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Research Article

Textual and contextual *commognitive* conflict students in solving an improper fraction

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Abstract

Commognitive conflict occurs when there are differences in the use of discourse by the interlocutor and becomes a challenge in mathematical communication. For sources of *commognitive* conflict that have not yet been discussed, therefore the purpose of this study was to reveal source of students' *commognitive* conflict in solving problems. This research was to determine the source of the *commognitive* conflict experienced by first-year students in solving mathematical problems, that is, improper fraction. *Commognitve* characteristics used in this study are word uses, visual mediators, narratives, and routines. A total of 38 students were given a math problem sheet, and two students were chosen as research participants because they had different and interesting answers. Then the participants participated in a semi-structured in-depth interview to find out the factors that cause *commognitive* conflict. The results of the research shown that the source of *commognitive* conflict lies in visual mediators and narratives produced by participants. The visual mediators that are produced should be interpreted contextually, but the indicators undergo a shift in meaning into the textual in the minds of participants.

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Introduction

Sfard (2001) says that thinking is conceptualized as communicating with oneself in verbal form or with symbolic assistance. Sfard (2007, 2008) views mathematics as a form of communication and refuses to separate thinking and communicating. To emphasize these two terms, Sfard combines cognitive and communication terms into a new adjective called "*commognitive*". In addition to recognizing communication as a component of thought, Sfard's position in communication is almost the same as thinking itself (Kieran, et al. 2002). Communication is used for thinking and can stand alone, described as "talking to yourself" (Sfard & Kieran, 2001). These conditions can be conceptualized as a type of communication activity. Communication through spoken and written language and manipulation of physical objects and artifacts are the main means employed for teaching and learning purposes. If the learning of mathematics marks a beginning in the discourse of mathematics, then in general it involves a shift in the ability of learners (Sfard, 2008).

Some researchers have used the *commognitive* concept to study changes or shifts in learner discourse abilities over time (Heyd-Metzuyanim, 2015; Nachlieli & Tabach, 2012, Zayyadi, et al. 2020). Viirman (2011) and Berger & Bowie

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(2012) use *commognitive* theory to study teacher discourse, and the results show that teachers can develop professionally through the exploration of changes or shifts in learners' thinking over time.

Specifically, discourse is unique to a community in the use of *word use*, *visual mediator*, *narrative*, and *routine*. Discourse is distinguished by the following four characteristics (Sfard, 2007; Sfard, 2008; Berger, 2013; Nardi, Ryve, Stadler, & Viirman, 2014):

Word use. One of the hallmarks of the keywords used in mathematics is words that signify number and form. Word use is very important because it is the same as what other people call "word meaning," in that users are responsible for what is said about something. Word use refers to words commonly used to communicate every day that have unique and specific meanings in mathematics.

Visual mediators are visible objects that are used as part of the communication process and are manifested in clear ways (such as graphics, diagrams, and symbols). Everyday discourse is usually mediated by physical objects that are used as props in the teaching of mathematics.

Narrative is the sequence of speech or text, whether oral or written, which describes the object and process and the relationship between the object and process. In the case of scientific mathematical discourse, the narrative used is known as a mathematical theory, and this includes constructing thinking such as definitions, proofs, and theorems.

Routines are characteristic of repetitive patterns of existing discourse. Characteristics of repetitive patterns include regularly employed and well-defined practices that are used specifically by a community such as defining, estimating, proving, and generalizing.

Based on the four characteristics of the previous discourse, mathematical communication depends a lot on the handling of the interlocutor's brand, such as the use of words. The appearance of differences in the use of words used by the interlocutor becomes a great challenge in mathematical communication and is one of the types of challenges that Sfard (2008) calls *commognitive* conflict.

