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ÖZGÜN ARAŞTIRMA

Comparing The Predictive Performances of Value at Risk Estimation Methods-Extreme Value Perspective

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Abstract
Various methodologies are developed to supervise and manage financial risks due to the risk management in the derivative market become highly important in the recent years in response to financial crisis. The Value at Risk (VaR) summarizes the worst loss over a target horizon with a given level of confidence. In 2008, extreme price fluctuations in the global financial disaster show that inefficiency of GARCH models whose main assumption is normality. Extreme value theory is a powerful and fairly robust framework that investigates the tail behavior of the distributions. The main objection of this paper is to compare performances of VaR estimations which are obtained by GARCH models and Extreme Value Theory (Generalized Pareto Distribution and Generalized Extreme Distribution) in process of 2008 global financial crisis by using secondly ISE30 index in between 02 January 2009 and 02 April 2012.

Key Words: Extreme Value Theory, Value-at-Risk, GARCH, Generalized Pareto Distribution, Generalized Extreme Distribution

Jel Codes: C22, C52, C53

Riske Maruz Değer Tahmin Yöntemlerinde Fiyat Tahmini Performanslarının Uç Değer Yöntemi ile Karşılaştırılması

Özet
Son yıllarda yaşanan finansal krizlere karşılık türev piyasalardaki risk yönetimi önem kazanmakla birlikte finansal riski denetlemek ve yönetmek için çeşitli yöntemler geliştirilmektedir. Riske maruz değer (RMD) verilen güven aralığında belirlenen zaman diliminde karşılaşılabilecek en büyük kaybı özetlemektedir. 2008 yılında, küresel finansal felakette görülen yüksek fiyat dalgalanmaları, temel varsayımı normal dağılım olan GARCH modellerinin yetersizliğini göstermiştir. Uç değerler yöntemi ise dağılımın kuyruk hareketlerini inceleyen güçlü ve oldukça sağlam bir yöntemdir. Bu çalışmanın temel amacı, 2008 küresel kriz sürecinde, 02 Ocak 2009 - 02 Nisan 2012 tarih aralığındaki ISE30 endeksi kullanılarak, GARCH modelleri ve Uç değerler yöntemi (Genelleştirilmiş Pareto Dağılımı, Genelleştirilmiş Uç Dağılımı) ile hesaplanan Riske Maruz Değer tahminlerinin karşılaştırılmasıdır.

Anahtar Kelimeler: Uç Değer Teorisi, Riske Maruz Değer, GARCH, Genelleştirilmiş Pareto Dağılımı, Genelleştirilmiş Uç Dağılımı

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1. INTRODUCTION

Risk term ensued from fluctuating financial and economic condition in 1970s. By the effects of rapidly developing capital and money market, risk was defined as term that is needed to manage. Along with the progress of financial markets and developing derivatives products, many researchers study on the price models related to the derivatives products and risk measurement became important.

Risk modeling is a popular approach for academic researchers from 1970s to the present. From the end of 1990s to 2000s, as consequences of the global crisis that exercises influence over the World, various models were

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developed to measure of risk. Within this scope, in 1994 RiskMetrics model was developed by JP Morgan and Value-at-risk (VaR) was introduced.

VaR summarizes the worst loss over a target horizon with a given level of confidence. (Gencay, Selcuk (2004)). There have also been a number of VaR studies in the finance literature in recent years. Hendricks (1996) studies VaR approach using exponential weighted average method, Alexander (1996) overviews the variance modeling using VAR approach, and Longin (2000) studies Extreme Value Theory (EVT). In addition, Gencay and Selcuk (2004) compare VaR estimations by using variance covariance model with Normal and Student t distributions, historical simulation, Generalized Pareto Distribution (GPD) and Pareto II models for emerging markets. Celik and Kaya (2010) compare VaR estimations by applying variance-covariance, EWMA, historical simulation and Fisher-Tippet theorem for 5 stocks in ISE 100 index. Gursakal (2007) compare VaR estimations by using variance-covariance and historical simulation for ISE 30 daily returns, Allen, Singh and Powell (2011) estimate and compare VaR, Conditional VaR and Expected Shortfall by applying EVT and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model for S&P-500 and ASX-All Ordinaries (Australia) index returns. Gencay and Selcuk (2006) estimate VaR by using Generalized Pareto Distribution (GPD) and Generalized Extreme Value Distribution (GED) for Turkey and U.S. overnight borrowing rates in the period before 2001 crisis.

