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Generalized Jacobi Elliptic Function Method for Traveling Wave Solutions of Some Nonlinear Schrödinger's Equations

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Generalized Jacobi elliptic function method, (3+1)-dimensional nonlinear Schrödinger's equation, Coupled nonlinear Schrödinger's equation, Traveling wave solutions

Abstract: In this study, we find the traveling wave solutions of these equations by applying (3+1)-dimensional nonlinear Schrödinger's equation and coupled nonlinear Schrödinger's equation to Generalized Jacobi elliptic function method. We have expressed these solutions both as Jacobi elliptical solutions and trigonometric and hyperbolic solutions. We present two and three dimensional graphics of some solutions we have found. We also state some studies on these equations.

Bazı Doğrusal Olmayan Schrödinger Denklemlerinin Hareketli Dalga Çözümleri İçin Genelleştirilmiş Jacobi Eliptik Fonksiyon Yöntemi

Anahtar Kelimeler
Genelleştirilmiş Jacobi eliptik fonksiyon yöntemi, (3 + 1) boyutlu doğrusal olmayan Schrödinger denklemi, Doğrusal olmayan Schrödinger denklem çifti, Hareketli dalga çözümleri

Öz: Bu çalışmada (3 + 1) boyutlu doğrusal olmayan Schrödinger denklemine ve doğrusal olmayan Schrödinger denklem çiftine Genelleştirilmiş Jacobi eliptik fonksiyon yöntemini uygulayarak bu denklemlerin hareketli dalga çözümlerini bulduk. Bu çözümleri hem Jacobi eliptik çözümler hem de trigonometrik ve hiperbolik çözümler olarak ifade ettik. Bulduğumuz bazı çözümlerin iki ve üç boyutlu grafiklerini sunduk. Ayrıca bu denklemler üzerine yapılan bazı çalışmaları ifade ettik.

1. INTRODUCTION

Nonlinear partial differential equations (NPDEs) have an important place in applied mathematics and physics. Many methods have been developed to solve these equations. In most of these methods, nonlinear partial differential equations are converted to ordinary differential equations. Various solutions for the considered PDE are found with the help of these ordinary differential equations. In this study, we obtained traveling wave solutions of the (3+1) dimensional nonlinear Schrödinger equation and coupled nonlinear Schrödinger equation using the generalized Jacobi elliptical function method [1]. Many authors applied Jacobi elliptical function method and similar methods on some nonlinear partial differential equations and obtained various solutions of these equations. Some of these applications are given in [2-29]. Some studies on (3+1) dimensional nonlinear Schrödinger equation are as follow: Najafi et al. [2] reached exact solutions of (3+1)-dimensional nonlinear Schrödinger's equation by means of using sine-cosine method. Bulut et al. [3] reached traveling wave solutions of (3+1)-dimensional nonlinear Schrödinger's equation using the Sine-Gordon expansion method. Arbabi et al. [4] reached exact solutions of (3+1)-dimensional nonlinear Schrödinger's equation using the $\left(\frac{G'}{G}\right)$ – expansion method. Bhrawy et al. [5] obtained exact solutions of (3+1)-dimensional nonlinear Schrödinger's equation using the Extended Jacobi elliptic function method. On the other hand, Esen et al. [6] reached the dark, bright, mixed dark-bright, singular and mixed singular optical solutions to the space-time fractional (1+1)-dimensional coupled nonlinear Schrödinger's equation using the Sinh-Gordon equation expansion method. Wazwaz [7] investigated optical bright and dark soliton solution of coupled nonlinear Schrödinger (CNLS) equations in the anomalous and dispersive regimes using the Variational Iteration Method (VIM).

2. ANALYSIS OF GENERALIZED JACOBI ELLIPTIC FUNCTION METHOD

First we will present a simple description of the generalized Jacobi elliptic function method, and then we will give a generalized Jacobi elliptic function method [1]. To this end, one can consider general form of a PDE with four variables

$$Q(u, u_t, u_x, u_{xx}, \dots) = 0 \tag{1}$$

and transform Eq. (1) with $u(x,t) = u(\xi)$, $\xi = x + vt$, where v is a constant. After transformation, we get a nonlinear ODE for $u(\xi)$,

$$Q'(u', u'', u''', \dots) = 0. \tag{2}$$

The solution of the equation (2) we are looking for is expressed in the form

$$u(\xi) = a_0 + \sum_{i=1}^{n}[a_i F^i(\xi) + b_i F^{-i}(\xi)], \tag{3}$$

where $\xi = x + vt$ and n positive integer that can be determined by balancing the highest order derivate and with the highest nonlinear terms in equation. Substituting solution (3) into Eq. (2) yields a set of algebraic equations for F^i and F^{-i} then, all coefficients of F^i and F^{-i} have to vanish. After this seperated algebraic equation, we could found coefficients a_0, a_i, b_i, v and ξ.

In this paper, we will consider solving the (3+1)-dimensional nonlinear Schrödinger's equation and coupled nonlinear Schrödinger's equation by using the generalized Jacobi elliptic function method which is introduced by Huai-Tang et al. [1]. The fundamental of their method is to take full advantage of the elliptic equation and use its solutions F. The desired elliptic equation is given as

$$F'^2 = A + BF^2 + CF^4, \tag{4}$$

where $F' = \frac{dF}{d\xi}$ and A, B, C are constants. The solutions of Eq. (4) are given in the paper [1].

3. THE (3+1)-DIMENSIONAL NONLINEAR SCHRODINGER'S EQUATION

We consider the (3+1)-dimensional nonlinear Schrödinger's equation

$$iq_t + q_{xx} + q_{yy} + q_{zz} + \gamma|q|^2 q = 0, \tag{5}$$

Let's use following conversion for the equation (5),

$$q(x,y,z,t) = e^{i\theta}u(\xi) \tag{6}$$

where $\xi = x + y + z + vt$, $\theta = \beta x + \alpha y + lz + rt$.

We obtain following ordinary differential equation when Substituting (6) into (5),

$$3u'' - (\beta^2 + \alpha^2 + l^2 + r)u + \gamma u^3 = 0. \tag{7}$$

where $v = -2(\beta + \alpha + l)$. In Eq. (7), balancing u'' with u^3 gives $n = 1$. Then, the solution (3) becomes as follows,

$$u = a_0 + a_1 F + b_1 F^{-1}, \tag{8}$$

when (8) and (4) are written into (7) equation, and when F^i and F^{-i} coefficients from the same degree are equalized to zero, then following linear algebraic equation system is obtained for $a_0, a_1, b_1, \beta, \alpha, l$ and r.