Commognitive conflict occurs when there are differences in the use of discourse between students and teachers which results in students adopting new teacher discourses (Sfard, 2007, 2008). *Commognitive* conflict itself encourages changes in the way someone defines words or identifies numbers (Sfard, 2008). *Commognitive* conflict is not a factual dispute that can be resolved with mathematical evidence but rather a dispute about the acceptance of an agreement governing discourse. Such conflict triggers meta-level learning because each newly introduced discourse is governed by different meta-rules from newcomers (Ioannou, 2018). Thus, students must gradually accept new discourses (Rabin, Fuller, & Harel, 2013). In contrast to cognitive conflict that occurs when disagreement arises which results in contradictions, interests, anxiety and revaluation (Lee & Byun, 2011). Pratiwi, et al. (2019) indicate that cognitive conflict occurs during information processing if the information received by sensory memory and transferred to short-term memory cannot be directly linked to information in long-term memory. In addition, according to Pratiwi, et al. (2019) the thinking process that occurs when students experience cognitive conflict, are namely: (1) students can perform assimilation processes to integrate the perception or new experiences into schemata, and (2) there are three stages of students' process of the accommodation, that is: first the students experience a lack of mastery of the concepts they have, then create a new conception that is easy to understand, and then the conception is used to solve the problem by providing a sensible answer.

Thoma & Nardi (2008) analyzed the result of first-degree students' answers and focused on the realization of unresolved *commognitive* conflict in students' involvement with the assignments given. The results of this research show that although lecturers' efforts can help students to equalize verbal communication differences, students could still experience *commognitive* conflict during the final exam. Thoma & Nardi (2017) in their research concluded that students' experiences of *commognitive* conflict occurred during mathematics transition from school to university. Furthermore, Ioannao (2017) in his research provides an example of commognitive conflict that occurs in students' learning experiences in mathematics. Analysis of the data shows that students may have a structural understanding of the evidence needed, but they still cannot do it practically. The analysis in Thoma & Nardi (2016) shows that the teacher's assessment aims to assist students in engaging in verbal communication. Specifically, this analysis shows that the lecturer designs assignments with an awareness of potential errors that might occur on students. Such errors are the impact of unresolved *commognitive* conflict.

Some of the above studies have not discussed the sources of the occurrence of *commognitive* conflict. Therefore, the objective of the research is to determine the source of the *commognitive* conflict experienced by first-year students in solving mathematical problems, that is, improper fraction. Commognitive characteristics that are used to see

commognitive conflict that is word uses, visual mediators, narratives, and routines. These findings provide data to guide further research on appropriate learning strategies to facilitate *commognitive* conflicts experienced by students.

Problem of Study

Commognitive conflict occurs when there are differences in the use of discourse by the interlocutor and becomes a challenge in mathematical communication. However, there are not many studies have examined the source of *commognitive* conflict of students in solving an improper fraction problem. Therefore, the research problem is "what is the source of students' *commognitive* conflict in solving an improper fraction problem?"

Method

Research Model

To express the *commognitive* conflict experienced by students, researchers use qualitative research methods to describe the data as it is in accordance with what is found in the field. All types of mathematical questions make it possible to bring up *commognitive* conflict. This study used instruments designed to bring up *commognitive* conflict by giving problems in the form of an improper fraction that is rarely done. Recent research reveals how students build knowledge about improper fractions is not an easy task (Olive & Steffe, 2001; Steffe, 2002; Tzur, 1999). The following improper fraction problem is given.

The rectangle above shows $\frac{6}{5}$. Draw a rectangle that shows $\frac{4}{3}$ based on the reference image above.

Figure1.
Improper Fraction Problem Sheet

Participants

The participants in this study were students who had taken the number concept course. There were 38 students who were given improper fraction problem sheet and only two students were selected as subjects because they met the criteria of experiencing *commognitive* conflict.

Data Collection

The data collected in this study were the data of students who experience *commognitive* conflict in solving mathematical problems in improper fractions. The research data collection was carried out after the participants were selected.