In first part of the paper a review of GED, GPD and GARCH models are covered. In the second part, descriptive statistics, comparison of popular approaches for VaR estimation are presented for ISE 30 secondly index returns in between 02.01.2009 and 02.04.2012.

2. METHODOLOGY

2.1. Value-at-Risk (VaR)

As a response of the financial disasters, risk management and risk modeling become popular research area especially in the emerging markets. VaR is defined as expected maximum loss in a certain level of confidence over a given time horizon. Recent developments in risk modeling techniques caused to survey accurate VaR methodologies for different risk statues. The major advantage of VaR is to provide overall market risks in a single parameter. In addition, VaR takes the correlations between risk factors into consideration. Different quantitative techniques which embed different models into VaR measurement are developed and applied in financial series for VaR estimations. GARCH model which is widely used for modeling of the nonlinear variance are the most popular models in VaR modeling. Conditional variance that is used to calculate VaR estimation is obtained by GARCH modeling. The main drawback of this model is that distribution of financial return series generally has a fatter tail than a normal distribution, due to the main assumption of GARCH model is that the return series distributed as normally. Moreover, normality assumption may cause bias VaR estimation. VaR-GARCH values are calculated as given in the following formula (Alexander, 1996):

$$\mathrm{VaR} - \mathrm{GARCH}_t(\alpha) = \mu_t + \mathrm{F}^{-1}(\alpha)\sigma_t \qquad (1)$$

where $F^{-1}(\alpha)$ is the q^{th} quantile ($q=1-\alpha$) of the sample distribution, μ_t and σ_t are conditional mean and conditional standard deviation respectively.

As a result of financial crisis, the risk measurement techniques are insufficient to calculate possible loss since financial crisis cannot be clarified normal distribution because of symmetric nature of normal distribution. Since financial crisis has a feature of extreme movements, EVT is most naturally developed as a theory of large losses (McNeil, 1999). EVT is only interested in extreme points of the dataset. There are two principal kinds of distribution for extreme values which are GED and GPD. The distribution of excesses is determined by EVT and obtained shape, scale and location parameters with maximum likelihood estimation and parameters are used to calculate VaR, as presented in the following formula (Celik & Kaya,2010; Gencay, & Selcuk, 2004):

$$VaR\_GEV_p = \mu + \frac{\sigma}{\epsilon}[(-\ln p)^{-\epsilon} - 1] \quad (2)$$

$$VaR\_GPD_\alpha = u + \frac{\sigma}{\epsilon}\left[(\frac{n}{n_u}\alpha)^{-\epsilon} - 1\right] \quad (3)$$

where α confidence level, μ location parameter, σ scale parameter and ϵ shape parameter, p value of the unknown distribution function F, u threshold value, n number of observation and n_u number of observation that excess threshold.

2.2. GARCH Model

Bollerslev (1986) introduced GARCH model which is the generalized Autoregressive Conditional Heteroscedasticity (ARCH) models, is used to forecast the volatility. In GARCH models conditional volatility does not only depend on lags of the error term but also depends on the lags of conditional volatility. The conditional variance of the GARCH (p, q) process is specified as follows:

$$\sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4)$$

with w > 0, $\alpha_i, \beta_j \geq 0$ for i = 1, . . . , p and j = 1, . . . , q to assure positivity of conditional variance. GARCH model is inappropriate to capture asymmetry effect. Brooks (2002) overviews that GARCH model is inefficient due to it cannot response symmetrically to the negative and positive volatility shocks. Engle and Ng (1993) promote that if a negative asymmetric effect exists, than GARCH model underestimates the volatility following negative news and overestimates the volatility following positive news. The main drawback of GARCH model is constraint of nonnegativity on the coefficients. Nelson (1991) evolved GARCH model and introduced Exponential GARCH (EGARCH) model to reflect the asymmetry effect.

$$\log(\sigma_t^2) = w + \alpha \log(\sigma_{t-1}^2) + \beta \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \gamma\left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \quad (5)$$

EGARCH model is constructed by lags of conditional variance as GARCH model. Superiority of EGARCH model is to observe asymmetric movements in the model in terms of β and σ^2 gets always positive values due to conditional volatility is modeled logarithmically. Only restriction In EGARCH model is that $\alpha + \beta \leq 1$, in order to ensure that the process is stationary.

2.3. Extreme Value Theory (EVT)

EVT which is based on the convergence of maxima, is firstly introduced by Fisher-Tippet (1928) from point of view Central Limit Theorem. The family of extreme value distributions that are Frechet, Gumbel and Weibul, can be presented under a single distribution known as the GED. In GED, limits of the extreme values are calculated by using determined distribution. EVT is used especially for the modeling of distribution of price movements in crisis period in financial markets. EVT has any assumption about the distribution, that is major advantage of it.

Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independently and identically distributed random variables which has an unknown distribution function F(x). Denote the maximum of the first k < t observations of X by $H_k = \max(X_1, X_2, \ldots X_t)$. Given a sequence of $a_k>0$ and b_k such that $(H_k - b_k)/ak$, the sequence of normalized maxima converges in distribution to the following so-called Generalized Extreme Value Distribution (GED) (Gencay & Selcuk, 2004):

$$H(x) = \begin{cases} e^{-(1+\epsilon\frac{x}{\beta})^{-1/\epsilon}} & if\ \epsilon \neq 0 \\ e^{-e^{-\frac{x}{\beta}}} & if\ \epsilon = 0 \end{cases} \quad (6)$$

where ϵ is shape parameter. The distribution of the maxima extreme values is determined by shape parameter (ϵ). If $\epsilon > 0$ then distribution is Frechet and it has fat tailed, if $\epsilon < 0$ then it is Weilbull and if $\epsilon = 0$ it is Gumbel.

In the framework of GPD, all the observations which are out of the band that is generated by threshold values (u) take in the consideration. In accordance with EVT purpose, probability of observations which are out of the threshold values, are less than or equal to determined y value is calculated by the following formula (Gencay & Selcuk, 2004):

$$F_u(y) = Prob\{X - u \leq y | X > u\} = \frac{F(y+u) - F(u)}{1 - F(u)} \quad (7)$$

Density function of the distribution based on the parameters which is given in the following formula (Smith, 1987):

$$f(x) = \frac{1}{\beta}(1 + \epsilon\frac{x}{\beta})^{-\frac{1}{\epsilon}-1} \quad (8)$$

Distribution parameters of the GPD which are shape parameter (ϵ) and scale parameter (β) is estimated by maximum likelihood estimation. Moreover, Balkema and De Haan (1974) and Pickands (1975) shows that the distribution of extreme values $F_u(y)$ converges to GPD if the number of extreme values increases (Gencay & Selcuk, 2004):

$$GPD(x) = \begin{Bmatrix} 1 - (1 + \epsilon\frac{x}{\beta})^{-1/\epsilon} & if\ \epsilon \neq 0 \\ 1 - e^{-x/\beta} & if\ \epsilon = 0 \end{Bmatrix} \quad (9)$$

Since, numbers of some distributions are involved in the GDP, shape parameter ϵ determines the form of the distribution such as for positive ϵ it is Pareto distribution. In addition, $\epsilon = 0.5$ indicates infinite variance and $\epsilon = 0.25$ indicates infinite fourth moment. Gencay et al. (2001) mention that in the financial time series, by the feature of heavy tail, Pareto distribution frequently viewed. For $\epsilon = 0$, it is closed to exponential distribution. For negative shape parameter ϵ the GDP implies Pareto II distribution.

There are two principal kinds of model for extreme values which are block maxima model (BMM) and peak over threshold (POT) model. In BMM, the distribution of the maxima and minima values of the blocks is determined. In POT model, distributions of the observations which exceed the defined thresholds are determined. The threshold value is highly important for accurate estimation of the distribution. Hosking and Wallis (1987) indicates that maximum likelihood maximum likelihood estimates are asymptotically normally distributed and approximate standard errors for the estimator can be obtained by maximum likelihood estimation.

3. EMPRICAL ANALYSIS

In this paper logarithmic returns of secondly ISE 30 index is used in between 02.01.2009-02.04.2012. Descriptive statistics of secondly returns are presented in the Table 1.

Table 1: Descriptive Statistics

Number of observation	298786
Mean	0.000257
Standard Deviation	0.099208
Kurtosis	97.19575
Skewness	0.790824
Maximum	4.532510
Minimum	-3.658906
JB **(Prob.)**	1.10e+08 (0.000)

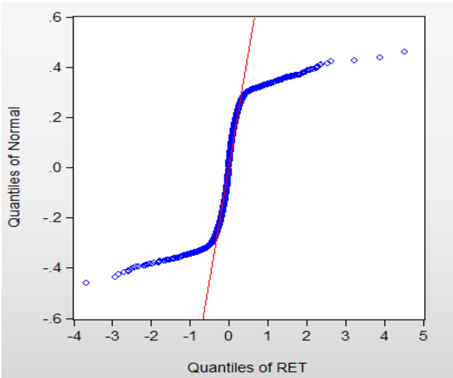

Figure 1: QQ plots of the returns

Based on the sample kurtosis estimates, the return distribution is far from being normally. In addition, QQ-plots of the returns and Jarque Bera statistics imply that the return distribution is not normal. The sample skewness shows that the secondly returns have an asymmetric distribution. Positive skewness indicates that the asymmetric tail extends more towards positive values than negative ones (Figure 1).