$3\gamma a_0 a_1^2 = 0,\ 6Ca_1 + \gamma a_1^3 = 0,\ 6Ab_1 + \gamma b_1^3 = 0,$
$3\gamma a_0 b_1^2 = 0,$

$-ra_0 - s^2 a_0 - \alpha^2 a_0 - \beta^2 a_0 + \gamma a_0^3 + 6\gamma a_0 a_1 b_1 = 0,$

$3Ba_1 - ra_1 - s^2 a_1 - \alpha^2 a_1 - \beta^2 a_1 + 3\gamma a_0^2 a_1 +$
$3\gamma a_1^2 b_1 = 0,$

$3Bb_1 - rb_1 - s^2 b_1 - \alpha^2 b_1 - \beta^2 b_1 + 3\gamma a_0^2 b_1 +$
$3\gamma a_1 b_1^2 = 0,$

If the algebraic equation system above is solved with the aid of Mathematica, a_0, a_1, b_1 and r are found as follow

$$a_0 = 0, \gamma \neq 0, a_1 = \frac{i\sqrt{6}\sqrt{C}}{\sqrt{\gamma}}, b_1 = \frac{i\sqrt{6}\sqrt{A}}{\sqrt{\gamma}}, r = 3B - s^2 - \alpha^2 - \beta^2 + 3\gamma a_1 b_1. \quad (9)$$

Substituting (9) into (8) and using solutions of (4), also considering conversion (6), the solutions of equation (5) are found as follow,

$i)$ $A = 1,\ B = -(1+m^2),\ C = m^2.$

$$q_1(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{6}\sqrt{m^2}}{\sqrt{\gamma}}\, sn(\xi,m) + \frac{i\sqrt{6}}{\sqrt{\gamma}} \frac{1}{sn(\xi,m)}\right), \quad (10)$$

$$q_2(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{6}\sqrt{m^2}}{\sqrt{\gamma}}\, cd(\xi,m) + \frac{i\sqrt{6}}{\sqrt{\gamma}} \frac{1}{cd(\xi,m)}\right), \quad (11)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \left(-3 - 3m^2 - 18\sqrt{m^2} - l^2 - \alpha^2 - \beta^2\right)t.$

$ii)$ $A = 1 - m^2,\ B = 2m^2 - 1,\ C = -m^2.$

$$q_3(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{6}\sqrt{-m^2}}{\sqrt{\gamma}}\, cn(\xi,m) + \frac{i\sqrt{6}\sqrt{1-m^2}}{\sqrt{\gamma}}\, i\, \frac{1}{cn(\xi,m)}\right), \quad (12)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \left(-3 + 6m^2 - 18\sqrt{-m^2}\sqrt{1-m^2} - l^2 - \alpha^2 - \beta^2\right)t.$

$iii)$ $A = m^2 - 1,\ B = 2 - m^2,\ C = -1.$

$$q_4(x,y,z,t) = e^{i\theta} \times \left(-\frac{\sqrt{6}}{\sqrt{\gamma}}\, dn(\xi,m) + \frac{i\sqrt{6}\sqrt{m^2-1}}{\sqrt{\gamma}} \frac{1}{dn(\xi,m)}\right), \quad (13)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \left(6 - 3m^2 - 18i\sqrt{-1+m^2} - l^2 - \alpha^2 - \beta^2\right)t.$

$iv)$ $A = -m^2(1-m^2),\ B = 2m^2 - 1,\ C = 1.$

$$q_5(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{6}}{\sqrt{\gamma}}\, ds(\xi,m) + \frac{i\sqrt{6}\sqrt{-m^2(1-m^2)}}{\sqrt{\gamma}}\, i\, \frac{1}{ds(\xi,m)}\right), \quad (14)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \left(-3 + 6m^2 - 18\sqrt{-m^2(1-m^2)} - l^2 - \alpha^2 - \beta^2\right)t.$

$v)$ $A = 1 - m^2,\ B = 2 - m^2,\ C = 1.$

$$q_6(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{6}}{\sqrt{\gamma}}\, cs(\xi,m) + \frac{i\sqrt{6}\sqrt{1-m^2}}{\sqrt{\gamma}} \frac{1}{cs(\xi,m)}\right), \quad (15)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \left(6 - 3m^2 - 18\sqrt{1-m^2} - l^2 - \alpha^2 - \beta^2\right)t.$

$vi)$ $A = \frac{1}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4}.$

$$q_7(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{m^2}}{\sqrt{\gamma}} \frac{sn(\xi,m)}{1 \pm dn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{1 \pm dn(\xi,m)}{sn(\xi,m)}\right), \quad (16)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \frac{1}{2}\left(-6 + 3m^2 - 9\sqrt{m^2} - 2l^2 - 2\alpha^2 - 2\beta^2\right)t.$

$vii)$ $A = \frac{m^2}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4}.$

$$q_8(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{m^2}}{\sqrt{\gamma}} \left(sn(\xi,m) \pm icn(\xi,m)\right) + \frac{i\sqrt{\frac{3}{2}}\sqrt{m^2}}{\sqrt{\gamma}} \frac{1}{sn(\xi,m) \pm icn(\xi,m)}\right), \quad (17)$$

$$q_9(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{m^2}}{\sqrt{\gamma}} \frac{dn(\xi,m)}{i\sqrt{1-m^2}\, sn(\xi,m) \pm cn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}\sqrt{m^2}}{\sqrt{\gamma}}\, i\, \frac{i\sqrt{1-m^2}\, sn(\xi,m) \pm cn(\xi,m)}{dn(\xi,m)}\right), \quad (18)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + (-3 - 3m^2 - l^2 - \alpha^2 - \beta^2)t.$

$viii)$ $A = \frac{1}{4},\ B = \frac{1-2m^2}{2},\ C = \frac{1}{4}.$

$$q_{10}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{dn(\xi,m)}{mcn(\xi,m) \pm i\sqrt{1-m^2}} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{mcn(\xi,m) \pm i\sqrt{1-m^2}}{dn(\xi,m)}\right), \quad (19)$$

$$q_{11}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \left(msn(\xi,m) \pm idn(\xi,m)\right) + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{1}{msn(\xi,m) \pm idn(\xi,m)}\right), \quad (20)$$

$$q_{12}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{sn(\xi,m)}{1 \pm cn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{1 \pm cn(\xi,m)}{sn(\xi,m)}\right) \quad (21)$$

$$q_{13}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{cn(\xi,m)}{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)}{cn(\xi,m)} \right), \quad (22)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + (-3 - 3m^2 - l^2 - \alpha^2 - \beta^2)t$.