Data Analysis

The selected participants were analyzed concerning their work results and participated in semi-structured in-depth interviews based on four *commognitive* characteristics, namely *word use*, *visual mediators*, *narrative*, and *routines*. The main data used in this study is work results. The process of semi-structured in-depth interviews conducted is as a triangulation of data in the study.

Findings

In this section, the differences between two participants (P1 and P2) about mathematical problem-solving activities, that is, the improper fraction will be explained.

Problem Resolution Results for P1

P1 solves the problem given using two steps, first by translating the image given in the problem as in Figure 1 below.

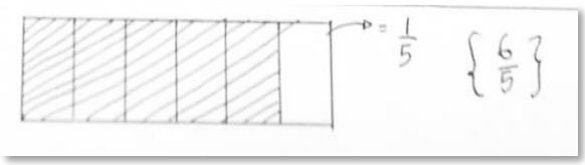

Figure 2.
P1's Work Results in the First Part

Second, P1 used visual mediators to solve the problem as in Figure 2 below.

Figure 3.
P1's Work Results in the Second Part

Based on Figure 1, P1 divides the rectangle into six equal parts and then shades the five divided rectangles. However, when confirmed through interviews, P1 argues that the section shows six parts as large or fractional as the denominator of six. The Table 1 are the results of interviews with P1 related to Figure 1 which he completed.

Table 1.
Interview about Confirmation of the Work of the First Part

Transcript	Word Uses	Visual Mediators	Narratives and Routines
R: can you tell us how the settlements process was done?			
P1: When I first read the problem, I saw the image that was in the problem shows $\frac{6}{5}$.	P1 draw attention to the statement that the rectangle shows $\frac{6}{5}$.		P1 focus on the given improper fraction on the image.
P1: Then I tried to redraw on the rectangle by showed $\frac{6}{5}$ by showing the parts in the image.		Rectangle: Iconic	
P1: I made an image with five equal parts.		Rectangle: Iconic	P1 decided to split the image according to the number of denominators.
P1: However, I experienced confusion after dividing the parts of the image. I understood that to show $\frac{6}{5}$ meant there were five equal parts, but the numerator in the fractions was six, so the initial image I made was lacking the numerator.			P1 agrees that to show 6/5 five equal parts are needed.
P1: To fulfill it, I drew another part that was equal to the one part I had drawn.		Rectangle: Iconic	
P1: However, I see that the image is not showing $\frac{6}{5}$ but as a fraction that contains six.			P1 realize that the results of the picture do not match what is expected i.e. show $\frac{6}{5}$.

The following is an illustration of the visualization of the P1's problem-solving process in the first part.

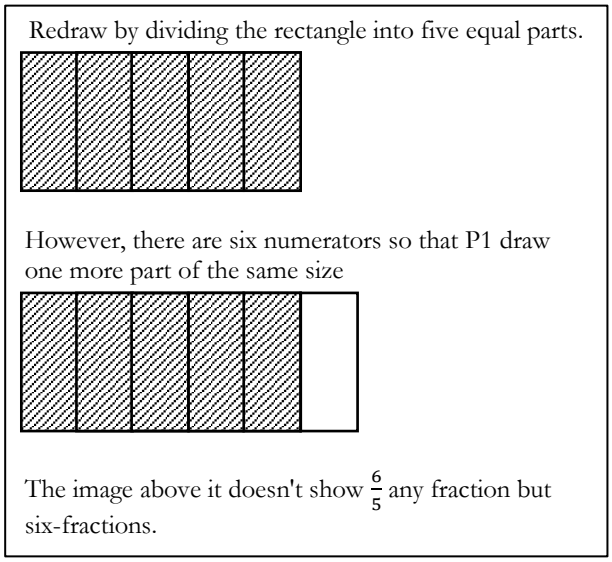

Figure 4.
Illustration of the Visualization of the P1's Problem-Solving Process in the First Part.

