



Research Paper / Article

A Comparative Study of Model Reduction Techniques for Specified Bandwidth on Aircraft

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Abstract: This paper presents a comparison of two popular model reduction techniques on an aircraft model. Large scale model of the aircraft is obtained via Ground Vibration Test (GVT). In vibration-based applications, natural frequencies are important, designers aim to focus on especially these frequencies. Therefore, a reduced-order model should be obtained adequately by approximating the original system at interested frequencies. To perform this, two popular model reduction methods; Frequency Weighted Balanced Model Reduction (FWBMR) and Rational Krylov based model reduction methods are resorted in this paper. The effectiveness of these two methods is discussed, and specifically, the flexibility of Rational Krylov based method is demonstrated on Body freedom flutter aircraft model.

Keywords: Model Order Reduction Methods, Large Scale Systems, Aircraft Model, System Identification

Hava Taşıtı Modeli Üzerinde Belirlemiş Frekans Aralığında Model İndirgeme Yöntemlerinin Karşılaştırılması

Öz: Bu çalışma, bir hava taşıtı modeli için iki popüler model indirgeme tekniğinin karşılaştırılması üzerinedir. Bu hava taşıtı modelinin büyük ölçekli yapı modeli Yer Titreşim Testi ile elde edilmiştir. Titreşim bazlı uygulamalarda bilindiği üzere doğal frekanslar önemli bir yer teşkil etmektedir ve tasarımcılar bilhassa bu frekanslara odaklanmayı hedeflemektedir. Bundan dolayı kestirilecek düşük dereceden modelin, orijinal sisteme bu frekansların etrafında yaklaşarak, buna benzer cevap vermesi gerekmektedir. Bu yaklaşımı gerçekleştirebilmek için iki popüler model indirgeme yöntemi olan Frekans Ağırlıklı Dengelenmiş Model İndirgeme yöntemi (Frequency Weighted Balanced Model Reduction (FWBMR)) ve Rational Krylov tabanlı model indirgeme yöntemlerine başvurulmuştur. Her iki metodun başarımları ele alınmış ve bilhassa Rasyonel Krylov metodunun hava taşıtı gövdesinin dalgalanma modeli üzerinde esnekliği gösterilmiştir.

Anahtar kelimeler: Model İndirgeme Yöntemleri, Büyük Ölçekli Sistemler, Hava Taşıtı Modeli, Sistem Tanımlama

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1. Introduction

In aviation industry, flutter effect is a well-known phenomenon which may occur when a structure is exposed by any aerodynamic forces. The damping of the structure at some speed may be insufficient to absorb the energy increasing due to the aerodynamic loads, as a result of this causes the amplitude of harmonic oscillations grows up rapidly. Loring [1] states that these unwanted oscillations may harm and destroy the structure. For aerospace applications, this phenomenon endangers flight safety.

Mathematical modeling of an aircraft often results in large scale models. Then, that yields a huge computational burden in order to design a controller for the system, also because of their high fragility on changes in systems parameters and noises, it is not quite easy to use such these large-scale systems in applications. In order to deal with high order one, a practical way is to truncate it a low order one which discussed at the paper of Antoulas [2]. In model reduction methods, it is basically aimed to reduce computational burden by deriving lower-dimensional models that represent the original high order system behavior similarly. The critical point is that low order model should have the same characteristics as high order one. In addition to this, in some applications, accuracy of a model is more important in a predefined frequency range.

In vibration-based applications, response of the system at natural frequencies is a compelling issue to investigate. The engineers account for these frequencies. In this context, deriving a low order model which encapsulates this frequency band is desired. By approximating at this predefined band, the response of reduced-order model should behave accurately as the original system. Therefore, designers can design a proper low order controller to make real-time systems behave in a desired way depending on predefined performance criteria.

In order to obtain an adequate model for complex systems like aircraft via structural dynamics principles, designers resort to finite element analysis, and that comes up with high order models. In order to deal with such these high order systems, there are many methods available in the literature and some of the most famous are collected in Gu [3] and Qu [4]. In addition to these structural dynamic model reduction methods, there are some other techniques in which the problem is taken into account as a control aspect at the paper of Lieu [5], Amsallem and Farhat [6], Kos [7]. Paper of Salimbahrami and Lohmann [8] discuss the order reduction with Krylov subspace methods for structural dynamics in their paper. In the work of Lieu [5], they are aimed to reduce the order of an F-16 aircraft model with proper orthogonal decomposition (POD) method for minimizing complexity effect for flutter analysis. In their work Kos [7], they compare Krylov, POD and generalized coordinate-based methods on F-16 aircraft method and flutter analysis. In another work Amsallem and Farhat [6], a new method is proposed for stabilizing reduced-order models and they show its performance on to mechanical vibration and aeroelastic model. Furthermore, most of the vibration and aeroelastic models are nonlinear and flexible aircraft control system models can be classified in this category. Cook, Goulart and Palacios apply model reduction techniques to linearized aeroelastic aircraft model [9]. Ronch and Badcock resort nonlinear model reduction on the nonlinear flexible aircraft control model [10]. Moreno studies efficient model reduction techniques on aeroservoelastic systems [11].