ix) $A = \frac{m^2-1}{4},\ B = \frac{m^2+1}{2},\ C = \frac{m^2-1}{4}$.

$$q_{14}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{-1+m^2}}{\sqrt{\gamma}} \frac{dn(\xi,m)}{1 \pm msn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}\sqrt{-1+m^2}}{\sqrt{\gamma}} \frac{1 \pm msn(\xi,m)}{dn(\xi,m)} \right), \quad (23)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + (6 - 3m^2 - l^2 - \alpha^2 - \beta^2)t$.

x) $A = \frac{1-m^2}{4},\ B = \frac{m^2+1}{2},\ C = \frac{1-m^2}{4}$.

$$q_{15}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{1-m^2}}{\sqrt{\gamma}} \frac{cn(\xi,m)}{1 \pm sn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}\sqrt{1-m^2}}{\sqrt{\gamma}} \frac{1 \pm sn(\xi,m)}{cn(\xi,m)} \right), \quad (24)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + (-3 + 6m^2 - l^2 - \alpha^2 - \beta^2)t$.

xi) $A = -\frac{(1-m^2)^2}{4},\ B = \frac{m^2+1}{2},\ C = -\frac{1}{4}$.

$$q_{16}(x,y,z,t) = e^{i\theta} \times \left(-\frac{\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} (mcn(\xi,m) \pm dn(\xi,m)) + \frac{i\sqrt{\frac{3}{2}}\sqrt{-(-1+m^2)^2}}{\sqrt{\gamma}} \frac{1}{mcn(\xi,m) \pm dn(\xi,m)} \right), \quad (25)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \frac{1}{2}\left(3 + 3m^2 - 9i\sqrt{-(-1+m^2)^2} - 2l^2 - 2\alpha^2 - 2\beta^2\right)t$.

xii) $A = \frac{1}{4},\ B = \frac{m^2+1}{2},\ C = \frac{(1-m^2)^2}{4}$.

$$q_{17}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{(-1+m^2)^2}}{\sqrt{\gamma}} \frac{sn(\xi,m)}{dn(\xi,m) \pm cn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{dn(\xi,m) \pm cn(\xi,m)}{sn(\xi,m)} \right), \quad (26)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \frac{1}{2}\left(3 + 3m^2 - 9\sqrt{(1-m^2)^2} - 2l^2 - 2\alpha^2 - 2\beta^2\right)t$.

xiii) $A = \frac{1}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^4}{4}$.

$$q_{18}(x,y,z,t) = e^{i\theta} \times \left(\frac{i\sqrt{\frac{3}{2}}\sqrt{m^4}}{\sqrt{\gamma}} \frac{cn(\xi,m)}{\sqrt{1-m^2} \pm dn(\xi,m)} + \frac{i\sqrt{\frac{3}{2}}}{\sqrt{\gamma}} \frac{\sqrt{1-m^2} \pm dn(\xi,m)}{cn(\xi,m)} \right), \quad (27)$$

where $\xi = x + y + z - 2(\beta + \alpha + l)t,\ \theta = \beta x + \alpha y + lz + \frac{1}{2}\left(-6 + 3m^2 - 9\sqrt{m^4} - 2l^2 - 2\alpha^2 - 2\beta^2\right)t$.

3.1. Remark

Here $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ are Jacobi elliptic functions and m denotes the modulus of the Jacobi elliptic functions. If $m \to 1$ then $sn\xi \to tanh\xi$, $cn\xi \to sech\xi$, $dn\xi \to sech\xi$ and If $m \to 0$ then $sn\xi \to sin\xi, cn\xi \to cos\xi, dn\xi \to 1$.

When $m \to 0$ ve $m \to 1$ are taken in the solutions (10)-(27) trigonometric and hyperbolic of (41) become as follow

$$q_{19}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (-3 - l^2 - \alpha^2 - \beta^2)t)} \times cosec(x + y + z - 2(\beta + \alpha + l)t), \quad (28)$$

$$q_{20}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (-3 - l^2 - \alpha^2 - \beta^2)t)} \times sec(x + y + z - 2(\beta + \alpha + l)t), \quad (29)$$

$$q_{21}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (-12 - l^2 - \alpha^2 - \beta^2)t)} \times cosec(x + y + z - 2(\beta + \alpha + l)t), \times sec[x + y + z - 2(\beta + \alpha + l)t], \quad (30)$$

$$q_{22}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (-3 - l^2 - \alpha^2 - \beta^2)t)} \times \frac{cot\left(\frac{x+y+z-2(\beta+\alpha+l)t}{2}\right)}{1 + cos(x+y+z-2(\beta+\alpha+l)t)}, \quad (31)$$

$$q_{23}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (-24 - l^2 - \alpha^2 - \beta^2)t)} \times cosh(2(x + y + z - 2(\beta + \alpha + l)t)) \times Cosech(x + y + z - 2(\beta + \alpha + l)t) \times sech(x + y + z - 2(\beta + \alpha + l)t), \quad (32)$$

$$q_{24}(x,y,z,t) = \sqrt{-\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (3 - l^2 - \alpha^2 - \beta^2)t)} \times sech(x + y + z - 2(\beta + \alpha + l)t), \quad (33)$$

$$q_{25}(x,y,z,t) = \sqrt{\frac{6}{\gamma}}\, i e^{i(\beta x + \alpha y + lz + (3 - l^2 - \alpha^2 - \beta^2)t)} \times cosech(x + y + z - 2(\beta + \alpha + l)t), \quad (34)$$

$$q_{26}(x,y,z,t)=\sqrt{\frac{6}{\gamma}}\,ie^{i(\beta x+\alpha y+lz+(-6-l^2-\alpha^2-\beta^2)t)}\times \frac{coth\left(\frac{x+y+z-2(\beta+\alpha+l)t}{2}\right)}{1+sech(x+y+z-2(\beta+\alpha+l)t)}, \quad (35)$$

$$q_{27}(x,y,z,t)=\sqrt{\frac{6}{\gamma}}\,ie^{i(\beta x+\alpha y+lz+(-6-l^2-\alpha^2-\beta^2)t)}\times tanh(x+y+z-2(\beta+\alpha+l)t), \quad (36)$$

$$q_{28}(x,y,z,t)=\sqrt{\frac{6}{\gamma}}\;ie^{i(\beta x+\alpha y+lz+(-6-l^2-\alpha^2-\beta^2)t)}, \quad (37)$$

