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Homeomorphic image of some kernels

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Abstract: Through this paper, we shall study of kernels of a set with the help of ideals and boundaries of these kernels. We also study the convergence of the associated filter in terms of a point which comes from the kernels. We shall also study the homeomorphic images of these kernels.

Keywords: Filter, associated filter, frontier points, local function, homeomorphism.

1. Introduction

The study of Kuratowski [1] and Vaidyanathaswamy's [2] local function on the topological space with ideal is the study of generalization the limit point of a set in the topological space. It was defined as: for a topological space (X,τ) with unideal I, $A^*(\mathrm{I},\tau)=\{x\in X \mid U_x\cap A\notin \mathrm{I}$ for every $U_x\in\tau(x)\}$, where $\tau(x)$ is the collection of all open sets containing x and $A\subseteq X$. $A^*(\mathrm{I},\tau)$ is simply denoted as $A^*(\mathrm{I})$ or A^*. Dual function of the local function $()^*$ is denoted as Ψ and defined by $\Psi(A)=X\setminus(X\setminus A)^*$ [3]. These two set functions have been studied by a good number of mathematicians (see [4-13]). Two simplest ideals on a topological space (X,τ) are $\{\phi\}$ and $\wp(X)$ (the power set of X) and we observe that $A^*(\{\phi\})=Cl(A)$ ($Cl(A)$ denotes the closure of A) and $A^*(\wp(X))=\phi$ for every $A\subseteq X$. Thus the study of the local function will be interesting when the ideal I is a proper ideal(an ideal I not containing X) on the topological space. Otherwise, when an ideal contains the whole set X, then it contains all subsets of X and the value of local function on any set is always empty.

In the field of ideal, the study of associated filter is a new part and it was introduced by Modak et al. [13]. For recognizing the associated filter, we recall the concept of the filter.

Let X be a nonempty set and $\mathrm{F} \subseteq \wp(X)$. Then F is called a filter [14, 15] on X if it satisfies the following:

1. $\phi \notin \mathrm{F}$,

2. $B \in \mathrm{F}$ and $B \subseteq A$ implies $A \in \mathrm{F}$,

3. $A, B \in \mathrm{F}$ implies $A \cap B \in \mathrm{F}$.

As for example, if we suppose I is a proper ideal on a topological space (X,τ),then $\mathrm{F} = \{A \subseteq X \mid X \setminus A \in \mathrm{I}\}$ forms a filter on X. This filter is called the associated filter on X and denoted as F_I.

Definition 1.1. [13] An ideal I_u on a set X is called a universal ideal if for any $A \subseteq X$, either $A \in \mathrm{I}_u$ or $X \setminus A \in \mathrm{I}_u$.

A filter F on a set X is called an ultrafilter [14, 15] if it is a maximal element in the collection of all filters on X, partially ordered by inclusion, that is, F is an ultrafilter if it is not properly contained in any filter on X .

Proposition 1.2. [14,15] For a filter F on a set X, the following statements are equivalent:

1. F is an ultrafilter;
2. For any $A \subseteq X$, either $A \in \mathrm{F}$ or $X \setminus A \in \mathrm{F}$;
3. For any $A, B \subseteq X$, $A \cup B \in \mathrm{F}$ if and only if either $A \in \mathrm{F}$ or $B \in \mathrm{F}$.

It is obvious that if I_u is a universal ideal on the set X, then its associated filter is a ultrafilter on X.

Throughout this paper we denote T and S as topological spaces (X,τ) and (Y,σ) respectively.

2. Kernels via ideal

Definition 2.1. Let I be an ideal on a topological space T and $A\in\wp(X)$. Define $Ker_{\Psi}(A)=\bigcap\{\Psi(B) \mid B\in\wp(X) \text{ and } \Psi(B)\supseteq A\}$.

The concept Kernel of a set in a topological space is not a new idea in the literature. For a subset A of a topological space T, $A^{\wedge}=Ker(A)$ [16] (resp. $A^{\vee}$ [16], $Ker_{spo}(A)$, $bker(A)$ [17])) $=\bigcap\{U\in\tau \mid A\subseteq U\}$ (resp. $=\bigcup\{F\subseteq X \mid X\setminus F\in\tau \text{ and } F\subseteq A\}$, $\bigcap\{U\in SPO(T) \mid A\subseteq U\}$, $\bigcap\{U\in BO(T) \mid A\subseteq U\}$), where $SPO(T)$ and $BO(T)$ denote the collection of semi-preopen sets [18] and the collection of b-open sets [19] respectively.