In the literature there exist abundant methods in order to reduce the order of the system. In the work of Antoulas [12], it's pointed out that these reduction methods are characterized into two main categories, Singular-Value Decomposition (SVD) methods and Krylov Subspace methods. One of the most preferable SVD methods is the balanced truncation method which is based on truncating lower Hankel singular values (HSV) of the system. Due to the simple and practical structure of this method, it still preserves its popularity. The fundamental idea of balanced truncation method is to

approximate the high order system for all frequency range. Frequency weighted balanced truncation is proposed to truncate the system model in the desired frequency band by Enns [13]. Kürschner shows that it's possible to reduce the model order with balanced truncation within the limited time intervals [14].

Rational Krylov which is based on matching the moments of the original high order and generated reduced order systems around predefined frequencies is another famous reduction method. These moments are obtained via Taylor expansion of transfer function at selected frequencies. The main idea behind Krylov methods is that reducing the dimensionality of the model is based on projecting the system onto expanding subspaces which is generated by the system matrices. In the work of Grimme [15], Krylov Subspace methods are mainly based on the moment choices. The matching of moments may be performed at any predefined frequencies. According to these frequencies, designers can resort to different approaches. One of the main drawbacks of Krylov based methods is that there is no global error bound in the reduced-order model. In addition to this Krylov methods can't guarantee the stability of the obtained system. In order to overcome these issues Heres studies on Robustness and efficiency on Krylov Subspace methods [16]. Alternatively, Bretien and Damm work on model reduction of bilinear control systems with Krylov subspaces [17]. A global error bounds for Krylov based method is developed by Panzer, and application of this method is demonstrated on the various models [18]. Ophem and Deckers work on model reduction of MIMO systems with Krylov subspace approach for vibration systems [19].

The natural frequencies of an aircraft can be found by experimentally for practical applications. One of the most favored experimental methods is known as GVT. Modal and structural natural frequencies can be derived by applying GVT to the aircraft model. In this paper, two popular model reduction methods are taken into account. As soon as the system model of the aircraft is derived via system identification, it is aimed to demonstrate the applicability of these model reduction methods onto this large-scale model. Based on obtained results the advantages and disadvantages of Rational Krylov and FWBMR method are compared.

The rest of the paper is organized as follows. Firstly, the aircraft model is derived via system identification methods. Section 3 covers the basics of FWBMR and Krylov Subspace methods. In Section 4 effectiveness of model order reduction methods on aircraft model are demonstrated, results are discussed.

2. System Description

In our work, the structural model of Body Freedom Flutter (BFF) aircraft is used. This BFF aircraft is designed by Lockheed Martin Aeronautics Company for testing new control technologies. BFF aircraft has a 22° sweptback wing platform, spanning 3.05m and chord length of 29.7cm as described in the work of Chicunque [20]. GVT is applied to this aircraft in order to derive the natural frequencies and modal shapes by a research team at University of Minnesota. These frequencies provide fundamental information in order to validate the system model. In order to excite the aircraft structure with a proper signal, Unholtz-Dickie Model 20 electrodynamic shaker is resorted in the work of Gupta [21]. Sinusoidal sweep wave is used as an excitation signal, and the frequency of this signal begins from 20 rad/s and increases up to 220 rad/s. The response of the aircraft from 34 different locations varying from tip to main body is measured via two PCB 353B16 miniature accelerometers as described in Moreno [22]. These locations can be seen from Figure 1. In our work, all experimental inputs outputs data are borrowed from these works [20]-[22].

After exciting the system with a proper signal, the selection of model structures is another important stage in system identification. In our work state-space model is chosen due to its practical and flexible structure. A 34th order state-space model that provides the same favorable behavior with experimental results, is obtained by a non-iterative subspace approach with the help of MATLAB System Identification Toolbox. The comparison between experimental data and the derived system at the 34th measurement point, which is one of the most swinging locations, are shown in Figure 2. Due to the lack of space, comparison of other points are not shown in the paper.

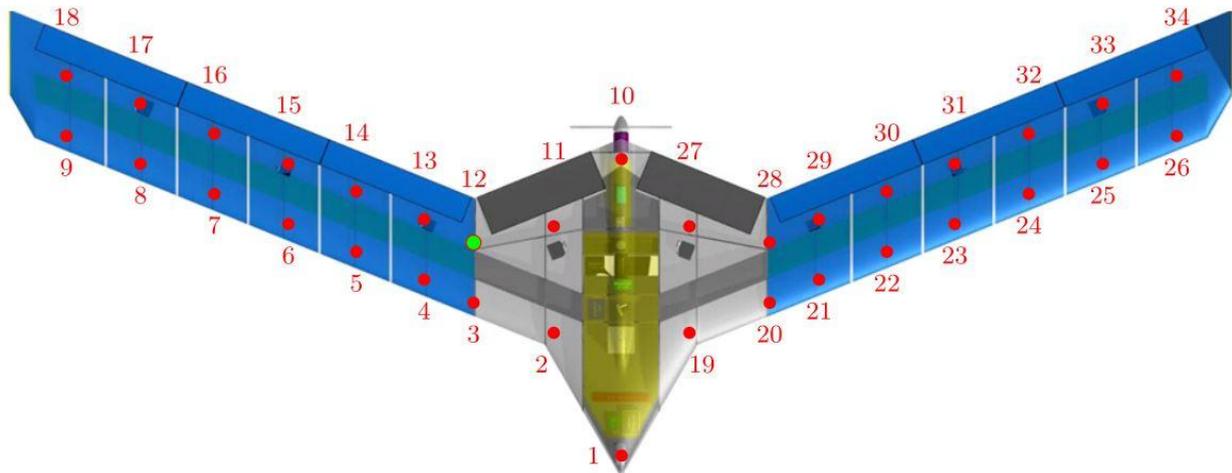


Figure 1. Top view of BFF aircraft, 12th point is where the excitation force is applied and other points where measurements taken

