 70% of the variance in MAT. The findings of the current study confirmed the previously obtained evidences that PVT contribute the general math achievement (Sari & Olkun, 2019; Sari & Aydogdu, 2020). We also investigated whether any gender differences existed. The only gender difference occurred in the MNL (0-1000) favoring boys. Among the MNL tests MNL3 (0-1000) predicted and contributed MAT most. In earlier studies (Olkun, Mutlu & Sari, 2017) showed that number line estimations (magnitude of larger numbers) of students were favoring boys. Compared to MNL tests, PVT contributed more to MAT. Based on these results we conclude that teaching place value concepts and relative magnitude of larger numbers contribute to each other and both contribute the general math achievement during primary school.

The findings of the current study confirmed the previously obtained evidences that there were strong relations between number line estimations, place value understanding, and overall mathematics achievement at the fourth-grade level. The stronger correlations between MNL3 (0-1000), which is relatively new number range for fourth graders, with both place value understanding and overall mathematics achievement as measured by curriculum based tools, may indicate that number line estimation acuity, which is considered to be more of an indigenous skill, supports the learning of formal mathematics. The relatively lower correlations between MNL1 and MNL2 with PVT and MAT may also indicate that the impact of familiar numbers decreases over time. Similar results were found by Fazio, Bailey, Thompson, and Siegler (2014). In their study the relations were stronger for number line estimation with fractions, which is relatively new for students, than with whole numbers.

A large body of findings in earlier studies (Dehaene, Izard, Spelke, & Pica, 2008) showed that number line estimations of students in the early grades map on a logarithmic number line, then they develop their estimations to map on a mental linear number line. In other words, their representations of the magnitudes of numbers become more precise through experience in an appropriate school education. The impact of ANS on math achievement diminishes over time as the children internalize the use of the symbolic representations of numbers. Importantly, number magnitude and place-value understanding are important precursor competencies for later arithmetic skills with other basic numerical competencies (Lambert & Moeller, 2019).

Moeller et al. (2009) also obtained similar results in a different study. They showed that number lines with a smaller range such as 0-to-10 scale, a linear function yielded a better fit than a logarithmic function. When the number range extents to 100, on the other hand a logarithmic function yielded a better fit than a linear function. It seems that a linear distribution of numbers is necessary in order to pass to a higher level of numerical understanding such as place value. Attaching relative magnitudes to symbols requires both conceptual and procedural understanding of representing numbers in symbols.

The place value understanding is a kind of structuring of numbers conceptually that requires one form of mental activity (Dehaene et al., 2008). A mental number line development is another form of mental activity that children develop with understanding place value conceptually. As the numbers are getting larger and larger, representations of these numbers on a mental number line requires an understanding and successful application of the base ten system in Arabic numerals in order to estimate the magnitude of a given number correctly in a number line task so that the intervals on the number line are supposed to be equidistant from each other. For instance, the interval between zero and 70 must be 10 times as large as the interval between zero and seven (Moeller et al., 2009). These processes show the comprehensive nature of children's numerical development. While developing conceptual place value children need to transition from concrete to abstract experiences, transition from small numbers to large numbers, rely upon a number as a structure, and rely upon their own mental number lines (MacDonald, Westenskow, Moyer-Packenham, & Child, 2018).

Children with good place-value understanding were more accurately place numbers on number-lines than their peers with poor understanding. The findings have implications for practitioners in making more explicit the connections between numbers and space (Bicknell & Young-Loveridge, 2015; Olkun, Sari, & Smith, 2019). For example, children's ability to place numbers on a number line is strongly related to their understanding of proportional reasoning and overall mathematical achievement (Rouder & Geary, 2014). If this is the case then young children who can easily discriminate the relative quantities of collections of objects will more easily achieve the understanding that different number symbols represent different quantities. Once children understand and establish the meaning of number symbols, the importance of the ANS for early mathematical learning may diminish (Chu et al., 2015). Then, symbolic ordinal ability fully mediates the relation between ANS acuity and mathematics achievement (van Marle et al., 2014).

In an effort to understand the complex nature of relationship between the ANS acuity and place value understanding, Moeller et al. (2009) proposed a somewhat different account of children's numerical development. They claim that rather than assuming a transition from logarithmic to linear coding, students may develop their numerical understanding by integrating the decomposed representations of tens and units complying with the place value structure of the Arabic number system with age and appropriate school experience. Student can get the feeling that changing the place of a digit one step in a multidigit number makes the value of it ten times larger or vice versa. Therefore, fully mastering of the first ten numbers become even more important in the subsequent learning of numbers from zero to hundred.

Considering the evidences from our current research and other studies in the literature we claim that number line estimation training should go hand in hand with place value teaching in order to help students construct a more vivid image of numbers represented symbolically starting from one digit on. Small numbers should be thoroughly mastered before introducing the larger numbers. However, longitudinal studies should be done in order to make reliable conclusions about this claim. Experimental studies are needed to confirm if number line estimation training result in an increase in estimation acuity, place value understanding and overall mathematics achievement both in the short and long run.

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