AJIT-e: Online Academic Journal of Information Technology
2015 Winter/Kış – Cilt/Vol: 6 - Sayı/Num: 18
DOI: 10.5824/1309-1581.2015.1.001.x
http://www.ajit-e.org/?menu=pages&p=details_of_article&id=136

Fractal Analysis of Stock Exchange Indices in Turkey

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ABSTRACT *The purpose of this study is to investigate possible fractal behavior in Istanbul Stock Exchange (BIST) indices. In particular evidence of chaotic and fractal behavior will be presented. To be able to analyze monofractality of given indices we are going to use Higuchi and Katz methods. In addition to this, we analyze the chaotic behavior of the investigated indices using Rescaled Range Analysis(R/S) and Detrended Fluctuation Analysis (DFA).*

Keywords: *Fractal Geometry, Higuchi, Katz, Detrended Fluctuation Analysis, Rescaled Range Analysis.*

ÖZET *Bu çalışmanın amacı İstanbul Menkul Kıymetler Borsası endekslerindeki muhtemel fractal davranışları araştırmaktır.Özellikle, kaotik ve fraktal davranışların kanıtları verilecektir.Verilen endekslerin monofraktal davranışlarını analiz etmek için Higuchi ve Katz metodlarını kullanacağız. Buna ek olarak,araştırılan endekslerin kaotik davranışlarını incelemek amacıyla Dönüştürülmüş Genişlik (R/S) ve Arındırlmış Dalgalanma (DFA) Analiz'leri kullanılmıştır.*

Anahtar Kelimeler: *Fraktal Geometri, Higuchi, Katz, Arındırılmış Dalgalanma Analizi, Dönüştürülmüş Genişlik Analizi.*

Introduction

Humanity has been looking for finding symmetry and smoothness in nature throughout its existence. In general, scientists search for patterns and call the events which do not conform to their conceptual framework as anomalies since they do not match their conceptual framework based on symmetry. However, most of the entities in our physical world do not obey Euclidean geometry at all.

Through the application of Euclidean geometry to a drawing, we can only create an approximation of a tree. In the real world, trees consist of a network of branches which are very similar to the overall shape of a tree but each branch is different. (Edgar E.Peters, 1994). Furthermore, there are other branches on branches on smaller scales (successive generation of branches). At individual branch level, each branch has a different size but share certain common properties. This "self-similar" property is one features of Fractal geometry. That means actual structure of tree includes both local randomness and deterministic point of view.

Another example could be fluid heated from below. Near the source, the fluid is heated by the way of convection, then the fluid is going to reach an equilibrium state in which maximum entropy occurs. During a heating period, all fluid molecules move independently. When the temperature passes a critical level, molecules which move independently start to behave coherently, that means heat flows by means of convection. In that case, the convection result is known by scientists but direction of roles of molecules is unknown. That means local randomness and global determinism coexist together.

Stock markets are also driven both by microeconomic considerations such as profit levels of firms and macroeconomic considerations such as employment and manufacturing data. The former can be compared to the individual molecules, the latter to the mass action of the molecules.

In that sense, the science of chaos theory and fractals are the places that chance (randomness) and necessity (determinism) seem together. Economic systems also exhibits complicated dynamic(chaotic) evidences by large amplitude and periodic fluctuation in economic indices, for instance, stock market prices, currency prices ,GDP(gross domestic product) (Edgar E.Peters,1994).Classical approach to economic anabolisms is the Newtonian one which economic fluctuations evaluated as linear perturbations near the equilibrium. However, large fluctuations in economic indicators shows that economic systems are driven from the equilibrium points such that nonlinearity takes roles and gives the clues of chaotic complex systems.

In economics stock markets show dynamic structures that can be examined through use of chaos theory and fractal analysis. The stock market consists of investors from different investment horizons. A stable market is one where all investors can make trade with each other, each confronted with the same risk level as others, adjusted for their investment horizon. However, forecasting using linear approaches to stock market values do not give reasonable explanations.

Theory

Monofractal Methods

Rescaled Range Analysis

Rescaled range analysis (R/S) was developed by Harold E. Hurst when he was working on the Nile River Dam Project in Egypt .After his work the technique was applied to financial time series by Mandelbrot and van Ness.