GED is estimated by applying BMM method. In BMM method firstly, sample is divided n blocks of equal length then the maximum values of each block is obtained finally GED distribution is fitted the set of maxima.

The critical point of BMM method is the appropriate choice of number and length of the blocks. By applying QQ plot and Scatter plots, number of the observations in each block is determined. QQ plot and scatter plots of the residuals that is obtained from GED by fitting block maxima of returns are presented in the following graphs for the length of blocks equals 100, 200 and 500 respectively (Figure 2).

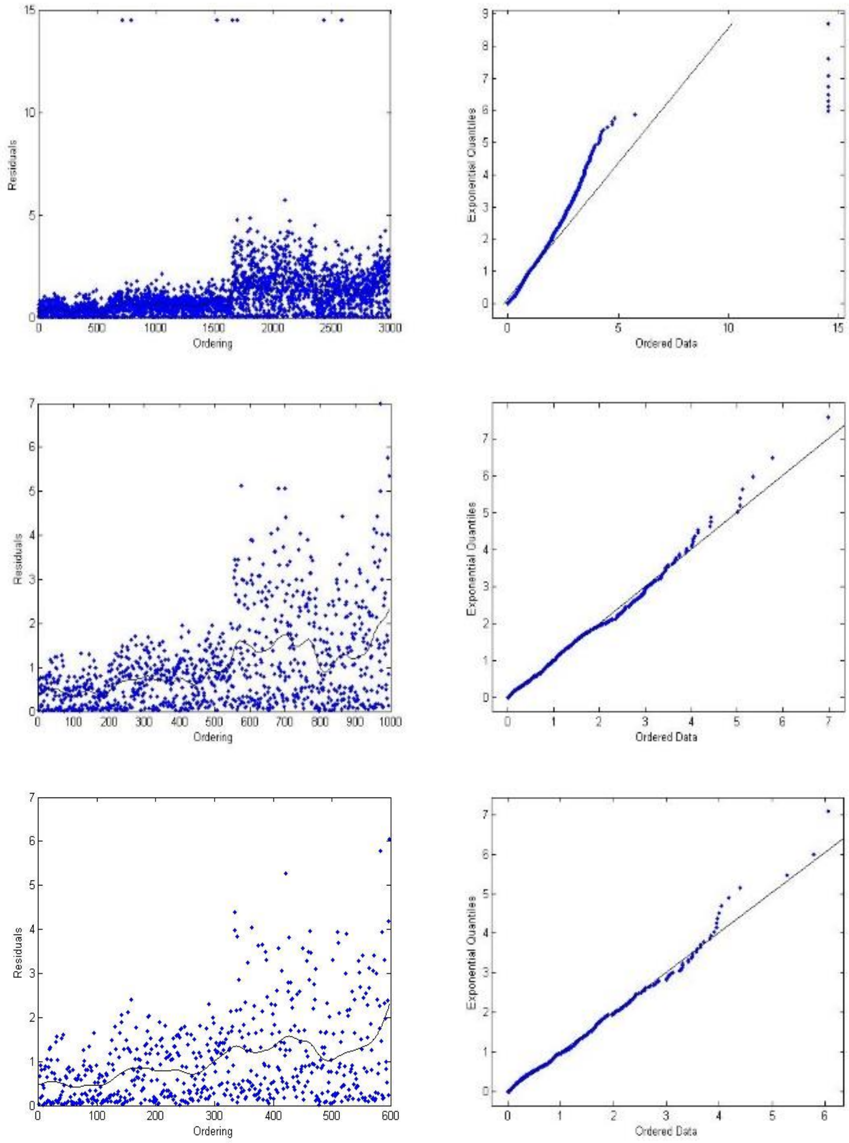

Figure 2: Scatter plots(left side) and QQ plots (right side) of the residuals that are obtained by fitting block maxima of returns for the length of blocks equals 100, 200 and 500 respectively.