$$q_{29}(x,y,z,t)=\sqrt{-\frac{6}{\gamma}}\,i\;e^{i(\beta x+\alpha y+lz+(6-l^2-\alpha^2-\beta^2)t)}, \quad (38)$$

$$q_{30}(x,y,z,t)=2\sqrt{\frac{6}{\gamma}}\;ie^{i(\beta x+\alpha y+lz+(-24-l^2-\alpha^2-\beta^2)t)}, \quad (39)$$

$$q_{31}(x,y,z,t)=2\sqrt{-\frac{6}{\gamma}}\,ie^{i(\beta x+\alpha y+lz+(24-l^2-\alpha^2-\beta^2)t)}. \quad (40)$$

a) **b)**

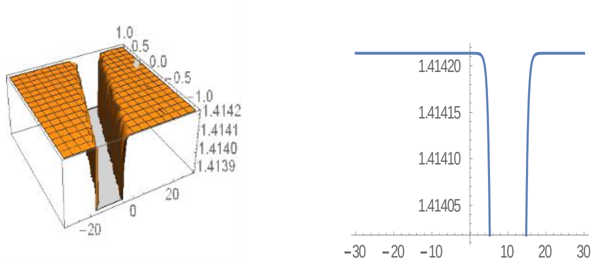

Figure 1. a) The 3D surfaces of |36| by considering the values $l=2,\alpha=1,\beta=2,\gamma=3,y=-2,z=2,-30<x<30,-1<t<1$. **b)** The 2D surfaces of |36| by considering the values $l=2,\alpha=1,\beta=2,\gamma=3,y=-2,z=2,t=1,-30<x<30$.

a) **b)**

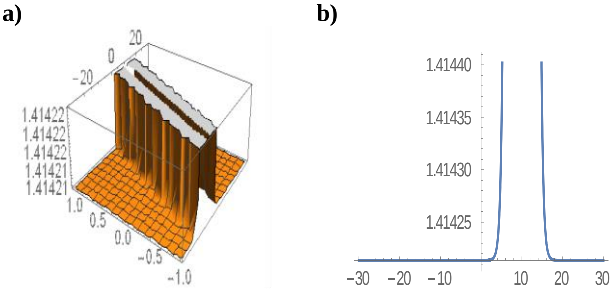

Figure 2. a) The 3D surfaces of |35| by considering the values $l=2,\alpha=1,\beta=2,\gamma=3,y=-2,z=2,-30<x<30,-1<t<1$. **b)** The 2D surfaces of |35|by considering the values $l=2,\alpha=1,\beta=2,\gamma=3,y=-2,z=2,t=1,-30<x<30$.

4. THE COUPLED NONLINEAR SCHRODINGER'S EQUATION

As a second example, we consider the coupled nonlinear Schrödinger's equation

$$\begin{cases} ip_t+p_{xx}+2p|p|^2+2p|q|^2=0,\\ iq_t+q_{xx}+2q|p|^2+2q|q|^2=0. \end{cases} \quad (41)$$

Let's use following conversions for (41),

$$\begin{cases} p(x,t)=e^{i\theta}u(\xi),\\ q(x,t)=e^{i\theta}v(\xi), \end{cases} \quad (42)$$

where $\xi=x+vt\,,\theta=\beta x+rt$.

We obtain following ordinary differential equation when conversions (42) are written in substitution for (41)

$$\begin{cases} u''-(\beta^2+r)u+2u^3+2uv^2=0,\\ v''-(\beta^2+r)v+2vu^2+2v^3=0, \end{cases} \quad (43)$$

where $v=-2(\beta+\alpha+l)$. In Eq. (43), balancing u'' with uv^2 and v'' with vu^2 gives $n_1=1$, $n_2=1$. Then, the solution (3) becomes as follow,

$$\begin{cases} u=a_0+a_1F+b_1F^{-1},\\ v=c_0+c_1F+d_1F^{-1}. \end{cases} \quad (44)$$

When (44) and (4) are written into (43) equation, and when F^i and F^{-i} coefficients from the same degree are equalized to zero, then following linear algebraic equation system is obtained for a_0,a_1,b_1,β and r,

$$-ra_0-\beta^2a_0+2a_0^3+12a_0a_1b_1+2a_0c_0^2+4b_1c_0c_1+4a_1c_0d_1+4a_0c_1d_1=0,$$

$$6a_0a_1^2+4a_1c_0c_1+2a_0c_1^2=0,\;\; 2Ca_1+2a_1^3+2a_1c_1^2=0,$$

$$Ba_1-ra_1-\beta^2a_1+6a_0^2a_1+6a_1^2b_1+2a_1c_0^2+4a_0c_0c_1+2b_1c_1^2+4a_1c_1d_1=0,$$

$$6a_0b_1^2+4b_1c_0d_1+2a_0d_1^2=0,$$

$$Bb_1-rb_1-\beta^2b_1+6a_0^2b_1+6a_1b_1^2+2b_1c_0^2+4a_0c_0d_1+4b_1c_1d_1+2a_1d_1^2=0,$$

$$2Ab_1+2b_1^3+2b_1d_1^2=0,$$

$$-rc_0-\beta^2c_0+2a_0^2c_0+4a_1b_1c_0+2c_0^3+4a_0b_1c_1+4a_0a_1d_1+12c_0c_1d_1=0,$$

$$2a_1^2c_0+4a_0a_1c_1+6c_0c_1^2=0,\;\; 2Cc_1+2a_1^2c_1+2c_1^3=0,$$

$$4a_0a_1c_0+Bc_1-rc_1-\beta^2c_1+2a_0^2c_1+4a_1b_1c_1+6c_0^2c_1+2a_1^2d_1+6c_1^2d_1=0,$$

$$2b_1^2c_0+4a_0b_1d_1+6c_0d_1^2=0,$$

$$4a_0b_1c_0+2b_1^2c_1+Bd_1-rd_1-\beta^2d_1+2a_0^2d_1+4a_1b_1d_1+6c_0^2d_1+6c_1d_1^2=0,$$

$$2Ad_1+2b_1^2d_1+2d_1^3=0.$$

If the algebraic equation system above is solved with the aid of Mathematica, $a_0,a_1,b_1,c_0,c_1,d_1,\beta$ and r are found as follow,

$a_0=0,$

$C\neq 0,\; b_1=\frac{\sqrt{A}a_1}{\sqrt{C}},\;\; c_0=0,\;\; c_1=\sqrt{-C-a_1^2},\;\; a_1\neq$