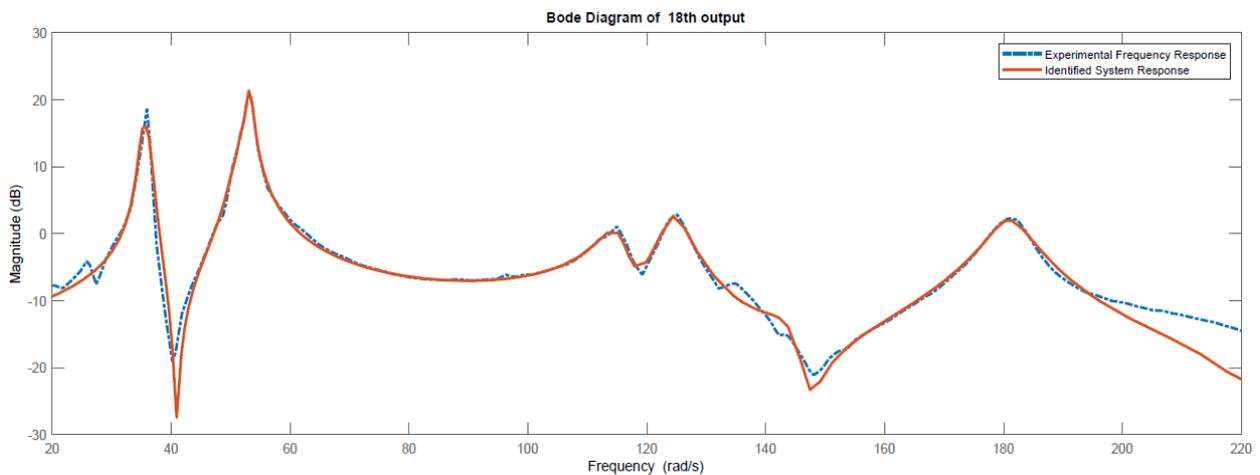


Figure 2. The response of experimental data and obtained system

From Figure 2, it's possible to determine the natural frequencies in the bode magnitude plot. Natural frequencies make picks at the bode magnitude plot. In Figure2 the picks at the magnitude plot can be seen at 35.94, 53.03 and 181.2 rad/s. These frequencies are important because systems tend to oscillate at these frequencies in the absence of any external input. When the external input is applied periodically at one of these frequencies, the system will vibrate at larger amplitude. In mechanical systems, resonance is the main unwanted phenomenon in most applications, due to its possibility of harming integrity of structure. In order to deal with resonance in vibration control applications, it's crucial to obtain low order model which behaves almost the same at these frequencies.

3. Model Reduction Methods

The state-space representation of a linear time-invariant system can be expressed as

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where the state vector $x(t) \in \mathbb{R}^n$, input vector $u(t) \in \mathbb{R}^m$, output vector $y(t) \in \mathbb{R}^p$, and where system matrices are $E \in \mathbb{R}^{n \times n}$, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$. When $E \neq I$ systems are known as descriptor systems where I is identity matrix. The reduced-order system can be obtained by projecting the system into a lower dimension.

Theorem 1. The reduced-order approximation of system represented at Equation (1) can be found as

$$\begin{aligned} E_r \dot{x}_r(t) &= A_r x_r(t) + B_r u(t) \\ y_r &= C_r x_r(t) + Du(t) \end{aligned} \tag{2}$$

Where $E_r \in \mathbb{R}^{q \times q}$, $A_r \in \mathbb{R}^{q \times q}$, $B_r \in \mathbb{R}^{q \times m}$, $C_r \in \mathbb{R}^{p \times q}$ are reduced system matrices. Reduced system matrices can be calculated as

$$\begin{aligned} E_r &= W^T E V & A_r &= W^T A V \\ B_r &= W^T B & C_r &= C V \end{aligned} \tag{3}$$

where W and V projection matrices.

The final reduced-order model can be found as

$$G_r = \begin{cases} \overbrace{W^T E V}^{E_r} \dot{x}_r(t) = \overbrace{W^T A V}^{A_r} x_r(t) + \overbrace{W^T B}^{B_r} u(t) \\ y_r(t) = \underbrace{C V}_{C_r} x_r(t) + \underbrace{D}_{D_r} u(t) \end{cases} \tag{4}$$

Proof of Theorem 1 can be found in the work of Antoulas [12]. For reducing the system order via projection it's necessary to find suitable V and W matrices. There are many methods proposed for finding this projection matrices. Most popular categories are SVD based methods and Krylov Subspace methods. SVD based methods try to generate projection matrices with the help observability and controllability gramians while Krylov Subspace methods use subspaces which are generated from system matrices.

There exist many SVD based model reduction methods. First SVD based method which known as balanced truncation, is introduced in the paper of Moore [23]. In the work of Liu and Anderson [24], by doing modifications on balanced truncating method, they are able to propose a new method called singular perturbation approximation. Optimal Hankel norm approximation which is another famous SVD based reduction method is introduced in the work of Glover [25]. These mentioned methods are used frequently in the control area. The main reason of their popularity comes from two important properties. First of all, they preserve the stability of the obtained reduced-order model if the original high order model is stable. Second important property is that they can provide a global error bound for obtained reduced-order system.