R/S analysis is a simple process which is highly data-intensive. To be able to understand R/S analysis, it is reasonable to follow the given sequential steps one by one below:

- Start with a time series of length of N. Then convert this time series to a time series with length **N'=(N-1)** with the following logarithmic ratios:

$$N_i' = \log\left(\frac{N_{i+1}}{N_i}\right)$$

- Then divide this generated time series into M number of adjacent sub-periods of length m and they satisfy the following condition M*m=N. Then give a name to each sub-period I_a where a=1, 2, 3... M. Each element in sub-period I_a can be named as $N_{k,a}$ where k=1,2,....m. For every I_a of length m the average value e_a is defined as:

$$e_a = \frac{1}{m}\sum_{i=1}^{m} N'_{k,a}.$$

- The time series of accumulated departures X from the mean value e_a for each I_a is

$$X_{k,a} = \sum_{i=1}^{k}(N'_{i,a} - e_a).$$

- The range is defined for each sub-period I_a as:

$$R_{I_a} = \max(X_{k,a}) - \min(X_{k,a})$$
$$k = 1,2,3,\dots,m.$$

- The standard deviation for each sub-period I_a can be calculated by:

$$S_{I_a} = \frac{1}{m}\sqrt{(N'_{k,a} - e_a)^{\,2}}\ .$$

- Each range can be normalized by dividing it by S_{I_a}. Then the rescaled range for each is equal to R_{I_a}/S_{I_a} .we had adjacent M sub-periods of length m. Then, the average R/S value of length m is:

$$(R/S)_m = \frac{1}{M}\sum_{a=1}^{M}(R_{I_a}/S_{I_a}).$$

- The length m is increased to the next higher value such that (N-1)/m is an integer value. We use values of m that includes start and final points of the time series and steps given above repeated until m=(N-1)/2.Then we can apply least square regression on log(m) vs. $\log\left(\frac{R}{S}\right)_m$ as:

$$\log\left(\frac{R_m}{S_m}\right) = \log c + H * \log m.$$

Note that R/S is the ratio of two different measures of dispersion, range and standard deviation. By these steps we calculate H, the Hurst Exponent. The Hurst exponent has a very close relationship to the fractal dimension. The following linear relation holds:

$$D = 2 - H$$

where D is fractal dimension. Using the Hurst exponent we can classify time series into types and gain some insight into their dynamics.

A value H in the range 0.5–1.0 indicates a time series with long-term positive autocorrelation, meaning both that a high value in the series will probably be followed by another high value and that the values a long time into the future will also tend to be high. A value in the range 0<H<0.5 indicates a time series with long-term switching between high and low values in adjacent pairs, meaning that a single high value will probably be followed by a low value and that the value after that will tend to be high, with this tendency to switch between high and low values lasting a long time into the future. A value of H=0.5 can indicate a completely uncorrelated series, but in fact it is the value applicable to series for which the autocorrelations at small time lags can be positive or negative but where the absolute values of the autocorrelations quickly decay exponentially to zero.

Detrended Fluctuation Analysis (DFA)

The detrended fluctuation analysis (DFA) algorithm is a scaling analysis method used to estimate long-range temporal correlations of power-law form (Peng et al., 1995) (Hardstone et al., 2012). Its advantage is the fact that extreme values are less likely to affect the result. DFA can be applied by the following four steps**:**

- Firstly, we need to determine the "profile" of the time series X_i of length N(where i=1..N) ($Y(i)$):

$$Y(i) = \sum_{i=1}^{k} (X_i - \bar{X})$$

where $\bar{X}$ is mean of the time series.

- In the second, step profile $Y(i)$ is divided into non-overlapping segments of length l where number of segments is integer $N_l = int(\frac{N}{l})$. At the end of this procedure the short part of the time series would remain. To overcome this problem, the second step can be repeated from the end of the time series. That's why,$2N_l$ segments are generated.
- In the third step for each segments local trend is calculated using least-square fitting.

$$F^2(t) = \frac{1}{l}\sum\nolimits_{i=1}^{l} \left[Y\big((\nu - 1)t + i\big) - p_\nu(i)\right]^2$$

For each segment ν, $\nu = 1,2, \dots, N_l$ and $p_\nu(i)$ is a fitting polynomials for each segment.