$0,\ d_1 = \frac{\sqrt{A}\sqrt{-C-a_1^2}}{\sqrt{C}},\ r = B - \beta^2 - 6\sqrt{A}\sqrt{C},\ A \neq 0.$ (45)

Substituting (45) into (44) and using solutions of Eq. (4), also considering conversion (42), the solutions of equation (41) are found as follow,

$i)\ A = 1,\ B = -(1+m^2),\ C = m^2.$

$$p_1(x,t) = e^{i\theta} \times \left(a_1\, sn(\xi,m) + \frac{a_1}{\sqrt{m^2}} \frac{1}{sn(\xi,m)}\right), \quad (46)$$

$$q_1(x,t) = e^{i\theta} \times \left(\sqrt{-m^2 - a_1^2}\, sn(\xi,m) + \frac{\sqrt{-m^2-a_1^2}}{\sqrt{m^2}} \frac{1}{sn(\xi,m)}\right), \quad (47)$$

$$p_2(x,t) = e^{i\theta} \times \left(a_1\, cd(\xi,m) + \frac{a_1}{\sqrt{m^2}} \frac{1}{cd(\xi,m)}\right), \quad (48)$$

$$q_2(x,t) = e^{i\theta} \times \left(\sqrt{-m^2 - a_1^2}\, cd(\xi,m) + \frac{\sqrt{-m^2-a_1^2}}{\sqrt{m^2}} \frac{1}{cd(\xi,m)}\right), \quad (49)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + (-(1+m^2) - \beta^2 - 6m)t$.

$ii)\ A = 1 - m^2,\ B = 2m^2 - 1,\ C = -m^2.$

$$p_3(x,t) = e^{i\theta} \times \left(a_1\, cn(\xi,m) + \frac{\sqrt{1-m^2}a_1}{\sqrt{-m^2}} \frac{1}{cn(\xi,m)}\right), \quad (50)$$

$$q_3(x,t) = e^{i\theta} \times \left(\sqrt{m^2 - a_1^2}\, cn(\xi,m) + \frac{\sqrt{1-m^2}\sqrt{m^2-a_1^2}}{\sqrt{-m^2}} \frac{1}{cn(\xi,m)}\right), \quad (51)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left((2m^2-1) - \beta^2 - 6\sqrt{(1-m^2)}\sqrt{-m^2}\right)t.$

$iii)\ A = m^2 - 1,\ B = 2 - m^2,\ C = -1.$

$$p_4(x,t) = e^{i\theta} \times \left(a_1\, dn(\xi,m) - i\sqrt{-1+m^2}a_1 \frac{1}{dn(\xi,m)}\right), \quad (52)$$

$$q_4(x,t) = e^{i\theta} \times \left(\sqrt{1-a_1^2}\, dn(\xi,m) - i\sqrt{-1+m^2}\sqrt{1-a_1^2} \frac{1}{dn(\xi,m)}\right), \quad (53)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left(2 - m^2 - 6i\sqrt{-1+m^2} - \beta^2\right)t.$

$iv)\ A = -m^2(1-m^2),\ B = 2m^2 - 1,\ C = 1.$

$$p_5(x,y,z,t) = e^{i\theta} \times \left(a_1\, ds(\xi,m) + \sqrt{-m^2(1-m^2)}a_1 \frac{1}{ds(\xi,m)}\right), \quad (54)$$

$$q_5(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-1-a_1^2}ds(\xi,m) + \sqrt{-m^2(1-m^2)}\sqrt{-1-a_1^2} \frac{1}{ds(\xi,m)}\right), \quad (55)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left(-1 + 2m^2 - 6\sqrt{m^2(-1+m^2)} - \beta^2\right)t.$

$v)\ A = 1 - m^2,\ B = 2 - m^2,\ C = 1.$

$$p_6(x,y,z,t) = e^{i\theta} \times \left(a_1\, cs(\xi,m) + \sqrt{1-m^2}a_1 \frac{1}{cs(\xi,m)}\right), \quad (56)$$

$$q_6(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-1-a_1^2}\, cs(\xi,m) + \sqrt{1-m^2}\sqrt{-1-a_1^2} \frac{1}{cs(\xi,m)}\right), \quad (57)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left(2 - m^2 - 6\sqrt{1-m^2} - \beta^2\right)t.$

$vi)\ A = \frac{1}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4}.$

$$p_7(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{sn(\xi,m)}{1 \pm dn(\xi,m)} + \frac{a_1}{\sqrt{m^2}} \frac{1 \pm dn(\xi,m)}{sn(\xi,m)}\right), \quad (58)$$

$$q_7(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{m^2}{4} - a_1^2} \frac{sn(\xi,m)}{1 \pm dn(\xi,m)} + \frac{\sqrt{-\frac{m^2}{4}-a_1^2}}{\sqrt{m^2}} \frac{1 \pm dn(\xi,m)}{sn(\xi,m)}\right), \quad (59)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \frac{1}{2}\left(-2 + m^2 - 3\sqrt{m^2} - 2\beta^2\right)t.$

$vii)\ A = \frac{m^2}{4},\ B = \frac{m^2-2}{2},\ C = \frac{m^2}{4}.$

$$p_8(x,y,z,t) = e^{i\theta} \times \left(a_1\, (sn(\xi,m) \pm icn(\xi,m)\,) + a_1 \frac{1}{sn(\xi,m) \pm icn(\xi,m)}\right), \quad (60)$$

$$q_8(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{m^2}{4} - a_1^2}\, (sn(\xi,m) \pm icn(\xi,m)\,) + \sqrt{-\frac{m^2}{4} - a_1^2} \frac{1}{sn(\xi,m) \pm icn(\xi,m)}\right), \quad (61)$$

$$p_9(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{dn(\xi,m)}{i\sqrt{1-m^2}\,sn(\xi,m) \pm cn(\xi,m)} + a_1 \frac{i\sqrt{1-m^2}\,sn(\xi,m) \pm cn(\xi,m)}{dn(\xi,m)}\right), \quad (62)$$

$$q_9(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{m^2}{4} - a_1^2}\, \frac{dn(\xi,m)}{i\sqrt{1-m^2}\,sn(\xi,m) \pm cn(\xi,m)} + \sqrt{-\frac{m^2}{4} - a_1^2}\, \frac{i\sqrt{1-m^2}\,sn(\xi,m) \pm cn(\xi,m)}{dn(\xi,m)}\right), \quad (63)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + (-1 - m^2 - \beta^2)t.$