3.1. Frequency Weighted Balanced Truncation

In order to approximate the original system at the desired frequency interval, frequency weighted balanced model reduction (FWBMR) is proposed by Enns [13]. Enns provides a development over classical balanced truncation method. After modifications are applied to classical methods, the obtained method can reduce the high order model into low order one with input weights, output weights, or using both of them. The main drawback of this method is that the obtained technique no longer guaranteeing stability when both weights are used. In order to deal with this issue, Lin and Chiu Lin [26] makes modifications on Enns method and so stability problem is solved.

In order to obtain a reduced model for the system given in 1, let us take $N(s)$ and $M(s)$ input and output weights respectively, and

$$\begin{aligned} N &= A_n, B_n, C_n, D_n \\ M &= A_m, B_m, C_m, D_m \end{aligned} \tag{5}$$

be their corresponding minimal realizations. It is needed to construct augmented systems with the following equations

$$\begin{aligned} G(s)M(s) &= \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} (A & BC_n) & \begin{pmatrix} 0 \\ B_n \end{pmatrix} \\ (C & DC_n) & 0 \end{bmatrix}, \\ M(s)G(s) &= \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} (A_m & B_m C) & \begin{pmatrix} B_m \\ B \end{pmatrix} \\ \begin{pmatrix} 0 \\ C_m \end{pmatrix} & A & 0 \end{bmatrix} \end{aligned} \tag{6}$$

The controllability and observability gramians of above realizations (A_i, B_i) and (A_o, C_o) can be expressed as

$$P_i = \begin{bmatrix} P_E & P_{12} \\ P_{12}^T & P_v \end{bmatrix} \quad Q_o = \begin{bmatrix} Q_w & Q_{12} \\ Q_{12}^T & Q_E \end{bmatrix} \tag{7}$$

where P_i and Q_o satisfy the following Lyapunov equations,

$$\begin{aligned} A_i P_i + P_i A_i^T + B_i B_i^T &= 0 \\ A_o^T Q_o + Q_o A_o + C_o^T C_o &= 0 \end{aligned} \tag{8}$$

If type the above equations explicitly we obtain

$$\begin{aligned} A P_E + P_E^T A + B C_v P_{12}^T + P_{12} C_v^T B^T + B D_v D_v^T B^T &= 0 \\ A^T Q_E + Q_E A + C^T B_w^T Q_{12} + Q_{12}^T B_w C + C^T D_0^T D_0 C &= 0 \end{aligned} \tag{9}$$

After diagonalizing the modified gramians which are $P - P_{12} P_{22}^{-1} P_{12}^T$ and $Q - Q_{12}^T Q_{22}^{-1} Q_{12}$, we get

$$T P - P_{12} P_{22}^{-1} P_{12}^T T^T = (T^{-1})^T T Q - Q_{12}^T Q_{22}^{-1} Q_{12} T^{-1} = \text{diag} (\sigma_1, \sigma_2, \dots, \sigma_r, \sigma_{r+1}, \dots, \sigma_n) \tag{10}$$

where matrix T is a transformation matrix that diagonalizes both P and Q matrices. Where $\sigma_1 > \sigma_2 > \dots > \sigma_n > 0$ are known as Hankel singular values of the system and they are

representing the effect of the corresponding states on the system behavior. In order to get a low order system with good approximation, it's crucial to include states with high HSV in reduced-order system. From another point of view, HSV tells us which states should be included in low order model for minimum error. One useful feature of this diagonalization is provided HSV are obtained in descending order. As a result of this, states with the highest contribution to system behavior will be in descending order in this transformation. After obtaining the transformation matrix and deciding the order of approximation by analyzing the HSV, it's possible to reduce the system with the following transforms.

$$\begin{bmatrix} TAT^{-1} & TB \\ CT^{-1} & D \end{bmatrix} = \begin{bmatrix} [A_{11} & A_{12}] \\ [A_{21} & A_{22}] \\ [C_1 & C_2] \end{bmatrix} \begin{bmatrix} [B_1] \\ [B_2] \\ [D] \end{bmatrix} \tag{11}$$

where A_{11}, B_1, C_1, D is our reduced-order system in interested frequency interval. Since states are in descended order it's guaranteed that omitted states give lower contribution than selected ones to system behavior. The main advantage of FWBMR, the stability of the reduced system is ensured. Furthermore, it's possible to compute the error bounds in terms of discarded Hankel singular values.

3.2. Rational Krylov Method

Another way to obtain suitable projection matrices is deriving the transformation which equalizes moments of the original and its reduced order model in predefined frequency points. The transfer function of the system given in equation (1) can be expressed as

$$G(s) = C(sE - A)^{-1}B + D \tag{12}$$

Assuming, system matrix A is non-singular, we can expand the transfer function with Taylor series around complex shift σ ,

$$G(s) = \sum_{i=0}^{\infty} \rho_i^\sigma (s - \sigma)^i \tag{13}$$

Where ρ_i^σ are called as moments of system $G(s)$ around σ . If we write ρ_i^σ explicitly

$$\rho_i^{\sigma_i} = -C[(A - \sigma E)^{-1}E]^{i-1}(A - \sigma E)^{-1}B \tag{14}$$

in this case problem of matching moments known as rational interpolation. One way to match moments while reducing the system is applying a projection to original system. In order to match moments projection matrices should be generated from specific Krylov subspaces. The definition of this Krylov subspaces can be given as

$$K_q(A, b) = span \{b, Ab, \dots, A^{q-1}b\} \tag{15}$$

where $A \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. Vector b is called as starting vector. The first independent basic vectors can be considered as a basis for the Krylov subspace. The following theorems state the suitable Krylov subspaces for model reduction with moment matching.