Followings are the properties of the operator Ker_{Ψ}.

Theorem 2.2. Let I be an ideal on a topological space T and $A,B\in\wp(X)$ (the power set of X). Then

1. $A\subseteq Ker_{spo}(A)\subseteq bKer(A)\subseteq A^{\wedge}\subseteq Ker_{\Psi}(A)$.

2. $Ker_{\Psi}(X)=X$.

3. For $A\subseteq B$, $Ker_{\Psi}(A)\subseteq Ker_{\Psi}(B)$.

4. For $\{A_i \mid i\in\Lambda\}\subseteq\wp(X)$, $Ker_{\Psi}(\bigcup_{i\in\Lambda}(A_i))\supseteq\bigcup_{i\in\Lambda}Ker_{\Psi}(A_i)$.

5. For $U\in\tau^{*}$ (the $*$-topology [8]), $U\subseteq Ker_{\Psi}(U)\subseteq\Psi(U)$.

6. For $U\in\tau$, $U\subseteq Ker_{\Psi}(U)\subseteq\Psi(U)$.

7. For regular open [20] set U, $Ker_{\Psi}(U)=\Psi(U)=U$.

Proof. 1. Proof is obvious from the fact that $\Psi(A)\in\tau$ for $A\in\wp(X)$ and $\tau\subseteq BO(T)\subseteq SPO(T)$.

4. Obvious.

5. $Ker_{\Psi}(U)=\bigcup\{\Psi(B) \mid U\subseteq\Psi(B), B\in\wp(X)\}$. Since for $U\in\tau^{*}, U\subseteq\Psi(U)$, so $Ker_{\Psi}(U)\subseteq\Psi(U)$.

6. Obvious from the relation $\tau \subseteq \tau^*$.

7. Obvious from the fact that $U = \Psi(U)$ [5, 9, 11] is true for each regular open set U.

Example 2.3. Let I_u be a universal ideal on a topological space T and $A \in \wp(X)$ with $\phi \neq Ker_\Psi(A) \neq X$. Then $p \in Ker_\Psi(A)$ if and only if p is not a limit point of the associated filter U.

Proof. Suppose $p \in Ker_\Psi(A)$. Then for all $B \in \wp(X)$ satisfying $A \subseteq \Psi(B)$, $p \in \Psi(B)$. This implies $p \notin (X \setminus B)^*$. Then there exists $U_p \in \tau(p)$ such that $U_p \cap (X \setminus B) \in \mathrm{I}_u$. This implies $(X \setminus U_p) \cup B \in \mathrm{U}$. Since U is an ultrafilter, so either $X \setminus U_p \in \mathrm{U}$ or $B \in \mathrm{U}$. Claim: $B \notin \mathrm{U}$. If $B \in \mathrm{U}$, then $X \setminus B \in \mathrm{I}_u$ and hence $(X \setminus B)^* = \phi$. So for all $B \in \wp(X)$ satisfying $A \subseteq \Psi(B)$, $\Psi(B) = X \setminus (X \setminus B)^* = X$ and consequently $Ker_\Psi(A) = X$, a contradiction. Thus $B \notin \mathrm{U}$ and hence $X \setminus U_p \in \mathrm{U}$. Therefore $p \in U_p \notin \mathrm{U}$ proving that p is not a limit point of U.

Conversely, suppose that p is not a limit point of U. Then there exists $U_p \in \tau(p)$ such that $U_p \notin \mathrm{U}$. This implies $X \setminus U_p \in \mathrm{U}$ as U is an ultrafilter. So $U_p \in \mathrm{I}_u$. Now $U_p \cap (X \setminus B) \subseteq U_p$ implies $U_p \cap (X \setminus B) \in \mathrm{I}_u$ for all $B \subseteq \wp(X)$. Consequently $p \notin (X \setminus B)^*$ and thus $p \in \Psi(B)$ for all $B \subseteq \wp(X)$ satisfying $A \subseteq \Psi(B)$. Therefore $p \in Ker_\Psi(A)$.