$viii)\ A = \frac{1}{4},\ B = \frac{1-2m^2}{2},\ C = \frac{1}{4}.$

$$p_{10}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{dn(\xi,m)}{mcn(\xi,m) \pm i\sqrt{1-m^2}} + a_1 \frac{mcn(\xi,m) \pm i\sqrt{1-m^2}}{dn(\xi,m)}\right), \quad (64)$$

$$q_{10}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{1}{4} - a_1^2}\, \frac{dn(\xi,m)}{mcn(\xi,m) \pm i\sqrt{1-m^2}} + \sqrt{-\frac{1}{4} - a_1^2}\, \frac{mcn(\xi,m) \pm i\sqrt{1-m^2}}{dn(\xi,m)}\right), \quad (65)$$

$$p_{11}(x,y,z,t) = e^{i\theta} \times \left(a_1 (msn(\xi,m) \pm idn(\xi,m)) + a_1 \frac{1}{msn(\xi,m) \pm idn(\xi,m)}\right), \quad (66)$$

$$q_{11}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{1}{4} - a_1^2}(msn(\xi,m) \pm idn(\xi,m)) + \sqrt{-\frac{1}{4}C - a_1^2}\, \frac{1}{msn(\xi,m) \pm idn(\xi,m)}\right), \quad (67)$$

$$p_{12}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{sn(\xi,m)}{1 \pm cn(\xi,m)} + a_1 \frac{1 \pm cn(\xi,m)}{sn(\xi,m)}\right), \quad (68)$$

$$q_{12}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{1}{4} - a_1^2}\, \frac{sn(\xi,m)}{1 \pm cn(\xi,m)} + \sqrt{-\frac{1}{4} - a_1^2}\, \frac{1 \pm cn(\xi,m)}{sn(\xi,m)}\right), \quad (69)$$

$$p_{13}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{cn(\xi,m)}{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)} + a_1 \frac{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)}{cn(\xi,m)}\right), \quad (70)$$

$$q_{13}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{1}{4} - a_1^2}\, \frac{cn(\xi,m)}{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)} + \sqrt{-\frac{1}{4} - a_1^2}\, \frac{\sqrt{1-m^2}\,sn(\xi,m) \pm dn(\xi,m)}{cn(\xi,m)}\right), \quad (71)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + (-1 - m^2 - \beta^2)t.$

$ix)\ A = \frac{m^2-1}{4},\ B = \frac{m^2+1}{2},\ C = \frac{m^2-1}{4}.$

$$p_{14}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{dn(\xi,m)}{1 \pm msn(\xi,m)} + a_1 \frac{1 \pm msn(\xi,m)}{dn(\xi,m)}\right), \quad (72)$$

$$q_{14}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{m^2-1}{4} - a_1^2}\, \frac{dn(\xi,m)}{1 \pm msn(\xi,m)} + \sqrt{-\frac{m^2-1}{4} - a_1^2}\, \frac{1 \pm msn(\xi,m)}{dn(\xi,m)}\right), \quad (73)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + (2 - m^2 - \beta^2)t.$

$x)\ A = \frac{1-m^2}{4},\ B = \frac{m^2+1}{2},\ C = \frac{1-m^2}{4}.$

$$p_{15}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{cn(\xi,m)}{1 \pm sn(\xi,m)} + a_1 \frac{1 \pm sn(\xi,m)}{cn(\xi,m)}\right), \quad (74)$$

$$q_{15}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{1-m^2}{4} - a_1^2}\, \frac{cn(\xi,m)}{1 \pm sn(\xi,m)} + \sqrt{-\frac{1-m^2}{4} - a_1^2}\, \frac{1 \pm sn(\xi,m)}{cn(\xi,m)}\right), \quad (75)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + (-1 + 2m^2 - \beta^2)t.$

$xi)\ A = -\frac{(1-m^2)^2}{4},\ B = \frac{m^2+1}{2},\ C = -\frac{1}{4}.$

$$p_{16}(x,y,z,t) = e^{i\theta} \times \left(a_1 (mcn(\xi,m) \pm dn(\xi,m)) - i\sqrt{-(-1+m^2)^2}\, a_1 \frac{1}{mcn(\xi,m) \pm dn(\xi,m)}\right), \quad (76)$$

$$q_{16}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{\frac{1}{4} - a_1^2}\,(mcn(\xi,m) \pm dn(\xi,m)) - \frac{1}{2} i\sqrt{-(-1+m^2)^2}\sqrt{1 - 4a_1^2}\, \frac{1}{mcn(\xi,m) \pm dn(\xi,m)}\right), \quad (77)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left(\frac{1}{2}\left(1 + m^2 - 3i\sqrt{-(-1+m^2)^2} - 2\beta^2\right)\right)t.$

$xii)\ A = \frac{1}{4},\ B = \frac{m^2+1}{2},\ C = \frac{(1-m^2)^2}{4}.$

$$p_{17}(x,y,z,t) = e^{i\theta} \times \left(a_1 \frac{sn(\xi,m)}{dn(\xi,m) \pm cn(\xi,m)} + \frac{a_1}{\sqrt{(-1+m^2)^2}}\, \frac{dn(\xi,m) \pm cn(\xi,m)}{sn(\xi,m)}\right), \quad (78)$$

$$q_{17}(x,y,z,t) = e^{i\theta} \times \left(\sqrt{-\frac{(1-m^2)^2}{4} - a_1^2}\, \frac{sn(\xi,m)}{dn(\xi,m) \pm cn(\xi,m)} + \frac{\sqrt{-(-1+m^2)^2 - 4a_1^2}}{2\sqrt{(-1+m^2)^2}}\, \frac{dn(\xi,m) \pm cn(\xi,m)}{sn(\xi,m)}\right), \quad (79)$$

where $\xi = x - 2\beta t,\ \theta = \beta x + \left(\frac{1}{2}\left(1 + m^2 - 3\sqrt{(-1+m^2)^2} - 2\beta^2\right)\right)t.$