The solid line corresponds to standard exponential quantiles in QQ plot and the solid line is the smooth of scattered residuals obtained by a spline method. Quantiles of the distribution by maximum likelihood estimation is compared by quantiles of empirical distribution. If estimated distribution is similar with empirical distribution, it would reflect to QQ plot as linearity. The scale parameter determines the slope. If data is from an exponential distribution, the points on the graph would lie along a positively sloped straight line. Concave presence is an indicator of fat tailed distribution; on the other hand convex presence is an indicator of short tailed distribution. Based on the graphics above, appropriate length of the blocks is determined as 200. By using maximum likelihood estimation shape, scale and location parameters are estimated as given in the following Table 2.

Table 2: Estimated GED parameters

	Parameters
Location Parameter (μ)	0.0038
Shape Parameter (ϵ)	0.4722
Scale Parameter (σ)	0.0023

Positive shape parameter shows that the distribution has fat tailed and known as Frechet distribution.

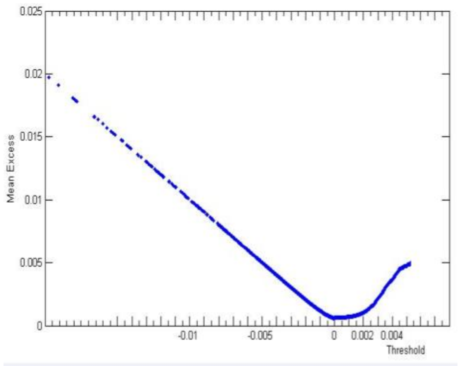

Figure 3: Mean excess function of excesses

Mean excess function (MEF) helps to determine the accurate threshold value which is critically important in GPD method. MEF shows the average values of the excesses values and is calculated as the dividing sum of the excesses to number of excesses (Figure 3).

According to the MEF graphics threshold value should be in between 0.001 and 0.004. Scale and location parameters of the distribution are estimated by maximum likelihood by using residuals.

In the graphic, positive sloped line for the above of the threshold shows that shape parameter is positive and the distribution converges to Pareto distribution. Otherwise, negative sloped line for the above of the threshold implies that short-tailed data If the series displays the exponential distribution, the horizontal line would be observed for the above a certain threshold u.

Table 3: Estimated GPD parameters (*refers the significance at 1% level)

Threshold	Location Parameter	Scale Parameter
0.001	0.2142*	0.000538*
0.002	0.4676*	0.000547*
0.004	0.5421*	0.0020*

The results indicates that the determined parameters of GDP distribution is fitted to the tails of the return distributions confirmed by QQ plot, scatterplot, tail of underlying distribution and exceedance distribution graphics (Figure 4).

For threshold=0.0021, positive slope and concave upward line shown in the QQ plot (right below) indicates that distribution has fat tailed and it is Pareto distribution. Density of the residuals in a specified interval indicates that the distribution is eligible for the residuals as shown in scatter plot in the Figure 4. Moreover, tail of underlying distribution and exceedance distribution graphics implies that GPD fits the tails of return distributions.

