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Research Article

Fuzzy Hypothesis Test for Cognitive Radios

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Abstract
This article presents a statistical approach called hypothesis testing for Cognitive Radio systems. Hyposthesis tests have a wide range of applications from detection theory to radar systems. Hypothesis testing is frequently used in Cognitive Radio systems, which are the solution to the spectrum shortage problem today. Hypothesis testing in Cognitive Radio systems is the basis of spectrum detection. By means of hypothesis testing, spectrum gaps are determined so that spectrum gaps are opened to different users' access. In this study, a fuzzy hypothesis test based spectrum detection method is proposed with the signal detected by Cognitive Radio users. Theoretical bases and simulation results of the proposed spectrum detection model are also given. Simulation studies prove the computational cost advantage and detection performance success of the proposed detection method.

Keywords: Cognitive Radio, Hypothesis Test, Fuzzy Hypothesis Test, Spectrum Sensing

Bilişsel Radyolar için Fuzzy Hipotez Test

Öz

Bu makale Bilişsel Radyo sistemleri de hipotez testi için istatistiksel bir yaklaşım sunmaktadır. Hipotez testleri, algılama teorisinden radar sistemlerine kadar çok çeşitli uygulamalara sahiptir. Günümüzde spektrum kıtlığı sorununun çözümü olan Bilişsel Radyo sistemlerinde hipotez testleri sıklıkla kullanılmaktadır. Bilişsel radyo sistemlerinde hipotez testleri spektrum algılamanın temelini oluşturmaktadır. Hipotez testi ile, spektrum boşluklarının farklı kullanıcıların erişimine açılması için spektrum boşlukları belirlenir. Bu çalışmada, Bilişsel Radyo kullanıcıları tarafından tespit edilen işaret ile bulanık hipotez testine dayalı spektrum algılama yöntemi önerilmiştir. Önerilen spektrum algılama modelinin teorik temelleri ve benzetim sonuçları da verilmiştir. Benzetim çalışmaları önerilen algılama yönteminin hesaplama maliyeti avantajını ve algılama performansındaki başarısını kanıtlamaktadır.

Anahtar Kelimeler: Bilişsel Radyo, Hipotez Testi, Fuzzy Hipotez Test, Spektrum Algılama

1. Introduction

Today, it is known that there is an increasing demand for radio frequency spectrum. Two important factors lead to increased spectrum demand. The first is the increase of the data sizes transmitted in the wireless communication systems, the second is the increase of the different applications used in wireless communication systems(Dahlman et al., 2013). Today, applications that are considered to be used for wireless

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communication systems cannot be used due to the lack of suitable / sufficient space in the spectrum(Bandari et al., 2018). This problem is called spectrum shortage in the literature. Despite the problem of spectrum shortage, measurements also reveal that the existing radio frequency spectrum was not used for much of the time(Chen & Zhang, 2018). The spectrum regions that are assigned to licensed users are not accessible to other users even if the related user is not active. Thus, inactive spectrum regions emerge in the spectrum(Abdalrazik et al., 2016). In accordance with fixed spectrum assignment policies, idle spaces cannot be assigned to another user. The purpose of Cognitive Radio (CR) systems is to determine the idle spectrum gaps and make user changes in these regions. Thus, the use of the spectrum is increased. Actually, the user change is to transfer the remaining spectrum from the licensed users to the unlicensed users. In the communication literature, the licensed user means the legal owner of a certain spectrum region. The unlicensed or cognitive user means the radio user who uses the empty spectrum regions as an opportunistic. Therefore, it is very important for CR users to find empty spectrum regions and this process is defined as spectrum detection(Ying-Chang Liang et al., 2008).

The determination of empty areas in the spectrum is defined as spectrum sensing. There are different methods in the literature for spectrum detection(Yonghong Z. et al., 2008). The most commonly used method is Energy Detection (ED) due to the advantage of computing cost(Shi-Qi et al., 2012). ED based methods is the most successful method in terms of detection performance if the noise variance existing in the environment is known exactly. In addition to this method, different methods such as covariance-based(Bao et al., 2012; Zeng & Liang, 2009), eigenvalue-based(Bao et al., 2012) and wavelet-based(Dibal et al., 2018) sensing are also used. The choice of detection method to be used should be determined by noise variance, bandwidth of the spectrum, and the number of cognitive users(Bazerque & Giannakis, 2010). The purpose of all these studies is to perform the most successful detection process in the shortest time, even at high noise levels(Çiflikli & Ilgin, 2018).