$xiii)\ A=\frac{1}{4},\ B=\frac{m^2-2}{2},\ C=\frac{m^4}{4}.$

$$p_{18}(x,y,z,t)=e^{i\theta}\times\left(a_1\frac{cn(\xi,m)}{\sqrt{1-m^2}\pm dn(\xi,m)}+\frac{a_1}{\sqrt{m^4}}\frac{\sqrt{1-m^2}\pm dn(\xi,m)}{cn(\xi,m)}\right), \quad (80)$$

$$q_{18}(x,y,z,t)=e^{i\theta}\times\left(\sqrt{-\frac{m^4}{4}-a_1^2}\frac{cn(\xi,m)}{\sqrt{1-m^2}\pm dn(\xi,m)}+\frac{\sqrt{-\frac{m^4}{4}-a_1^2}}{\sqrt{m^4}}\frac{\sqrt{1-m^2}\pm dn(\xi,m)}{cn(\xi,m)}\right), \quad (81)$$

where $\xi=x-2\beta t,\ \theta=\beta x+\left(\frac{1}{2}\left(-2+m^2-3\sqrt{m^4}-2\beta^2\right)\right)t.$

4.1. Remark

Here $sn(\xi,m), cn(\xi,m), dn(\xi,m)$ are Jacobi elliptic functions and m denotes the modulus of the Jacobi elliptic functions. If $m\to1$ then $sn\xi\to tanh\xi$, $cn\xi\to sech\xi$, $dn\xi\to sech\xi$ and If $m\to0$ then $sn\xi\to sin\xi, cn\xi\to cos\xi, dn\xi\to1$.

When $m\to0$ ve $m\to1$ are taken in the solutions (46)-(81) trigonometric and hyperbolic solutions of equation (41) become as follow

$$p_{19}(x,t)=a_1e^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t) \quad (82)$$

$$q_{19}(x,t)=a_1ie^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t), \quad (83)$$

$$p_{20}(x,t)=a_1e^{i(\beta x+(-1-\beta^2)t)}\times cos(x-2\beta t), \quad (84)$$

$$q_{20}(x,t)=a_1i\,e^{i(\beta x+(-1-\beta^2)t)}\times cos(x-2\beta t), \quad (85)$$

$$p_{21}(x,t)=a_1e^{i(\beta x+(-8-\beta^2)t)}\times cosh[2(x-2\beta t)]cosech(x-2\beta t)sech(x-2\beta t), \quad (86)$$

$$q_{21}(x,t)=\sqrt{-1-a_1^2}e^{i(\beta x+(-8-\beta^2)t)}\times cosh[2(x-2\beta t)]cosech(x-2\beta t)sech(x-2\beta t), \quad (87)$$

$$p_{22}(x,t)=2a_1\times e^{i(\beta x+(-8-\beta^2)t)}, \quad (88)$$

$$q_{22}(x,t)=2\sqrt{-1-a_1^2}\times e^{i(\beta x+(-8-\beta^2)t)}, \quad (89)$$

$$p_{23}(x,t)=a_1e^{i(\beta x+(1-\beta^2)t)}\times sech(x-2\beta t), \quad (90)$$

$$q_{23}(x,t)=\sqrt{1-a_1^2}\,e^{i(\beta x+(1-\beta^2)t)}\times sech(x-2\beta t), \quad (91)$$

$$p_{24}(x,t)=2a_1\times e^{i(\beta x+(8-\beta^2)t)}, \quad (92)$$

$$q_{24}(x,t)=2\sqrt{1-a_1^2}\times e^{i(\beta x+(8-\beta^2)t)}, \quad (93)$$

$$p_{25}(x,t)=a_1e^{i(\beta x+(1-\beta^2)t)}\times cosec(x-2\beta t), \quad (94)$$

$$q_{25}(x,t)=\sqrt{-1-a_1^2}e^{i(\beta x+(1-\beta^2)t)}\times cosec(x-2\beta t), \quad (95)$$

$$p_{26}(x,t)=a_1e^{i(\beta x+(1-\beta^2)t)}\times cosech(x-2\beta t), \quad (96)$$

$$q_{26}(x,t)=\sqrt{-1-a_1^2}e^{i(\beta x+(1-\beta^2)t)}\times cosech(x-2\beta t), \quad (97)$$

$$p_{27}(x,t)=a_1e^{i(\beta x+(-4-\beta^2)t)}\times cosec(x-2\beta t)sec(x-2\beta t), \quad (98)$$

$$q_{27}(x,t)=\sqrt{1-a_1^2}e^{i(\beta x+(-4-\beta^2)t)}\times cosec(x-2\beta t)sec(x-2\beta t), \quad (99)$$

$$p_{28}(x,t)=\frac{a_1}{2}e^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t), \quad (100)$$

$$q_{28}(x,t)=\frac{a_1i}{2}e^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t), \quad (101)$$

$$p_{29}(x,t)=2a_1e^{i(\beta x+(-2-\beta^2)t)}\times coth(x-2\beta t), \quad (102)$$

$$q_{29}(x,t)=\sqrt{-1-4a_1^2}e^{i(\beta x+(-2-\beta^2)t)}\times\frac{coth\left[\frac{1}{2}(x-2\beta t)\right]}{1+sech(x-2\beta t)}, \quad (103)$$

$$p_{30}(x,t)=2a_1e^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t), \quad (104)$$

$$q_{30}(x,t)=2a_1ie^{i(\beta x+(-1-\beta^2)t)}\times sin(x-2\beta t), \quad (105)$$

$$p_{31}(x,t)=2a_1e^{i(\beta x+(-1-\beta^2)t)}\times cos(x-2\beta t), \quad (106)$$

$$q_{31}(x,t)=2a_1ie^{i(\beta x+(-1-\beta^2)t)}\times cos(x-2\beta t), \quad (107)$$

$$p_{32}(x,t)=a_1e^{i(\beta x+(-2-\beta^2)t)}\times\frac{\left[1+(i\,sech(x-2\beta t)+tanh(x-2\beta t))^2\right]}{i\,sech(x-2\beta t)+tanh(x-2\beta t)}, \quad (108)$$

$$q_{32}(x,t)=\sqrt{-1-4a_1^2}e^{i(\beta x+(-2-\beta^2)t)}\times tanh(x-2\beta t), \quad (109)$$

$$p_{33}(x,t)=2a_1\times e^{i(\beta x+(-2-\beta^2)t)}, \quad (110)$$

$$q_{33}(x,t)=2\sqrt{-\frac{1}{4}-a_1^2}\times e^{i(\beta x+(-2-\beta^2)t)}, \quad (111)$$

$$p_{34}(x,t)=2a_1e^{i(\beta x+(-1-\beta^2)t)}\times cosec(x-2\beta t), \quad (112)$$

$$q_{34}(x,t)=\sqrt{-1-4a_1^2}e^{i(\beta x+(-1-\beta^2)t)}\times\frac{cot\left[\frac{1}{2}(x-2\beta t)\right]}{1+cos(x-2\beta t)}, \quad (113)$$