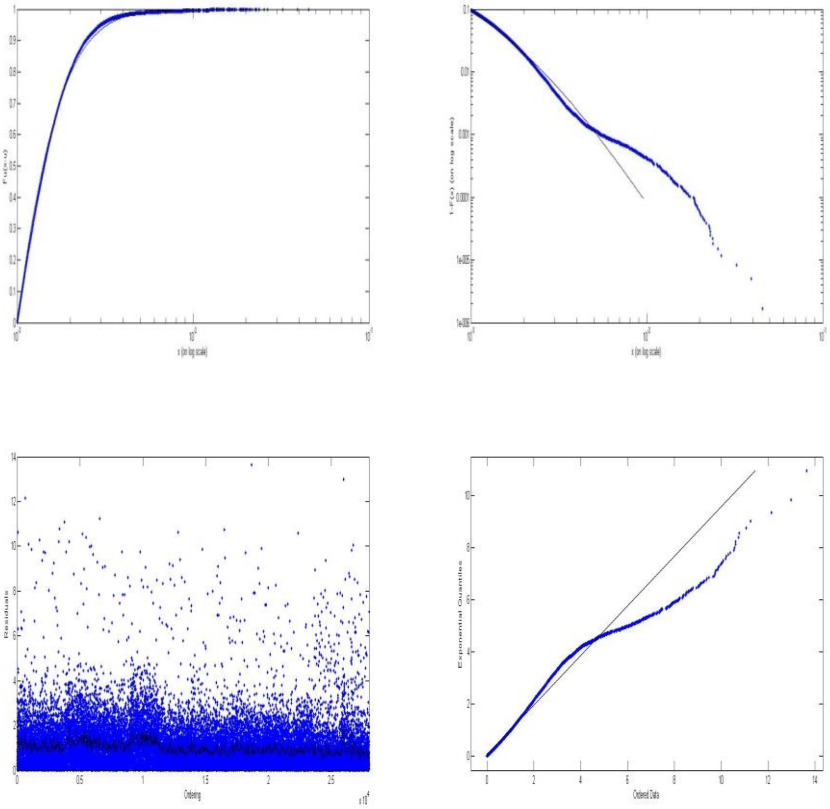

Figure 4: QQ plot (right below), scatterplot (left below), tail of underlying distribution (right above) and exceedance distribution graphics (left above).

Finally, the results of GARCH(1,1) and EGARCH(1,1) models with student t and normal distributions are given in the following Table 4.

As a result of asymmetry effect on the returns, EGARCH(1,1) Student t distribution best fitted compared with other models given in the table. One period ahead return prediction are estimated by applying GED, GDP and EGARCH models for various tail quantiles, block sizes and thresholds. In forecasting method moving window which has a fixed size is used for different window sized. For example, for 100 windows size in adaptive method first 100 returns are used to forecast 101th return, then 2^{nd} and forecasted 101th are used to forecast the 102th return. The performance of the constructed models is compared by using back testing method which is based on the violation ratio. Gencay and Selcuk (2004) define the violation as a situation that occurs when the realized return is greater that estimated one in a given second. Also, Gencay and Selcuk (2004) define the variation ratio dividing the total number of violation to number of one step ahead forecasted returns.

For various block sizes and/or threshold values, performances of value at risk estimations differ greatly. In consideration of overestimation or under estimation of risk, the accurate model is the one which has violation ratio closes to 5% for 95% quantile. This indicates that the only 5% of the total forecasted data is far away from realized returns.

Table 4: Results of GARCH(1,1) and EGARCH(1,1) models with student t and normal distributions

	GARCH(1,1) Normal Distribution	GARCH(1,1) Student t Distribution	EGARCH(1,1) Normal Distribution	EGARCH(1,1) Student t Distribution
W	0.000545*	0.0000000000000759	-0.746587*	-0,228393*
A	0.196568*	0.216904*	0.296436*	0.172244*
B	0.787737*	0.859098*	0.053263*	-0.035647*
ϒ	-	-	0.884323*	0.977953*
Degree of Freedom	-	3.191538*	-	3.596091*
AIC	-2.002048	-2.406603	-1.981142	-2.356812
SCI	-2.001906	-2.406426	-1.980965	-2.356599
HQ	-2.002007	-2.406552	-1.981091	-2.356750

In crisis period, minimizing the block size implies the accurate estimations since it reflects the minimum shocks to the GED model. The result of this study, the GED model is best performed whose block size is 200 for 5% significance level. Moreover, the violation ratios 0.0464, 0.0498 and 0.0596 are obtained respectively by using three different threshold ratios (1.5%, 1.55% and 1.6%) to predict accurate GPD model in 5% significance level. The best fitted GPD model is chosen whose violation ratio is closed to 5% significance level. On the other hand EGARCH(1,1) with student t distribution estimates VaR values higher than GED and GPD approaches.

4. CONSLUSIONS

In this study, extreme value theory and GARCH models are compared with regards to the value at risk by using GPD and GED. The distribution parameters of both GED and GPD are estimated by maximum likelihood method. Since shape parameter is positive, the generalized extreme distribution has a similar feature with Frechet distribution. On the other hand, excesses have fat tailed and converge to Pareto distribution. Since the returns incorporates the, EGARCH is used to model the asymmetry affect in returns.

We conclude that, extreme value theory can be useful for assessing the extreme events. Backtesting results indicate that the extreme value theory tends to outperform the EGARCH approach at 5% confidence levels, for the same confidence interval, expected risk by using GPD is higher than obtained by GED. GPD is very effective approach especially in crisis periods that investors exhibit unstable and abstainer behaviors. Moreover, due to the normality assumption in EGARCH models, EGARCH shows the risk of tails higher than actual. Moreover, this study implies that EGARCH model is insufficient for modeling extreme movements particularly for intraday returns. Especially for the derivative market, the risk measurement is highly important for determination of margins of derivative instruments. For short positions, over forecasting causes to higher margin which defines as amount of capital that should be allocated to cover the possible loss, as a consequence loss of interest rate income. On the other hand, under forecasting causes to lower margin this means less than required capital allocation.

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