- The final step is finding the average over all segments and taking the square root to get the fluctuation function $F(l)$:

$$F(l) = \sqrt{\frac{1}{2N_l}\sum_{\nu=1}^{2N_l} F_t^2(\nu)}$$

If the data are long-range power-law correlated, $F(l)$ increases, for large values of l, as a power-law:

$$F(l) \sim l^{\alpha}$$

where α, the fluctuation exponent can be obtained by finding the slope of the logarithmic graph of $F(l)$ vs. l .

Power (Fourier) Spectral Analysis

The purpose of spectral analysis is to study the properties of an economic variable over the frequency spectrum, i.e. in the frequency-domain. In particular, the estimation of the population spectrum or the so-called power spectrum (also known as the energy-density spectrum) aims at describing how the variance of the variable under investigation can be split into a variety of frequency components. (Masset,2008). A deterministic signal has few Fourier components, signals coming from a non-deterministic process has many frequencies.

In Fourier analysis, the given time series (or signal) is demonstrated as a family of sinusoidal functions. In Fourier transform, the time series X(t) converted to "frequency-domain" representation X(t).The set of values X(t) for each frequency f is called as spectrum of (t) .(Ricardo Gutierrez-Osuna).

The Fourier Spectrum can be calculated mathematically as follows:

$$X(f) = \int_{-\infty}^{\infty} X(t)e^{2\pi i f t}\, dt$$

Katz Method for analyzing fractal behavior

Katz's method calculates the fractal dimension of a time series as follows:

- The sum of (Euclidean) distances between the successive points of the time series are calculated as:

$$d = \max\big(distance(1, i)\big).$$

- The fractal dimension of the time series is given as:

$$D = \frac{\log_{10} L}{\log_{10} d}.$$

The fractal dimension compares the actual number of units that compose a curve with the minimum number of units required to reproduce a pattern of the same spatial extent. Fractal dimensions computed in this fashion depend upon the measurement units used. If the units are different, then so are these dimensions.

DOI: 10.5824/1309-1581.2015.1.001.x

Katz's approach solves this problem by creating a general unit or yardstick: the average step or average distance $\underline{a}$ between successive points.

$$D = \frac{\log_{10} \frac{L}{a}}{\log_{10} \frac{d}{a}}$$

If we define n as the number of steps in the curve then $n = L/a$ and the fractal dimension D is:

$$D = \frac{\log_{10} n}{\log_{10} n + \log_{10} d/L}.$$

Higuchi Method

This is a slightly different method for determining fractal dimension. We have the time series X(i) with a length N where i =1... N and the data is taken at regular intervals.

We create the new time series from the given time seriesX(i):

$$\mathrm{X(m), X(m+k), X(m+2k), \dots, X(m + \left[\frac{N-m}{k}\right].k)}$$

In this representation m shows the initial time and k indicates the time interval and [ ] represents greatest integer function. By this way, we will have constructed k sets of time series. We can calculate the length of the curve of the constructed time series:

$$\mathrm{L_m(k) = \{(\sum_{i=1}^{\frac{N-m}{k}} |X(m+ik) - X(m+(i-1)k)|) \frac{N-1}{\left[\frac{N-m}{k}\right].k}\}/k.}$$

The average length of the curve $< \mathrm{L(k)} >$ is defined as:

$$< \mathrm{L(k)} > = \frac{1}{\mathrm{k}} \sum_{\mathrm{m=1}}^{\mathrm{k}} \mathrm{L_m(k)}$$

If the curve has a fractal behavior:

$$< \mathrm{L(k)} > \sim \mathrm{k^{-D}}$$

We can get the fractal dimension D from the slope of the best fitted line corresponding to the plot of log $(< L(k) >)$ against log (k).

Observational Results and Analysis for Stock Market Indices

We use the following data sets for our experimental evaluations: BIST 100 index, BIST 50 Index, BIST 30 Index, Dow-Jones 30 Industrial Index. All the data sets are between the dates 2005-2015 each of which consist of 2516 data points.

In Table 1, the result of fractal dimension calculations according to Katz's and Higuchi's methods are presented. In addition to this, corresponding Hurst exponents are given. When we compared to this result for Indian stock exchange results presented in (Sammader et al.,2013), we obtain very close results for both Turkish Stock Market Indexes and the Dow-Jones Index. Since the fractal dimensions are between1<D<2, it can be said that self-similar property of fractal geometry is observed. It is one of the supporting results that Efficient Market Hypothesis does not represent the realistic view of financial time series. When the Hurst exponents calculated by (R/S) method are taken into account all the Hurst exponents are bigger than 0.5.That means this time series are persistent or trend reinforcing series rather than a series where information from the previous step dominates over information from parallel processes, however, all processors scale in similar ways. In other words, long memory structures exist for this time series. Since this time series are persistent, they presents fractional Brownian motion, or biased random walk. However, since the Hurst exponents are not much bigger than 0.5 it can be said that there will be a noise in the given the series due to possible seasonal fluctuations (economic, social or political crisis).