Likewise, when P1 moved to solve the next problem, there were differences in the resolution process, as shown in Figure 2. To evaluate these differences, further interviews were conducted with P1. The Table 2 are the results of the P1 interview related to Figure 2 which he completed.

Table 2.
Interview about Confirmation of the Work of the Second Part

Transcript	Word Uses	Visual Mediators	Narratives and Routines
R: What about the settlement process to answer the questions given to the problem?			
P1: The initial settlement is the same, First, I made an image with three equal parts.	P1 draw attention to the statement that the rectangle shows $\frac{4}{3}$.	Rectangle: Iconic	P1 focus on the given improper fraction on the image.
P1: Then I thought, if I did it like the first one, in the end I would be confused again, whether the image showed a fraction $\frac{4}{3}$ or later it would be four. Therefore, I decided to divide one section in what I had drawn into three equal sizes.		Rectangle: Iconic	P1 changes the resolution strategy based on previous problem solving experience.
P1: After that, I took a new part that I split and added it to the first image that I made (while pointing to an image that is divided into three equal parts).		Rectangle: Iconic	
R: What do you think the last image shows $\frac{4}{3}$?			
P1: Yes, ma'am. But I was surprised because the image $\frac{4}{3}$ is smaller than $\frac{6}{5}$, whereas I thought it would be greater than $\frac{6}{5}$.			Image results do not match what is expected results show $\frac{4}{3}$ smaller than $\frac{6}{5}$, and that's wrong.

The following is an illustration of the visualization of the P1's problem-solving process in the second part.

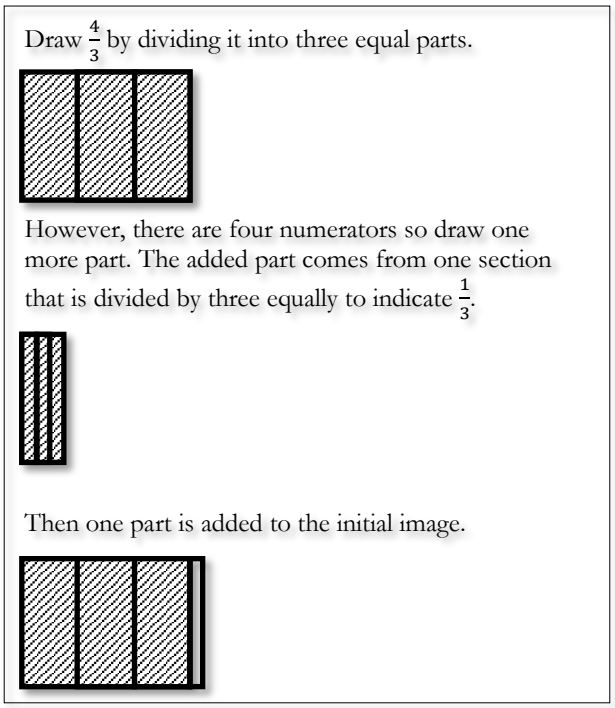

Figure 5.
Illustration of the Visualization of the P1's Problem-Solving Process in the Second Part

Problem Resolution Results for P2

P2 solved the problem given using two steps, first by translating the image given in problem as in Figure 5 below.

Figure 6.
P2's Work Results in the First Part

Second, P2 used visual mediators to solve the problem as in Figure 6 below.

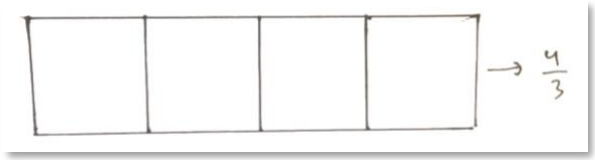

Figure 6.
P2's Work Results in the Second Part

In Figure 5, P2 divides the rectangle into six equal parts. In Figure 6, P2 divides the rectangle into four equal sizes. However, when confirmed through interviews, P2 cannot determine the part of the rectangle that shows the improper fraction $\frac{6}{5}$ and $\frac{4}{3}$. The Table 3 are the results of interviews conducted with P2 related to the image he constructed in Figures 5 and 6.