In this study, a fuzzy based sensing method is proposed for spectrum detection. For the proposed method, detailed statistical analyzes were made and the test statistic and threshold value were calculated theoretically. In addition, theoretical studies have been verified by simulation studies and compared with ED based detection method to evaluate the performance of the proposed method.

In this study, uppercase(X) and lowercase (x) letters represent matrices and vectors, respectively.

2. Fuzzy Hypothesis Test

Basically the signal detection problem is to determine whether there is a embedded communication signal in the noise. In detection theory, this decision mechanism is explained by binary hypothesis testing. In binary hypothesis test, H_0 indicates that there is only a noise signal, H_1 indicates that it is a noise + communication signals. Mathematical decision making process is given below(Akyildiz et al., 2011).

$$H_0 \ : y(n) = \eta(n) \qquad (1)$$

$$H_1 \ : y(n) = s(n) + \eta(n) \qquad (2)$$

Where $s(n)$ and $\eta(n)$ are zero-mean Gaussian noise signal and the signal to be received by the CR user respectively. $y(n)$ is the observation signal at the CR user. Fuzzy Hypothesis Test (FHT) decides between these two hypotheses based on the average of the observation signal(Parchami et al., 2016). Suppose the mean of the Probability Distribution Function (PDF) of the observation signal is Θ. The measurements show that the received signal has a normal distribution under the H_1 hypothesis. In the FHT,we use the hypotheses expressions as $H_0: \Theta = \Theta_0$ versus $H_1: \Theta \neq \Theta_0$. Since there is also noise noise factor in wireless communication environments, this expression can be expressed mathematically as follows(Parchami et al., 2016).

$$\begin{cases} H_0: & \Theta \text{ is close to } \Theta_0 \\ H_1: & \Theta \text{ is away from } \Theta_0 \end{cases} \quad (3)$$

Let $Y = (Y_1, \ldots, Y_N)$ be a random sample with the observed value $y = (y_1, \ldots, y_N)$, where y_i has the PDF namely, $f(x_i, \Theta)$ and it will be assumed that the PDF of $f(x, \Theta)$ is known. The most commonly used statistical tests in FHT are given below.

$$H_0: \Theta = \Theta_0 \, versus \;\; H_1: \Theta = \Theta_1 \quad (4)$$

$$H_0: \Theta = \Theta_0 \, versus \;\; H_1: \Theta \neq \Theta_0 \quad (5)$$

$$H_0: \Theta > \Theta_0 \, versus \;\; H_1: \Theta \leq \Theta_0 \quad (6)$$

To summarize, FHT decides between the two hypotheses based on the average of the received signal.

2.1 Spectrum Sensing with FHT

The proposed spectrum sensing model is given in Fig. 1. Where Primary User (PU) and Cognitive User (CU) defines the licensed user and unlicensed user, respectively(Kortun et al., 2014). The task of CR users is to determine whether the primary PU is active / passive by performing a FHT. When the PU transmitter is inactive, this spectrum region will be used by CR users.

Where M defines the number of CU's in the spectrum sensing model, and h_1, h_2 and h_3 define the channel coefficient vector. Thus, the signal detected at m.th CR user is defined as follows.

$$y_m(n) = \psi h_m s(n) + \eta(n) \quad (7)$$

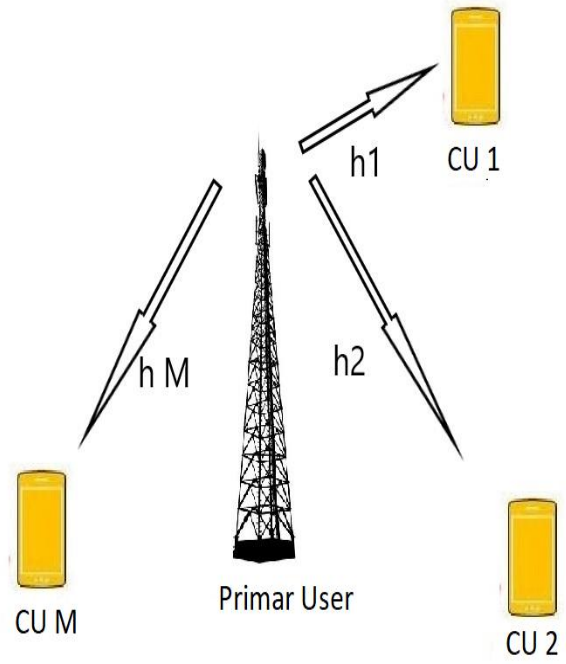

Figure 1. Proposed Detection Scenario

Where $\psi = 0$ under H_0, and $\psi = 1$ under H_1; h_m denotes the Independent and Identically Distributed (IID) circularly symmetric complex Gaussian (CSCG) white noise with mean zero and covariance $\sigma_\eta^2 I_M$. In addition Where $m = 1,2, \ldots .. M$ and $n = 1,2, \ldots .. N$ are number of CU's and received sample size respectively. FHT based proposed detection model is given below.

$$E = \sum_{n=1}^{N} y_m(n) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{FHT} \qquad H_0 \; or \; H_1 \quad (8)$$

Where E represents the signal received by the *m*. th CR user. The PDF of the signal received under H_0 has normal distribution but differs under the H_0 or H_1 hypotheses. Fig. 2 shows these differences.