Theorem 2. If the matrix V used in Equation (3), is a basis of Krylov subspace $K_{q_1}(A^{-1}E, A^{-1}b)$ with rank q and matrix W is chosen such that the matrix A_r is nonsingular, then the first q moments of the original and reduced order systems match.

The subspace $K_{q_1}(A^{-1}E, A^{-1}b)$ is called input Krylov subspace and order reduction can be done by using the basis of this subspace as a projection matrix. This method is classified as one-sided Krylov subspace method because only one Krylov subspace is used in the procedure. With this approach, only q moments of reduced-order system and the original system will be matched. Another projection matrix W is chosen as $V = W$. In order to match more moments different Krylov subspaces are needed.

Theorem 3. If the matrix V and W used in Equation (2), are bases of Krylov subspace $K_{q_1}(A^{-1}E, A^{-1}b)$ and $K_{q_2}(A^{-T}E^T, A^{-T}c)$ respectively, both with rank q , then the first $2q$ moments of the reduced and original order system will match.

Proof of these theorems can be found in the work presented by Lohmann and Salimbahrami [27]. These theorems show that the presented approach is able to solve model order reduction problems in a flexible way, meaning that it's possible to tackle different order reduction problems with the most appropriate subspace. On the other hand, the main drawback of this result is that there are not any formulations for error bounds. Consequently, the approximation quality can be determined only around local points in moment matching methods, and it is not possible to make a foresight about error bounds apart from these predefined local points.

Calculation of basis vectors of Krylov subspace is a very compelling issue for application. In the work Bai [28] it's showed that generally these basic vectors tend to linearly dependent even for moderate values of q . As a result of the multiplication $A_i B$, Krylov subspace is quickly converging to the dominant eigenvector of $(A^{-1}E, A^{-1}b)$. In the literature there exist two popular algorithms, Arnoldi and Lanczos, in order to derive the projection matrices V and W .

Lanchzos algorithm introduced as a model reduction technique by Grimme, Van Dooren and Gallivan [29]. Lanchzos algorithms are based on finding two bases for input and output Krylov subspaces that are orthogonal to each other. The numerical accuracy of this algorithm is not good as Arnoldi algorithm. In Arnoldi algorithm, the orthogonal basis for Krylov subspace is constructed as a first step. In this algorithm generally, projection matrices are chosen as $V = W$. After taking these matrices the same, this equality yields one sided method, so it is able to match only q moments. On another hand, two-sided model reduction method can be obtain by small modifications. Two-sided model reduction can be applied twice to the system using Arnoldi algorithm. In this work one and two sided Arnoldi algorithm is used because of its numerical robustness and quickness.

4. Model Order Reduction of BFF Aircraft Model

In model order reduction process, the order of truncation is strongly related with computational burden and there exists a trade-off between this order and performance specifications. In the literature, there is not a systematic way to decide the order of reduction.

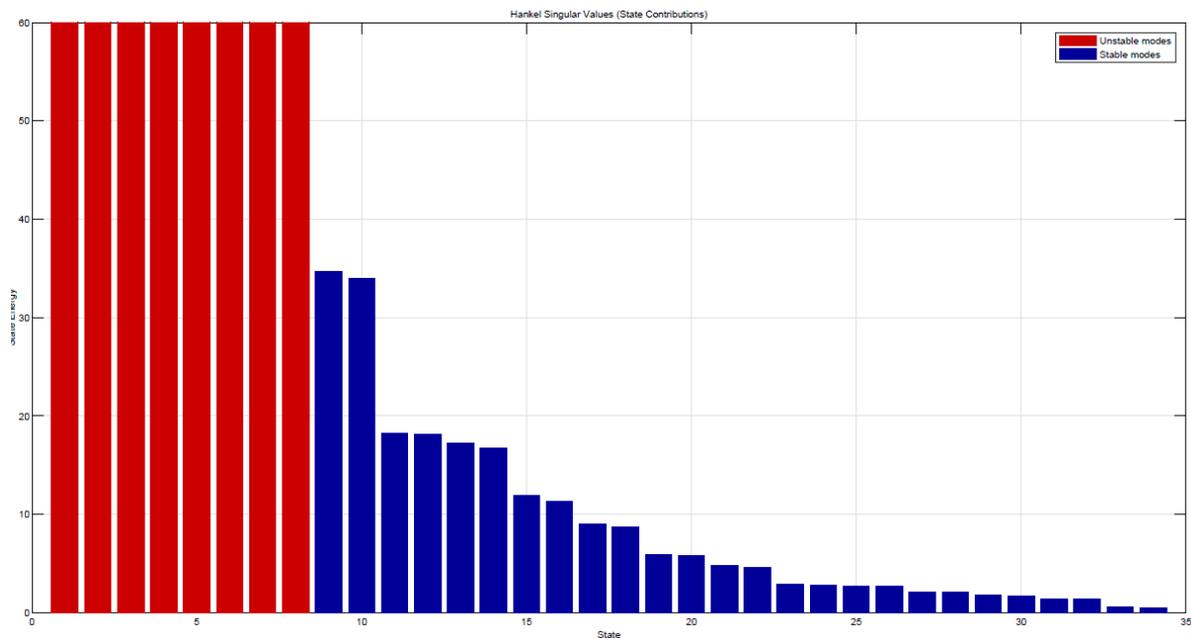


Figure 3. Hankel Singular Values of 34th order original system

A conventional way is trial and error approach. On the other hand, detecting the contribution of states to output provides reasonable information about reduction order. The contribution of these states to output is relative to the energy of each state. This energy can be measured as the square roots of the eigenvalues of the product of controllability and observability gramian which are known as Hankel Singular Values (HSV). It is possible to determine where to stop the reduction process using HSV. HSV of BFF aircraft model can be seen from Figure 3. Another valuable information included in HSV is that the stability of the system can be seen from it. States with infinite HSV correspond to unstable states. As can be seen from Figure 3 there are 8 unstable states.