Table 3.
Interview about Confirmation of the Work

Transcript	Word Uses	Visual Mediators	Narratives and Routines
R: Can you tell us about the workflow? P2: First, I tried to draw back the image that was given in the problem.	P2 draw attention to the statement that the rectangle shows $\frac{6}{5}$.	Rectangle: Iconic	P2 focus on the given improper fraction on the image.
P2: I drew six parts equally because I saw that there would be six parts. This refers to the numerator, which is six.		Rectangle: Iconic	P2 decided to split the image according to the number of numerators.
P2: After describing the six equal parts, I could not determine which part to shows $\frac{6}{5}$, because the results of my drawing showed $\frac{6}{6}$ as if I had decided not to shade the parts in the image.			P2 cannot decide which part shows $\frac{6}{5}$ because the resulting image can also look like $\frac{6}{6}$.
P2: Likewise, when I made a drawing to show $\frac{4}{3}$, the completion process was the same. After dividing the image into four equal parts, again I could not determine which $\frac{4}{3}$ because the image that I made appears to show $\frac{4}{4}$.		Rectangle: Iconic	P2 does not change the strategy to solve the problem so that it still cannot determine the part that shows $\frac{4}{3}$ as the previous problem solving. For P2 the resulting image can also be $\frac{4}{4}$.

The following is an illustration of the visualization of the P2's problem-solving process in the first and second parts.

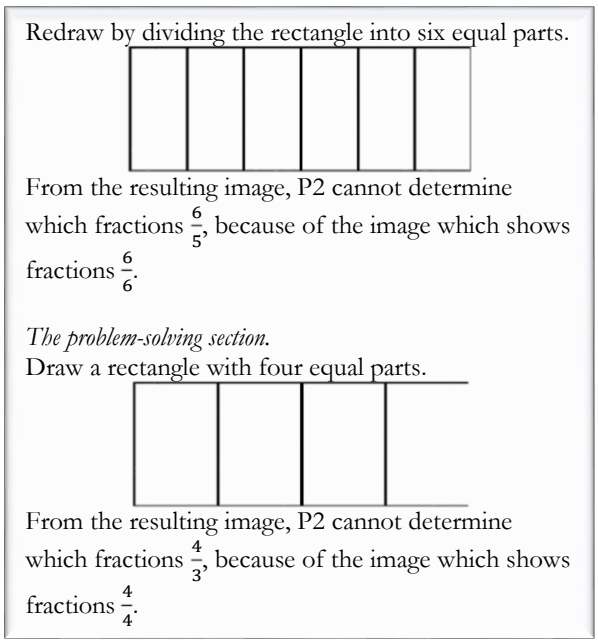

Figure 7.
Illustration of the Visualization of the P2's Problem-Solving Process

Based on the above data exposure, the subjects experienced *commognitive* conflict on two commognitive characteristics namely visual mediators and narrative while the characteristics of word uses and routines did not experience *commognitive* conflict.

Discussion and Conclusion

The results of both the participants' work and the interviews revealed problem-solving activities of improper fraction through *commognitive* analysis. "*Commognitive*" derived from two terms, "communication" and "cognition," and can serve as a reminder that communicating with others and thinking "in someone's head" belongs to one category. Sfard (2018) says that thought is considered the same as communicating with oneself and not necessarily with words. The analysis shows that there are two sources of *commognitive* conflict based on the results of commognitive analysis, namely: first, *commognitive* conflict occurs on the characteristics of visual mediators; secondly, *commognitive* conflict occurs with narrative characteristics. Both *commognitive* conflicts that occur are caused by differences in the textual and contextual subjects in solving problems. The following will discuss the *commognitive* conflict experienced by research subjects in solving problems.