Data Set	Higuchi Method	Katz Method	Hurst Exponent
BIST 100	1.4647	1.6886	0.5951
BIST 50	1.4694	1.7075	0.6368
BIST 30	1.4694	1.7075	0.6368
Dow Jones 30 Industrial	1.5064	1.7586	0.6386

Table 1: Fractal dimensions and Hurst Exponents

In Figure 1, the graphical representation of the (R/S) analysis is given. In this graph, it can be seen that there is breakdown after the first 1200 days of observation. That means there would be two different time scales. However, the slope of Turkish Stock Markets increasing after the breakdown, while the slope of Dow-Jones Index decreasing which is very similar to the result presented in (Cakar, Aybar, Hacinliyan, Kusbeyzi, 2010).

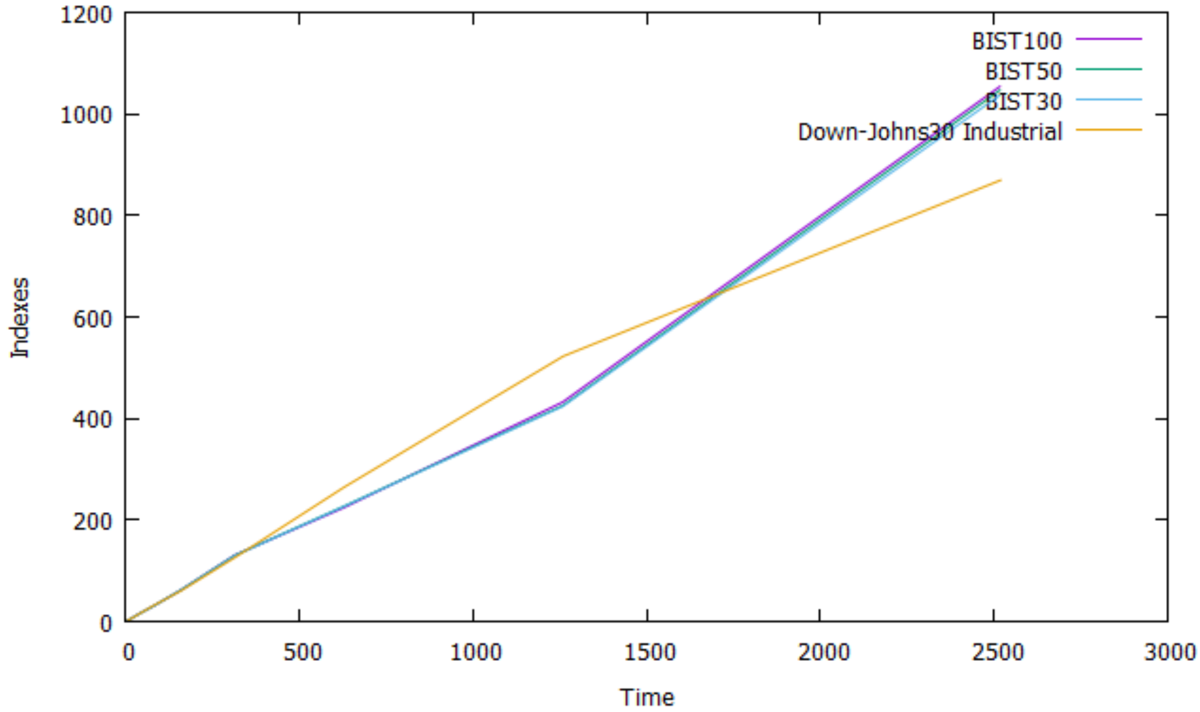

Figure 1: Rescaled Range Analysis

The Detrended Fluctuation Analysis results of given indices, Figure 2, shows identical behavior in terms of fluctuation. However, in this analysis two different time scales observed in (R/S) cannot be seen. It is probable that a short term nonstationarity present in the original data has been smoothed out because of the detrending. Therefore, DFA is not a suitable tool to understand the existence of multiple time scales or regimes in this sense, if these trends are due to possible nonstationarity.