In this section, two popular reduction methods are used to reduce the dimension of 34th order BFF aircraft around natural frequencies. As mentioned before identified BFF aircraft model has 8 unstable states. Therefore, the lowest possible order for FWBMR is 8, because balanced truncation-based methods are unable to make manipulation on unstable states. In brief, it aimed to focus on two natural frequencies, 35.4 rad/s and 181.2 rad/s, and the orders of truncation are selected as 8 and 10 by observing HSV.

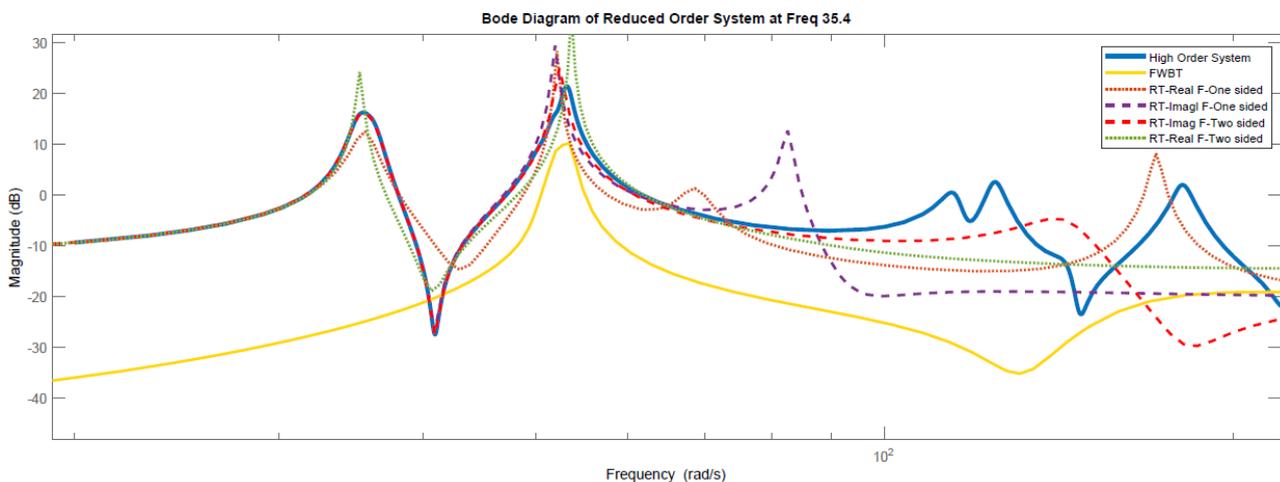


Figure 4. Performances of approximated 8th order models focusing at frequency 35.4 rad/s

Primarily, we focus on two natural frequencies, models with 8th orders are derived using Rational Krylov and FWBMR. As can be seen from the Figure4 and Figure 5 FWBMR is not successful to approximate the system because this method is unable to deal with unstable states as mentioned before and directly takes all these unstable states into reduced-order model.

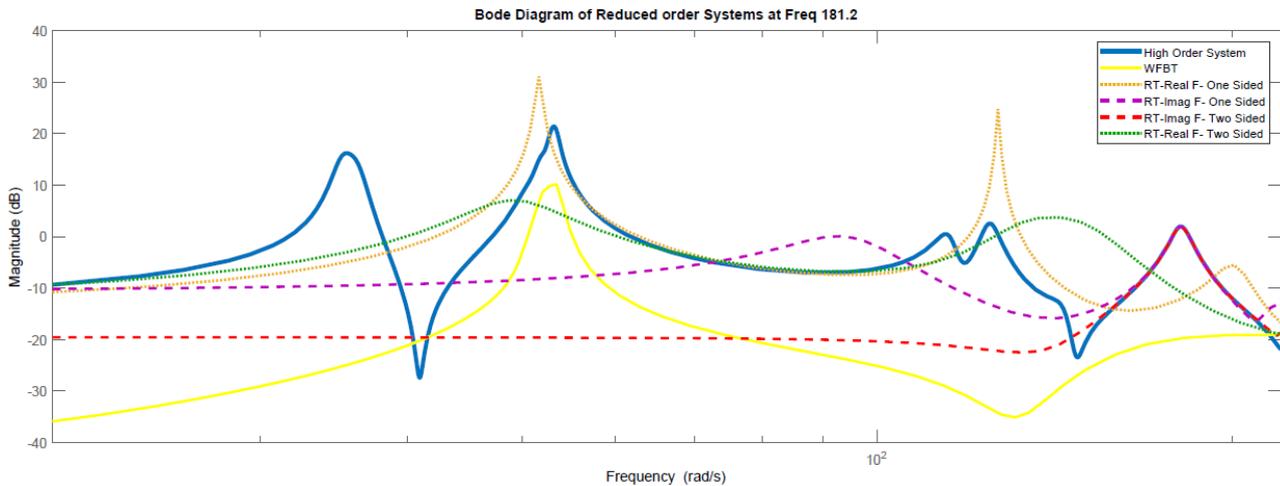


Figure 5. Performances of approximated 8th order models focusing at frequency 181.2 rad/s

This means that it’s not possible to get a reduced model without unstable states by using FWBMR. In order to get an adequate model with FWBMR, it is essential to increase the order of the truncated model.