$$p_{35}(x,t)=2a_1e^{i(\beta x+(-1-\beta^2)t)}\times\frac{[sec(x-2\beta t)+tan(x-2\beta t)]}{1+sin(x-2\beta t)}, \quad (114)$$

$$q_{35}(x,t)=\sqrt{-1-4a_1^2}e^{i(\beta x+(-1-\beta^2)t)}\times sec(x-2\beta t), \quad (115)$$

$$p_{36}(x,t)=2a_1\times e^{i(\beta x+(2-\beta^2)t)}, \quad (116)$$

$$q_{36}(x,t) = \sqrt{1-4a_1^2} \times e^{i(\beta x+(2-\beta^2)t)}, \quad (117)$$

$$p_{37}(x,t) = 2a_1 e^{i(\beta x+(1-\beta^2)t)} \times cosh(x-2\beta t), \quad (118)$$

$$q_{37}(x,t) = 2a_1 i\, e^{i(\beta x+(1-\beta^2)t)} \times cosh(x-2\beta t), \quad (119)$$

$$p_{38}(x,t) = 2a_1 e^{i(\beta x+(1-\beta^2)t)} \times sech(x-2\beta t), \quad (120)$$

$$q_{38}(x,t) = \sqrt{1-4a_1^2}\, e^{i(\beta x+(1-\beta^2)t)} \times sech(x-2\beta t), \quad (121)$$

$$p_{39}(x,t) = \frac{1}{2} a_1 e^{i(\beta x+(1-\beta^2)t)} \times sinh(x-2\beta t), \quad (122)$$

$$q_{39}(x,t) = \frac{1}{2} a_1 i\, e^{i(\beta x+(1-\beta^2)t)} \times sinh(x-2\beta t), \quad (123)$$

$$p_{40}(x,t) = \frac{1}{2} a_1 e^{i(\beta x+(-1-\beta^2)t)} \times cos(x-2\beta t), \quad (124)$$

$$q_{40}(x,t) = \frac{1}{2} a_1 i\, e^{i(\beta x+(-1-\beta^2)t)} \times cos(x-2\beta t), \quad (125)$$

a) **b)**

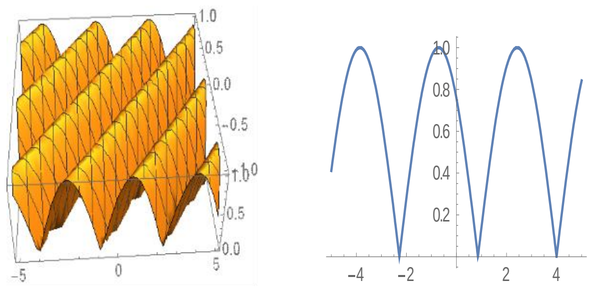

Figure 3. a) The 3D surfaces of |82| by considering the values $a_1 = 1, \beta = 2, -10 < x < 10, -1 < t < 1$. **b)**The 2D surfaces of |82| by considering the values $a_1 = 1, \beta = 2, t = 1, -10 < x < 10$.

a) **b)**

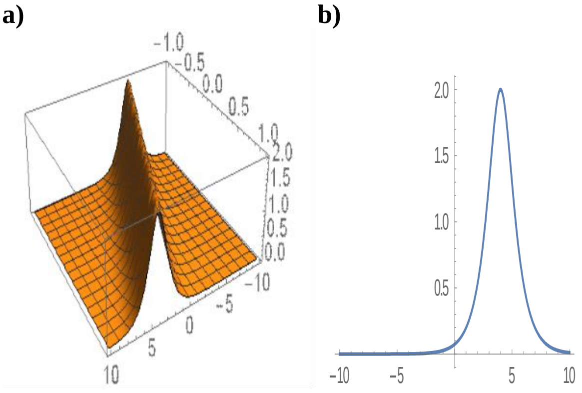

Figure 4. a) The 3D surfaces of |90| by considering the values $a_1 = 2, \beta = 2, -10 < x < 10, -1 < t < 1$. **b)**The 2D surfaces of |90| by considering the values $a_1 = 2, \beta = 2, t = 1, -10 < x < 10$.

a) **b)**

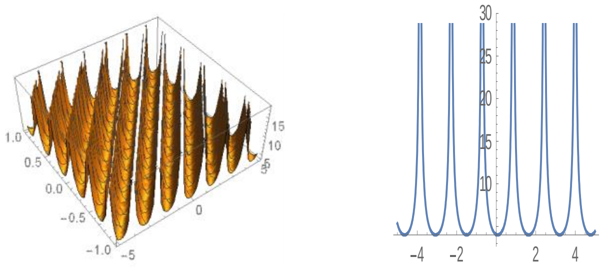

Figure 5. a) The 3D surfaces of |98| by considering the values $a_1 = 2, \beta = 2, -5 < x < 5, -1 < t < 1$. **b)**The 2D surfaces of |98| by considering the values $a_1 = 2, \beta = 2, t = 1, -5 < x < 5$.

a) **b)**

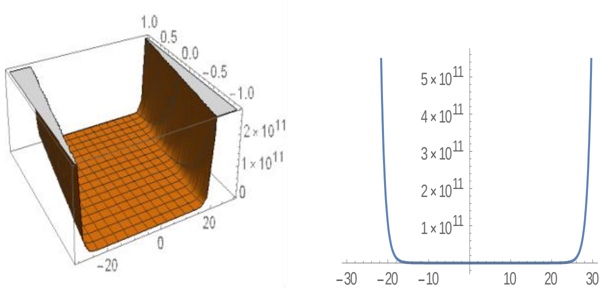

Figure 6. a) The 3D surfaces of |118| by considering the values $a_1 = 2, \beta = 2, -30 < x < 30, -1 < t < 1$. **b)**The 2D surfaces of |118| by considering the values $a_1 = 2, \beta = 2, t = 1, -30 < x < 30$.

5. CONCLUSION

In this study, we obtained traveling wave solutions of the (3+1) dimensional nonlinear Schrödinger equation and coupled nonlinear Schrödinger equation using the generalized Jacobi elliptical function method. We have expressed these solutions both as Jacobi elliptical solutions and trigonometric and hyperbolic solutions. . Later, using Mathematica 11.2 program, we saw that these solutions satisfy the equation. We present two and three dimensional graphics of some solutions we have found. This method has been successfully applied to solve some nonlinear wave equations and can be used in many other nonlinear equations or combined equations.

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