Characteristics *Commognitive* that Do Experience *Commognitive* Conflict

Visual Mediators

Visual mediators, namely the discursive impulse used in interviews to communicate about operations and their relationships (Sfard, 2008), in this case, are limited to images showing fraction expressions (iconic mediators). The participants resolved the problem by redrawing the visual mediators given in the problem. Redrawing was undertaken using information previously known by the participants so that they could contextually solve it. However, when the visual mediators existing in the problem were redrawn in detail, this caused a shift in the meaning of the textual. "Textual" in this study refers to the work of students that have been produced in the form of writing and drawing to solve mathematical problems. This understanding refers to the textual meaning (Smith, 2017), which requires a researcher to concentrate on the text rather than the context. This shows that researchers must try to understand that what the participants wanted to convey through the text in this study were visual mediators. On the other hand, "contextual" (Suprapto, 2010) means using everything that is known to provide deeper meaning. In this research, the contextual aspects in question is linking information provided to mathematical problems with information already possessed so that the mathematical problem is solved in accordance with preexisting knowledge. The shift from textual to contextual meaning experienced by these participants caused *commognitive* conflict with regard to the visual characteristics of mediators in solving problems.

Early *commognitive* conflicts occur when students try to translate visual mediators into a given problem. The observed *commognitive* conflict is that the participants use the definition of a fraction that is worth one but does not use fractions that are worth more than one (improper fraction). The participants tried to translate the visual mediators present in the problem by dividing according to the numbers in the numerator. In this process, the participants do not think of using the definition of an improper fraction so that the process of translating visual mediators into the problem is confused. Textually, visual mediators that are generated are not suitable for showing the fraction given in the problem. *Commognitive* conflict in translating visual mediators in the problem given occurred after the participants divided according to the numbers in the numerator. The participants did not notice that the visual mediators that were redrawn showed the fraction that was in the problem. This shows that the textual visual mediators produced were not suitable to show the fraction given in the problem. Contextually, the steps taken by the participants to redraw were already believed to be correct. This condition shows the differences between the textual and contextual in translating visual mediators to the given problem. This happens because the participants only use the definition of a fraction for the value of one. The participant's condition shows that in translating the visual mediators that are in the problem, there is a gradual adjustment in his mind. Nardi, Ryve, Stadler, & Viirman (2014) say *commognitive* conflict is not always recognized by the interlocutor and is often resolved in an unseen way by gradual adjustment of the interlocutor by way of thinking. In this study, the interlocutor in question is the researcher himself with his mind. This is in accordance with Rabin, Fuller, & Harel (2013), who said that learning mathematics is verbal communication, which is a process of communication that occurs not only with others but also with yourself. Thoma & Nardi (2018) said that there is *intra-commognitive* conflict, which is a condition where differences occur between words or symbols used by the interlocutor (writing). In addition, Sfard (2008) said that communication can occur through written or spoken language which constitute the primary means for teaching purposes and changes in abilities in learning. The *commognitive* conflict that occurs when the participants redraw the visual mediators given in the problem affects the process of solving the next problem.

There was a change in decision to resolve the problem in this case, the repetition of the situation experienced by the participants when translating visual mediators that existed in the problem. This condition requires the participants to describe visual mediators using references to visual mediators that have been given in the problem. Participants