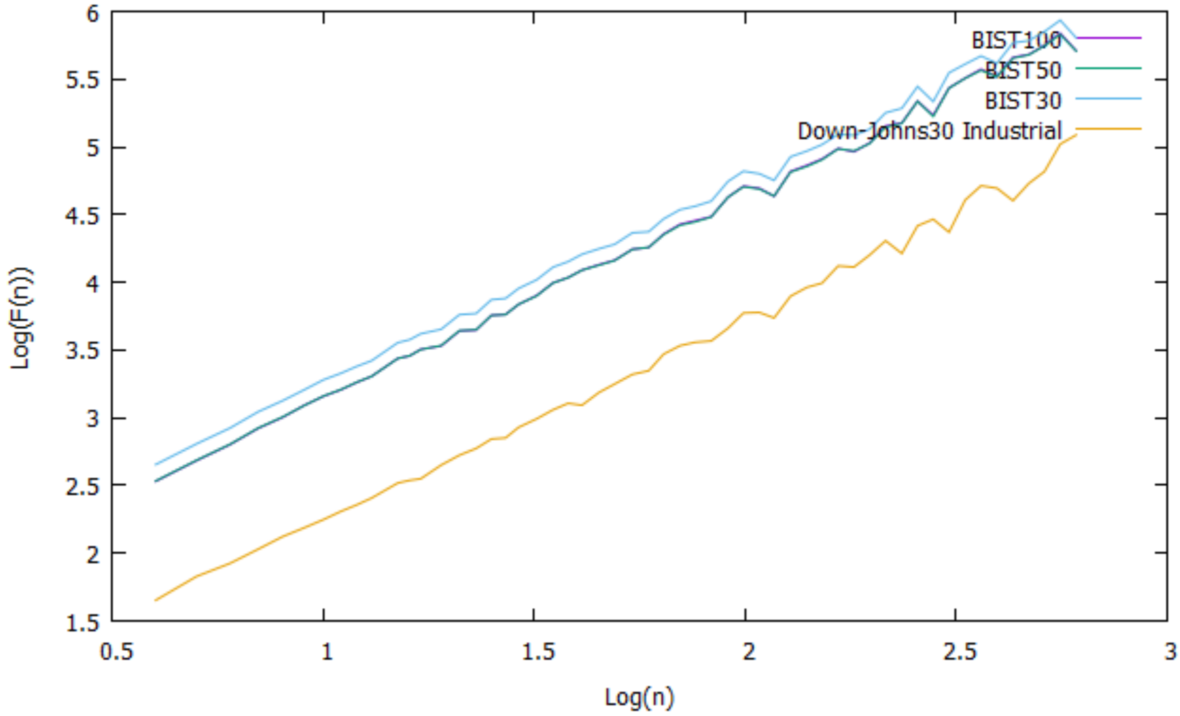

Figure 2 : Detrended Fluctuation Analysis

In Figure 3, power spectrum of BIST 100 data set versus frequency is plotted in logarithmic scale. The best fit line is $1/f^{1.678}$.(this exponent estimated from f=5 to f=267). This relation which is close to $1/f^2$ implies the Brownian motion as indicated by Hurst analysis.