Corollary 2.4. Let I_u be a universal ideal on a topological space T and $A \subseteq X$ with $\phi \neq Ker_\Psi(A) \neq X$. Then $p \in Ker_\Psi(A)$ if and only if p is not a cluster point of the associated filter U.

Definition 2.5. Let I be an ideal on a topological space T and $A \in \wp(X)$. Define $Ker^*(A) = \bigcup \{F^* | F \in \wp(X) \text{ and } F^* \subseteq A\}$.

Theorem 2.6. Let I be an ideal on a topological space I and $A, B \in \wp(X)$. Then

1. $Ker^*(\phi) = \phi$.

2. $Ker^{*}(A) \subseteq A$.

3. for $A \subseteq B$, $Ker^{*}(A) \subseteq Ker^{*}(B)$.

4. $Ker^{*}(A) \cup Ker^{*}(B) \subseteq Ker^{*}(A \cup B)$.

5. $Ker_{\Psi}(X \setminus A) = X \setminus Ker^{*}(A)$.

6. $Ker^{*}(A \cap B) \subseteq Ker^{*}(A) \cap Ker^{*}(B)$.

7. $Ker^{*}(A) \subseteq A^{\vee}$.

8. for $I \in \mathcal{I}$, $Ker^{*}(I) = \phi$.

9. $Ker^{*}(X) = X$ if and only if the space satisfies the condition $\mathcal{I} \cap \tau = \phi$.

10. $Ker^{*}(A) \cap Ker^{*}(X \setminus A) = \phi$.

Proof. 5. follows from the facts:

(i). $X \setminus \bigcup\{F^{*} : F \in \wp(X), F^{*} \subseteq A\} = \bigcap\{X \setminus F^{*} : F^{*} \subseteq A\} =$

$\bigcap\{\Psi(X \setminus F) : X \setminus F^{*} \supseteq X \setminus A\}$

$= \bigcap\{\Psi(X \setminus F) : (X \setminus A) \subseteq \Psi(X \setminus F)\} = Ker_{\Psi}(X \setminus A)$.

(ii). $X \setminus \bigcap\{\Psi(B) : A \in \wp(X), A \subseteq \Psi(B)\} = \bigcup\{X \setminus \Psi(A) : A \subseteq \Psi(B)\} =$

$\bigcup\{(X \setminus B)^{*} : (X \setminus B)^{*} \subseteq (X \setminus A)\} = Ker^{*}(X \setminus A)$.

8. Follows from the following facts:

For $I \in \mathcal{I}$ and $F^{*} \subseteq I$, $F^{*} = \{\phi\}$.

9. Suppose $Ker^{*}(X) = X$. If $\mathcal{I} \cap \tau \neq \{\phi\}$, let U be anon-empty subset of X such that $U \in \mathcal{I} \cap \tau$. Then for each $x \in U$, $x \notin U^{*}$, a contradiction towards the fact $x \in X$. Hence $\mathcal{I} \cap \tau = \{\phi\}$.

Converse is trivial.

Theorem 2.7. Let I be an ideal on a topological space T. Then $Ker_{\Psi}(\phi)=\phi$ if and only if the space satisfies the condition $\mathrm{I} \cap \tau = \{\phi\}$.

Theorem 2.8. Let I be an ideal on a topological space T and $A \in \wp(X)$. Then $Ker^{*}(A)$ is closed in X.

Proof. We prove the result by showing that $X \setminus Ker^{*}(A)$ is open.

Let $x \in X \setminus Ker^{*}(A)$. Then $x \notin Ker^{*}(A)$. This implies for all $F^{*} \subseteq A$, $x \notin F^{*}$. So there exists an open set $U_x \in \tau(x)$ such that $U_x \cap F \in \mathrm{I}$, for all $F^{*} \subseteq A \cdots\cdots\cdots(1)$. Claim: $U_x \subseteq X \setminus Ker^{*}(A)$. For each $y \in U_x$, $y \notin F^{*}$, for all $F^{*} \subseteq A$, by (1). This implies $y \notin Ker^{*}(A)$ showing that $y \in X \setminus Ker^{*}(A)$. Therefore $U_x \subseteq X \setminus Ker^{*}(A)$. This shows that x is an interior point of $X \setminus Ker^{*}(A)$. Since $x \in X \setminus Ker^{*}(A)$ is arbitrary, so $X \setminus Ker^{*}(A)$ is open.