In Rational Krylov methods reduction is based on substituting real number into ρ or substituting imaginary value into ρ where the ρ value is the interested frequency to approximate. In addition to this, it is possible to approximate the system with one-sided or two-sided method due to its flexible structure. Combination of these choices generates 4 possible approaches to our system. As can be seen in Figure 4-5 approximating with Krylov methods yields reasonably good results, especially when the moments calculated with imaginary number. Two sided methods approximate better because they are matching 2 times more moments than one sided method. Krylov methods are approximating better than FWBMR method for this case because they are matching moments. They don’t have to include the unstable mods. This difference in their working principle makes a great advantage for Krylov subspace-based methods when reducing unstable systems since order of the reduced model can be chosen lower than unstable mod number. As a result of that there are no restrictions on the degree of the reduced system while reducing with Krylov methods.

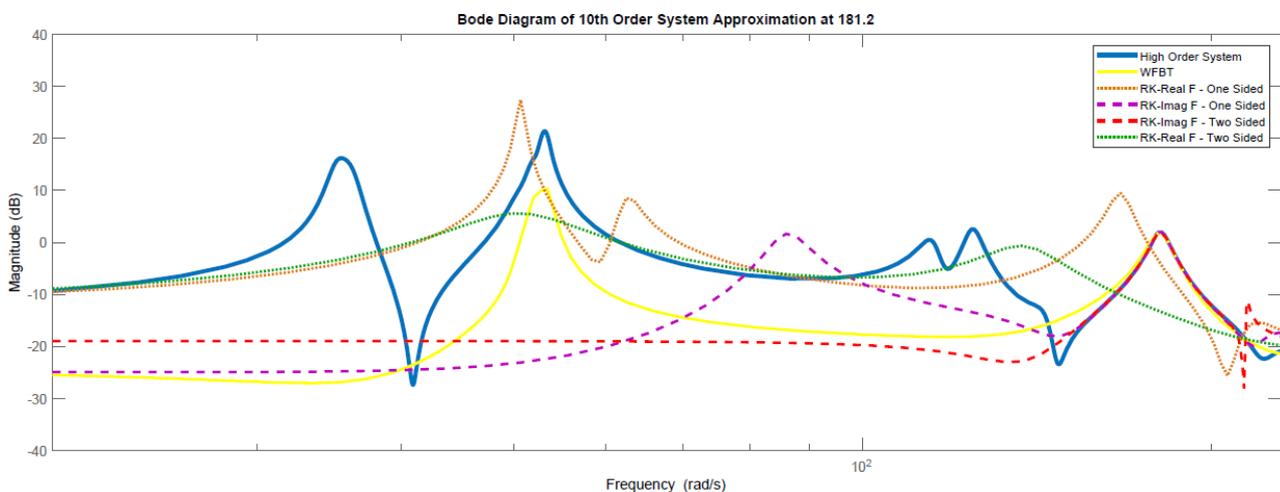


Figure 6. Performances of approximated 10th order models focusing at frequency 181.2 rad/s

As we increase truncation order, FWBMR is able to match response at the interested frequency as shown in Figure 6. According to HSV, it is observable that, first two stable modes have an important impact on the system. Therefore, the truncated model should include these modes in order to behave adequately as the original system. In Krylov methods, it is expectable that, increasing order of the system improves the accuracy of response near the interested frequency.

If we have an interest in more than one frequency, we can still approximate the system with the mentioned methods. The frequency responses of 10th order obtained model when interested frequencies are chosen as 35.4 rad/s and 181.1 rad/s given in Figure 7. In this figure only Krylov methods with imaginary expansion points are able to match the response of reduced-order system with the original system with good accuracy at interested frequencies. FWBMR method is able to match the system response at 35.4 rad/s but can't approximate the response at 181.1 rad/s. The reduced-order models obtained with real expansion points provide a worse approximation to the original system at interested frequencies but they are able to match the response better in all frequency range than others.

In some applications, it is desired to focus on more than one frequency points. In Figure 7 the successes of Krylov and FWBMR is showed. Although the truncated model with FWBMR encapsulates first natural frequency, it behaves unlikely as the original system at other natural frequencies. On the other hand, the model which is approximated with Krylov method provides almost the same characteristic as the original system at both natural frequencies.