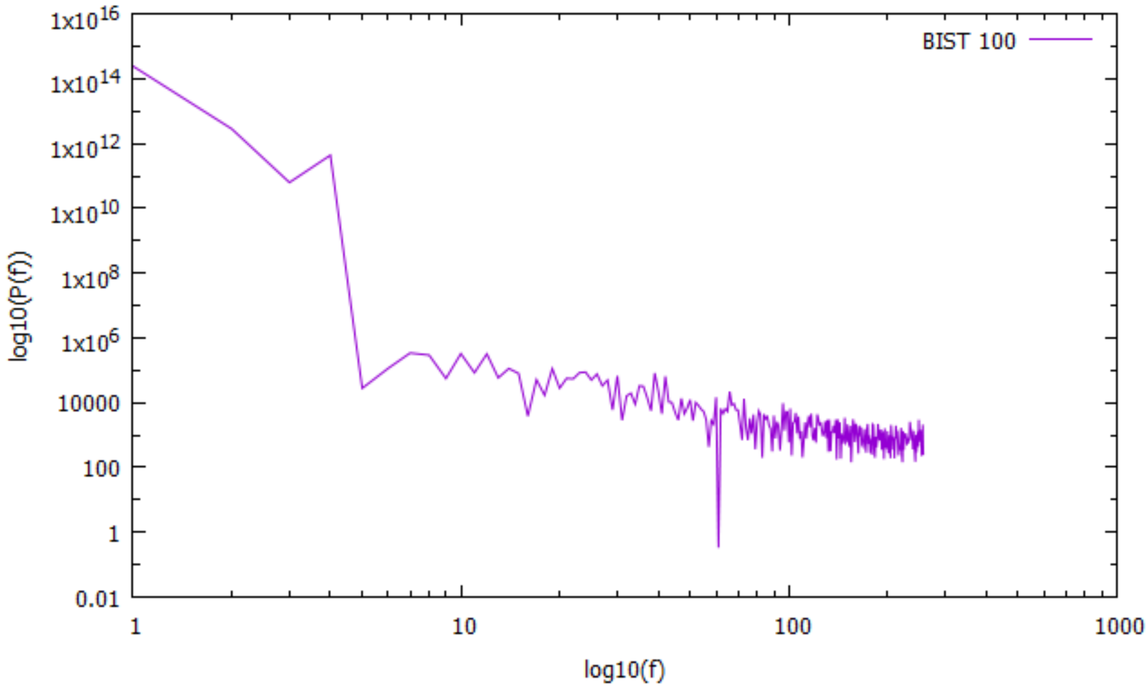

Figure 3: Power Spectrum Analysis of BIST 100

DOI: 10.5824/1309-1581.2015.1.001.x

The behavior of the mutual information analysis (Figure 4) shows that when all sets of indexes are very close to each other and they have almost the same delay time of 5 days (a week).

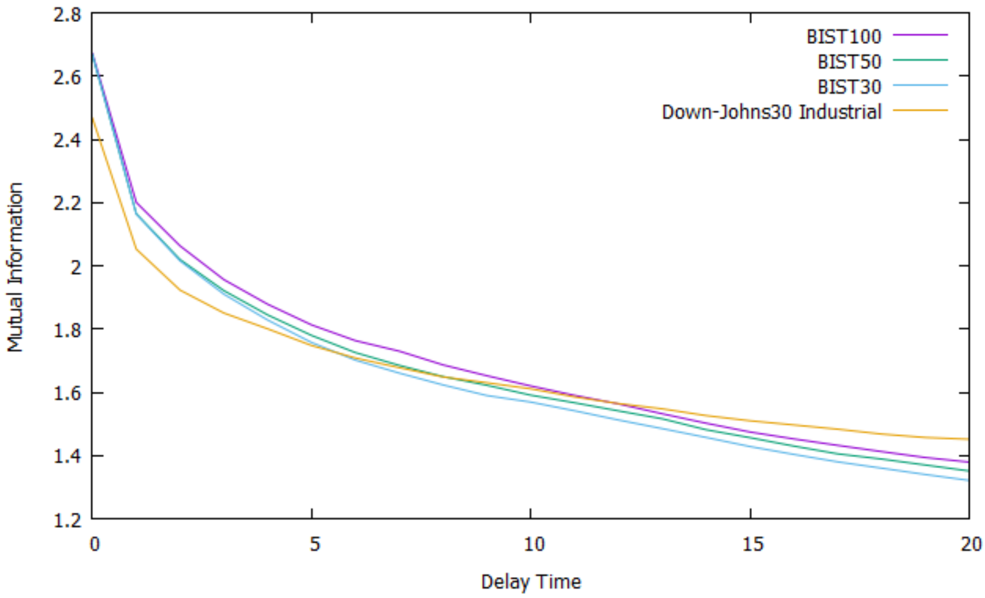

Figure 4: Mutual Information

After determining delay times, embedding dimensions can be determined. To get a meaningful value for the embedding dimension, false nearest neighbors' method offer a good estimate. After finding delay time for all data sets, the fraction of false nearest neighbors are calculated. In Figure 5, the fraction of false nearest neighbors versus embedding dimension are plotted. All regions embedding dimension graphs' are stabilizing at more than 4 dimensions, implying that at least a two dimensional model is needed.