Theorem 2.9. Let I be an ideal on a topological space T and $A \in \wp(X)$. Then $Ker_{\Psi}(A)$ is open in X.

Theorem 2.10. Let I_u be a universal ideal on a topological space T and $A \in \wp(X)$ with $\phi \neq Ker^{*}(A) \neq X$. Then for $p \in Ker^{*}(A)$ if and only if p is a limit point of the associated filter U.

Proof. Suppose $p \in Ker^{*}(A)$. Then $p \in F^{*}$ for some $F \in \wp(X)$ satisfying $F^{*} \subseteq A$. This implies for all $U_p \in \tau(p)$, $U_p \cap F \notin \mathrm{I}_u$. Since I_u is a universal ideal, so $X \setminus (U_p \cap F) \in \mathrm{I}_u$ and hence $U_p \cap F \in \mathrm{U}$. Now $U_p \cap F \subseteq U_p$ implies $U_p \in \mathrm{U}$ for all $U_p \in \tau(p)$. Therefore p is a limit point of U.

Conversely, let p is a limit point of U. If $p \notin Ker^{*}(A)$, then $p \in Ker_{\Psi}(X \setminus A)$. Since $\phi \neq Ker^{*}(A) \neq X$, so $\phi \neq Ker_{\Psi}(X \setminus A) \neq X$. Then by Theorem 2.3., p is a limit point of U, a contradiction. Therefore $p \in Ker^{*}(A)$.

Corollary 2.11. Let $\mathcal{I}_u$ be a universal ideal on a topological space $\mathcal{T}$ and $A \subseteq X$ with $\phi \neq Ker^*(A) \neq X$. Then $p \in Ker_\Psi(A)$ if and only if p is a cluster point of the associated filter $\mathcal{U}$.

Now, we discuss homoeomorphic images of the above two kernels. To do this, we recall two lemmas from [13].

Lemma 2.12. Let $f : X \to Y$ be a bijective function. If $\mathcal{I}$ is a proper ideal on X, then $f(\mathcal{I}) = \{f(I) \mid I \in \mathcal{I}\}$ is a proper ideal on Y.

Lemma 2.13. Let $f : X \to Y$ be a surjective function.If $\mathcal{J}$ is a proper ideal on Y, then $f^{-1}(\mathcal{J}) = \{f^{-1}(J) \mid J \in \mathcal{J}\}$ is a proper ideal on X.

For following theorem, we denote Ker_{Ψ_I} as the kernel evaluated under the ideal $\mathcal{I}$.

Theorem 2.14. Let $\mathcal{T}$ and $\mathcal{S}$ be two topological spaces and $\mathcal{I}$ be a proper ideal on $\mathcal{T}$. If $f : \mathcal{T} \to \mathcal{S}$ is a homeomorphism, then for $A \in \wp(X)$, $f(Ker_{\Psi_I}(A)) = Ker_{\Psi_{f(I)}}(f(A))$.

Theorem 2.15. Let $\mathcal{T}$ and $\mathcal{S}$ be two topological spaces and $\mathcal{I}$ be a proper ideal on $\mathcal{T}$. If $f : \mathcal{T} \to \mathcal{S}$ is a homeomorphism, then for $A \in \wp(X)$, $f(Ker^{*I}(A)) = Ker^{*f(I)}(f(A))$ (here $Ker^{*I}(A)$ means that the kernel of A with respect to the ideal $\mathcal{I}$).

Hence we conclude that Ker_Ψ and Ker^* of a set A remain invariant under homeomorphism.

3. Frontier points via kernels

The idea 'frontier points' of a set in a topological space has been introduced by Bourbaki [14]. In this section we shall consider some new types of frontier points with the help of the kernels of this paper.

At first, we shall show that $Ker_\Psi(A) \cap Ker_\Psi(X \setminus A) = \phi$ is not always true.