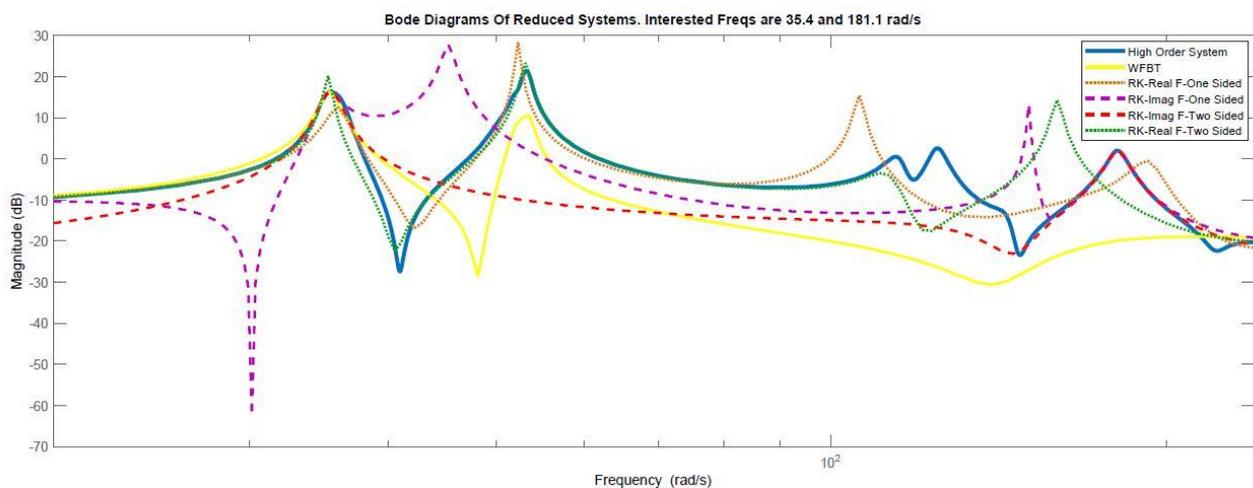


Figure 7. Performances of approximated 10th order models focusing at natural frequencies 35.4 and 181.2 rad/s

In addition to this, due to the flexible structure of Krylov based model reduction method, it is possible to distribute moments onto different frequency points. If the response of the system is more important at some frequency points, we can focus that interval while reducing the system at other frequency points. Therefore, designers can supervise the trade-off between different frequency points according to the importance of them with choosing appropriate moment weights. In Figure 7, the distribution of moments weight is chosen as, 4 moments are matched at $\pm 35.4i$ and 6 moments are matched at $\pm 181.1i$. Changing these weights basically provides to approximate the original system better at the important frequencies. It can be observable from Figure 8, moment weights of these two natural frequencies are changed, chosen as 8 moments for $\pm 35.4i$ 2 moments for $\pm 181.1i$, so the approximated model can now represent the original system behavior much more adequately at 35.1 rad/s.

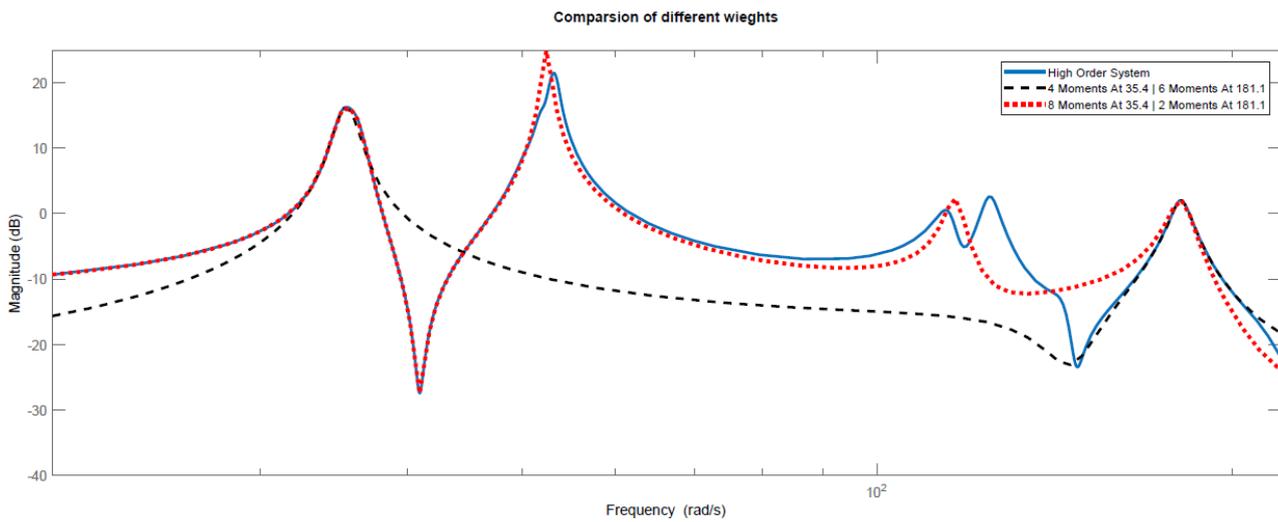


Figure 8. Performances of approximated 10th order models with different moment weights focusing at frequencies 35.4 and 181.2 rad/s

5. Conclusion

The response of an aircraft at natural frequencies is an important issue to investigate in vibration-based applications. In this work, 34th order state-space model of BFF aircraft is derived using experimental data with the help of system identification tools. FWBMR and Rational Krylov methods are used for reducing the system order to deal with computation complexity. In this context, reduced-order models are constructed in the predefined frequency band. The effectiveness of these two reduction methods is compared. Obtained results show that Rational Krylov methods have a great advantage over FWBMR. It's possible to arrange the number of matched moments at interested frequencies with Krylov methods. Krylov method provides different reduced models with the same order while focusing on different frequency points. This advantage may be useful for practical applications. For example, if the system is affected by noise at some frequencies it's possible to match less moment at these frequencies and reduce the effect of noise in reduced-order system model.

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