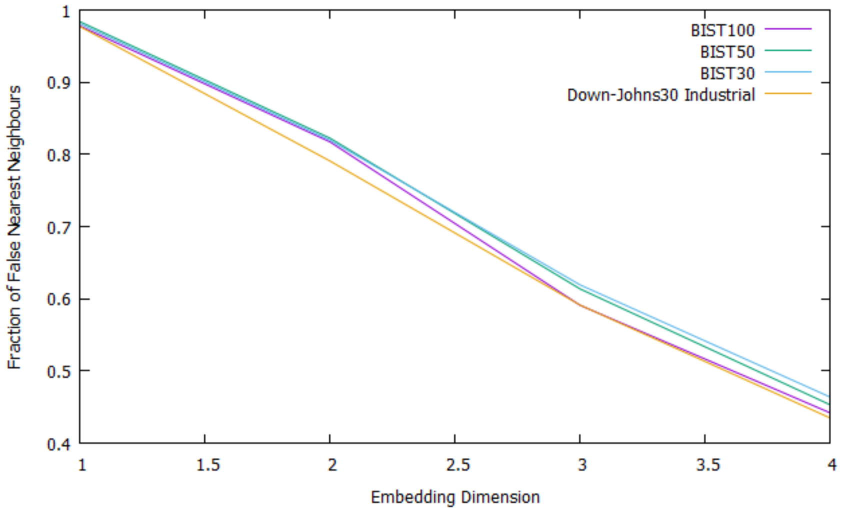

Figure 5: False Nearest Neighborhood

The Lyapunov exponents are invariants of the dynamics. All the slopes in Figure 6 are calculated. For BIST 100, Lyapunov exponent is 0.291683, for BIST 50 is 0.292167, for BIST 30 is 0.295938 and for Dow-Jones 30 Industrial is 0.26603. As a conclusion a positive Lyapunov exponent is indicated from studied indices.Since, all Lyapunov exponents are positive , they are not stable fixed points . Consequently, they do not indicate random noise. However, they are positive and this shows that this time series is chaotic .

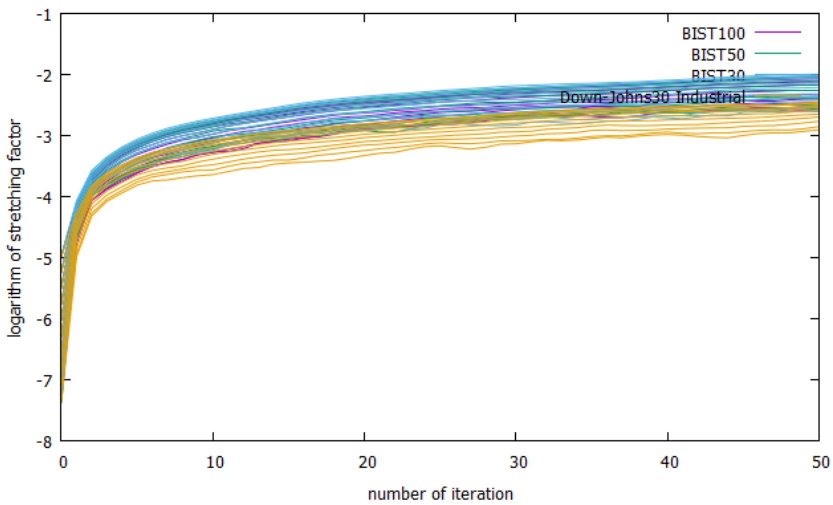

Figure 6: Maximum Lyapunov Exponent

Conclusion

All of the stock market data show Fractal Brownian motion trends, meaning that there is a close correlation between each successive step accompanied with positive indicators of chaos. The time period of approximately one week is indicated by three different observations, namely the stabilization of the false nearest neighbors at approximately five periods, Lyapunov exponents around 0.25 indicating a prediction horizon of 3-4 days and disappearance of the two components indicated by Hurst analysis upon detrending. The positive indicators of chaotic behavior are compatible for the findings concerning parallel research in the Indian, Tel Aviv stock markets, dollar and Euro prices and gold prices. ((Sammader et al., 2013), (A. S. Hacinliyan et al., 2010), (A. S. Hacinliyan et al., 2013), (Alan, İ. Kuşbeyzi Aybar and O.O Aybar, Hacınlıyan,2013).

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