experience *commognitive* conflict which has not been resolved when translating visual mediators into a given problem. Thus, in the process of resolving this problem, participants still experience the same *commognitive* conflict with the translating condition, which is still a difference between the textual and contextual in solving problems. However, there is something unique that the participants resolve with a process other than translating the problem. This difference indicates the existence of *intra-commognitive* conflict that is trying to be resolved. The participants still approache the problem using the definition of a fraction that is worth one. Departing from this definition, the participants proceeds by dividing again one part of the whole. The participants trie to make adjustments in stages due to the repetition of the settlement process (Sfard, 2007; Nardi, et al., 2014; Thoma & Nardi, 2018). Adjustments made by the participants still produce errors so that the *commognitive* conflict experienced is still unresolved. This shows such errors are the impact of unresolved *commognitive* conflict (Thoma & Nardi, 2016). The *commognitive* conflict experienced by the participants continues because the previous *commognitive* conflict has not been resolved. The manifestation of unresolved *commognitive* conflict is thought to occur because there is involvement in one conversation (Sfard, 2007; Sfard, 2008; Nardi, Ryve, Stadler, & Viirman, 2014; Rabin, Fuller, & Harel, 2013; Thoma & Nardi, 2018).

Narrative

In formal mathematical discourse, narratives approved by the academic mathematics community are called mathematical theories. These theories consist of several discursive objects such as axioms, theorems, and definitions (Berger, 2013). *Commognitive* conflict on narrative characteristics was revealed when the interview process was carried out, namely a shift in the textual and contextual meaning experienced by the participants when trying to explain the strategies used to resolve the problem with the results obtained (in the form of visual mediators). The participants explained that the beginning of solving the problem of improper fraction by dividing the rectangle into equal parts according to the given denominator. Then, add a portion of the number with the given numerator. The passage of the improper fraction settlement activity, according to Hackenberg (2007), the improper fraction completion activity carried out by the participants has the potential to mark the initial construction of an iterative fractional scheme, that is, build the concept of improper fraction starting by making a part of the whole and then adding the part as necessary. It is also known that there is a double-check of the results of the problem solving conducted by the participants. Re-checking is done to ascertain whether the idea used in solving the problem is wrong or correct. In addition, the participants also want to think of appropriate reasons related to the shift in the textual and contextual meaning they experience. The participants also make a strategy change in problem solving after checking again when translating visual mediators given in the problem section. Changes in strategy result in new *commognitive* conflicts, and the participants knows that there are errors in the visual mediators that are built because after performing manual calculations it is known that visual mediators do not match the manual calculations ($\frac{4}{3}$ greater than $\frac{6}{5}$). This is also in accordance with Ioannou (2016), who said that *commognitive* conflict is caused by errors related to understanding the problem given and the use of mathematical statements used for problem solving.

Characteristics *Commognitive* that do not Experience *Commognitive* Conflict

Word Uses

The participants employed word use well. By redrawing the image given in the problem, they showed that word use performs certain functions in mathematical discourse. This is in accordance with Zayyadi, et al. (2019) using word uses with special terms in mathematics used by students in solving problems. The indication of their compliance with the settlement rules is shown in the images that were made to facilitate the problem-solving process. This shows that there were no *commognitive* conflicts occurring in the characteristics of word uses experienced by the participants.

Routines

Routine is a series of sequential steps to solve a problem and refers to each step in the solution process. Sfard (2008) states that truly exploratory thinking will be characterized by the ability of learners to use more than one way to find solutions to problems. The participants carry out the strategies used to solve the given problem. This shows that the participants use a type of exploration routine that is the routine whose purpose is to verify the narrative, for example, to solve problems. In this study, with regard to routines characteristics, the participants did not experience *commognitive* conflict because a strategy had already been chosen to solve the problem. In this condition, the participants of the study did not experience any shift in meaning to what they thought with the results obtained because they only completed what was in their mind based on the strategy that had been previously chosen. This illustrates that the participants had appropriate discursive actions (Sfard, 2008).

Recommendations

As this study focuses on revealing *commognitive* conflict, the results obtained have not yet reached the follow-up to stage to eliminate *commognitive* conflict experienced by students. This is an opportunity for further research for lecturers or researchers.

Limitations

The main limitation is the lack of data collection in the form of think aloud when the participants resolve the improper fraction problem. Data analysis would be better if the recording of think aloud as part of triangulation of data in this study.

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All authors in this study have the same contribution and following their expertise.

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