Example 3.1. Let $X = \{o_1, o_2, o_3\}$, $\tau = \{\phi, \{o_1, o_2\}, X\}$ and $\mathcal{I} = \{\phi, \{o_1\}\}$. Then for ϕ, $\Psi(\phi) = X \setminus X^* = X \setminus X = \phi$; for $\{o_1\}$, $\Psi(\{o_1\}) = X \setminus \{o_2, o_3\}^* = X \setminus X = \phi$; for $\{o_2\}$, $\Psi(\{o_2\}) = X \setminus \{o_1, o_3\}^* = X \setminus \{o_3\} = \{o_1, o_2\}$; for $\{o_3\}$, $\Psi(\{o_3\}) = X \setminus \{o_1, o_2\}^* = X \setminus X = \phi$; for $\{o_1, o_2\}$, $\Psi(\{o_1, o_2\}) = X \setminus \{o_3\}^* = X \setminus \{o_3\} = \{o_1, o_2\}$; for $\{o_1, o_3\}$,

$\Psi(\{o_1,o_3\}) = X \setminus \{o_2\}^* = X \setminus X = \phi$; for $\{o_2,o_3\}$, $\Psi(\{o_2,o_3\}) = X \setminus \{o_1\}^* = X \setminus \phi = X$. Let $A=\{o_1,o_2\}$. Then $X \setminus A=\{o_3\}$. Now $Ker_\Psi(A)=\{o_1,o_2\}$ and $Ker_\Psi(X \setminus A)=X$. Therefore $Ker_\Psi(A)\cap Ker_\Psi(X \setminus A)=\{o_1,o_2\}\neq\phi$.

We define the frontier operator Fr_Ψ on a topological space T with an ideal I in the following way: for $A \in \wp(X)$, $Fr_\Psi(A)=Ker_\Psi(A)\cap Ker_\Psi(X \setminus A)$.

Theorem 3.2. Let I be an ideal on a topological space T and $A,B \in \wp(X)$. Then

1. $Fr_\Psi(A)=Fr_\Psi(X \setminus A)$.

2. $Fr_\Psi(X)=Fr_\Psi(\Psi)=Ker_\Psi(\phi)$. In fact, $Fr_\Psi(X)=Fr_\Psi(\phi)=\phi$ if and only if the space satisfies the condition $\mathrm{I} \cap\tau=\{\phi\}$.

3. $Fr_\Psi(A)=Ker_\Psi(A)\setminus Ker^*(A)$.

4. For $I \in \mathrm{I}$, $Fr_\Psi(I)=Ker_\Psi(I)$.

5. $Fr_\Psi(A)$ is open.

6. For $U \in \tau^*$, $Fr_\Psi(U)\subseteq\Psi(U)\setminus Ker^*(U)$.

7. For $U \in \tau$, $Fr_\Psi(U)\subseteq\Psi(U)\setminus Ker^*(U)$.

8. For regular open set U, $Fr_\Psi(U)=U\setminus Ker^*(U)$.

Proof. 3. $Fr_\Psi(A)=Ker_\Psi(A)\cap Ker_\Psi(X \setminus A)=Ker_\Psi(A)\cap(X \setminus Ker^*(X \setminus A))$, by Theorem 2.6.(5). Thus $Fr_\Psi(A)=Ker_\Psi(A)\setminus Ker^*(A)$.

4. Obvious from (3) and the fact $I \in \mathrm{I}$ implies $Ker^*(I)=\phi$.

6. Obvious from the fact $Ker_\Psi(U)\subseteq\Psi(U)$ for $U \in \tau^*$.

7. Obvious from the fact $\tau\subseteq\tau^*$.

8. Obvious from the fact $Ker_\Psi(U)=\Psi(U)=U$.

We have seen that $Ker^*(A) \cap Ker^*(X \setminus A) = \phi$. However, following example shows that $Ker^*(A) \cup Ker^*(X \setminus A)^* = X$ is not true always.