In Fig.2, It is understood from the PDF of the received signal that there is only noise in the environment under the H_0 hypothesis. Because the noise signal is zero mean complex Gaussian noise.

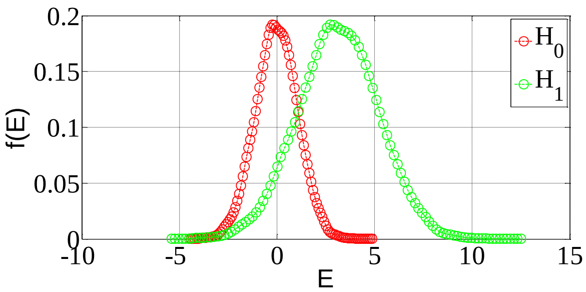

Figure 2. Received signal PDFs under H_0 and H_1 hypotheses

Our aim is to make a spectrum decision using the mean differences in H_0 and H_1 hypotheses. Means of PDF's are known under both hypotheses. Then our aim is to apply hypothesis testing according to the average value of PDFs. Let $x = 2.25$ be an observation from $N(\mu, 1)$ distribution.We wish to test;

$$\begin{cases} \widetilde{H}_0: & \mu \; smaller \; than \quad 1.5 \\ \widetilde{H}_1: & \mu \; bigger \; than \quad 1.5 \end{cases} \qquad (9)$$

where $\widetilde{H}_0$ and $\widetilde{H}_1$ have membership functions, namely;

$$H_{0b}^{*}(\mu) \begin{cases} 1 & if & \mu < 0 \\ \frac{3-\mu}{3} & if & 0 < \mu < 3 \\ 0 & if & \mu > 3 \end{cases} \qquad (10)$$

and $H_0(\mu) = 1 - H_0(\mu)$ (see Fig. 2). Considering Equ. 3, the membership function of the boundary of fuzzy null hypothesis is Equ. (10). Where the intersection point of FHTs is also the intersection point of PDFs under H_1 and H_0 hypotheses. Also, 2.25 is a randomly selected value. In reality, this value is the value determined by the International Communication Committee. In simulation studies, calculations are made for different values.

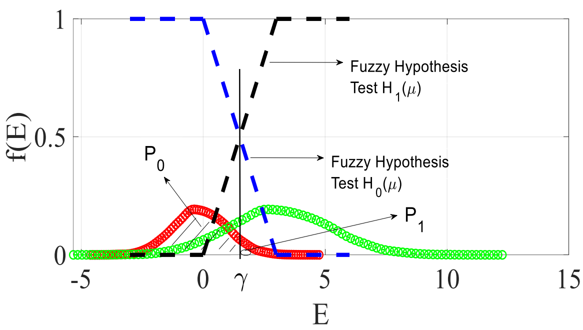

Figure 3. Fuzzy hypothesis test for proposed method

Where P_1 and P_0 are defined as the Probability of False Alarm (P_{fa}) and Probability of Detection (P_d) in classical detection theory and is expressed mathematically as follows. It should be noted that P_1 and P_0 are the area of the shaded areas. These shaded areas are on the right and left of gamma

$$P_d = P_1 = P(E \leq \gamma | H_0) \qquad (11)$$

$$P_{fa} = P_0 = P(E > \gamma | H_0) \qquad (12)$$

Unlike conventional hypothesis testing, in FHT the P_1 is calculated as follows(Torabi & Behboodian, 2007).

$$P_1 = \int H_{0b}^{*}(\theta) P_\theta \, (E \leq \gamma) d\theta \qquad (13)$$

and

$$P_0 = \int H_{0b}^{*}(\theta) P_\theta \, (E \geq \gamma) d\theta \qquad (14)$$