Example 3.3. Let $X = \{o_1, o_2\}, \tau = \{\phi, \{o_1\}, \{o_2\}, X\}$ and $I = \{\phi, \{o_1\}\}$. Then $\phi^* = \phi$, $\{o_1\}^* = \phi$, $\{o_2\}^* = \{o_2\}$ and $X^* = \{o_2\}$. Therefore $Ker^*(\{o_2\}) \cup Ker^*(\{o_1\}) = \{o_2\} \neq X$.

We define the frontier operator Fr_* on a topological space T with ideal I in the following way: for $A \in \wp(X)$, $Fr_*(A) = Ker^*(A) \cup Ker^*(X \setminus A)$.

Theorem 3.4. Let I be an ideal on a topological space T and $A, B \in \wp(X)$. Then

1. $Fr_*(\phi) = Ker^*(X)$.

2. $Fr_*(\phi) = X$, if and only if the space satisfies the condition $I \cap \tau = \{\phi\}$.

3. $Fr_*(X) = Fr_*(\phi)$.

4. $Fr_*(X) = X$, if and only if the space satisfies the condition $I \cap \tau = \{\phi\}$.

5. for $I \in I$, $Fr_*(I) = Ker^*(X \setminus I) = X \setminus Ker_\Psi(I)$.

6. $X \setminus Fr_*(A) = Fr_\Psi(A)$.

7. $Fr_*(A) = Fr_*(X \setminus A)$.

8. $Fr_*(A)$ is closed.

Proof. 1. Obvious from the fact $Ker^*(\phi) \subseteq Ker^*(X)$.

2. Obvious from the fact that $Ker^*(X) = X$ if and only if the space satisfies $I \cap \tau = \{\phi\}$.

3. Obvious from (1).

4. Obvious from (2) and (3).

5. $Fr_*(I) = Ker^*(I) \cup Ker^*(X \setminus I) = \phi \cup (X \setminus Ker_\Psi(I) = X \setminus Ker_\Psi(I)$.

6. $X \setminus Fr_*(A) = X \setminus (Ker^*(A) \cup Ker^*(X \setminus A)) = (X \setminus Ker^*(A)) \cap (X \setminus Ker^*(X \setminus A)) =$

$(X \setminus Ker^*(A)) \cap (X \setminus Ker_{\Psi}(A)) = Ker_{\Psi}(A) \setminus Ker^*(A) = Fr_{\Psi}(A)$.

Now, we discuss about the homeomorphic images of the above two frontier operators.

Theorem 3.5. Let T and S be two topological spaces and I be an ideal on T. If $f: \mathrm{T} \to \mathrm{S}$ is a homeomorphism, then for $A \in \wp(X)$, $f(Fr_{\Psi_I}(A)) = Fr_{\Psi_{f(I)}}(f(A)$ (here $Fr_{\Psi_I}(A))$ means the set of frontier points A with respect to the ideal I).

Theorem 3.6. Let T and S be two topological spaces and I be an ideal on T. If $f: \mathrm{T} \to \mathrm{S}$ is a homeomorphism, then for $A \in \wp(X)$, $f(Fr_{*I}(A)) = Fr_{*f(I)}(f(A))$ (here $Fr_{*I}(A))$ means the set of frontier points A with respect to the ideal I).

4. Conclusions

1. The value of the Kernel of a set A in a topological space depends upon the collection. If the collection A is larger (with respect to set inclusion) than τ of a topological space (X, τ), then the value of the Kernel of A (when the Kernel is defined in terms of the collection A) is smaller than the original value of the Kernel of A. Furthermore, if the collection B is smaller than τ , then the value of the Kernel of A (when the kernel is defined in terms of B) is bigger (with respect to the set inclusion) than the original value of the Kernel of A.

2. The local function $()^*$ and the set operator Ψ are not distributive over arbitrary union or intersection but the Kernels related to $()^*$ and Ψ operators, and the frontier points via $()^*$ and Ψ operators remain invariant under homeomorphism.

Authorship contribution statement

Shyamapada Modak: Supervision, Writing, Reviewing and Editing, Conceptualization, Methodology. **Jiarul Hoque:** Data creation, Writing, Reviewing, Draft preparation, Investigation. **Sk Selim:** Writing, Reviewing, Draft preparation, Investigation.

Declaration of Competing Interest

The authors declare that there is no competing financial interests or personal relationships that influence the work in this paper.

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