Where $H_{0b}^{*}(\theta) = H_{0b}(\theta)/\int H_{0b}(\theta)d(\theta)$ is the normalized membership function of the boundary in the fuzzy null hypothesis, E is the observed value of test statistic. Then

$$P_o = \int H_{0b}^{*}(\theta)P_\theta\,(\gamma \geq 2.25)d\theta \quad (15)$$

from Equ. 9;

$$P_0 = \int_0^3 \frac{(3-\mu)/3}{1.5}[1-\Gamma(2.25-\mu)]d\mu \quad (16)$$

then;

$$P_0 = 0.422 \quad (17)$$

P_1 is calculated as follows.

$$P_1 = \int H_{1b}^{*}(\theta)P_\theta\,(\gamma \leq 2.25)d\theta \quad (18)$$

$$P_1 = \int_0^3 \frac{1-[\frac{3-\mu}{3}]}{1.5}\Gamma(2.25-\mu)d\mu \quad (19)$$

$$P_1 = 0.846 \quad (20)$$

The threshold (γ) used to decide between the H_1 and H_0 hypotheses is expressed as follows.

$$\gamma = \frac{P_1}{P_1+P_0} = \frac{0.843}{0.843+0.42} = 0.336 \quad (21)$$

3. Simulation Studies

In this section, we evaluate the performance of the proposed algorithm using the detection probability versus SNR curves by Monte Carlo simulations. Channel coefficients and PU signal were generated randomly for the detection model given in Fig. 1 by Monte Carlo simulation. In simulations, we assume the number of samples(N) is equal to 200.

It can be seen that the change of gamma changes the sensing performance considerably. The γ is the threshold value corresponding to a P_{fa} determined by FCC and has different standard values for CR systems. Therefore, there may be different threshold values for different P_{fa}. For example, when gamma = 5, the most successful detection occurred.

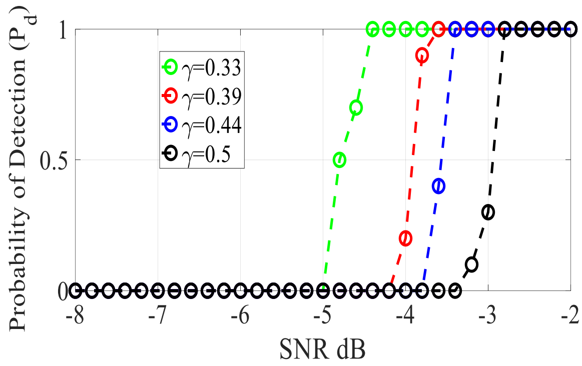

Figure 4. P_d versus SNR for FHT

Threshold(γ) corresponding to different P_1 are given in Table 1. These values are calculated by Equ. given by Equ. 15 and Equ. 18.

Table 1. P_{fa} versus γ

P_{fa}	P_1	P_0	γ
2.25	0.846	0.422	0.336
2.0	0.791	0.503	0.391
1.75	0.731	0.588	0.445
1.50	0.66	0.66	0.5

In cognitive radio systems, spectrum detection should be done as soon as possible. Because as soon as the spectrum is empty, the cognitive user should enter it immediately, and when it is full, he should empty it immediately. otherwise, access by users to the spectrum may be restricted. Fig. 5 shows that increasing the number of samples also increases the detection performance. However, it should be remembered that the increase in the number of samples increases the detection time. Fig. 5 gives the detection performance for the ED based detection method. ED based detection method is very advantageous in terms of calculation cost. However, in this method, the noise variance must be known for successful detection. If the noise variance is unknown, it must be found using estimation methods.

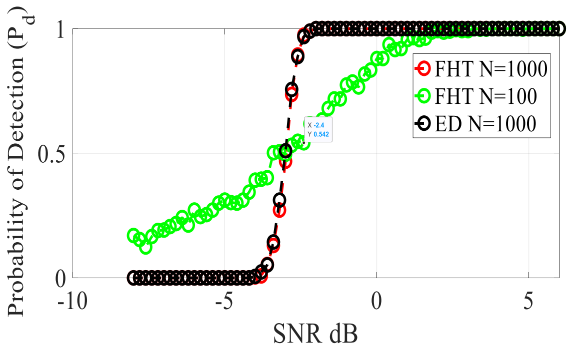

Figure 5. SNR versus P_d for ED and FHT

Detection times for different sample size are given in Table 2. These times are for Monte Carlo simulation (1000 times).

Table 2. Detection time for different *N*

Sample size (N)	100	1000	5000
Detection time	1.04 sec	3.2 sec	9.56 sec

4. Conclusion

In this study, a fuzzy-based detection model is proposed for spectrum sensing in Cognitive Radio systems. Theoretical analysis of test statistics and threshold values were performed for the proposed detection method. Theoretical findings have been proven by simulation studies. It was seen that the proposed detection method showed similar results in terms of detection performance with ED based method.

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