INTEGRAL INVARIANTS OF PARALLEL P-EQUISTANTE RULED SURFACES WHICH ARE GENERATED BY INSTANTANEOUS PFAFF VECTOR

Süleyman ŞENYURT*

Ordu University, Faculty of Arts and Sciences,Department of Mathematics, Ordu - Turkey

Abstract

In this study, relationships between integral invariants of parallel p-equidistant ruled surfaces which are generated by instantaneous pfaff vector are examined in 3-dimensional Euclidean space E^3.
Key words: Instantaneous pfaff vector, ruled surface, integral invariant.

AMS Mathematical Subject Classification(2000): 53A04, 53A05

Özet

Bu çalışmada Öklid uzayında iki regle yüzeyin striksiyon eğrileri boyunca teğet vektörleri paralel ve uygun noktalardaki polar düzlemler arasındaki uzaklık sabit kabul edilerek elde edilen kapalı regle yüzeylere bağlı olarak Darboux vektörü yönündeki birim vektörün meydana getirdiği kapalı regle yüzeylerin integral invaryantları arasındaki bağıntılar bulunmuştur.

Anahtar Kelimeler: Ani pfaff vektör, regle yüzey, integral invaryant.

1. INTRODUCTION

Let $\alpha : I \to E^3$ be a differentiable curve with arc-length parameter. Here,

(1.1) $$u_1(s) = \alpha'(s), \quad u_2(s) = \frac{\alpha''(s)}{\|\alpha''(s)\|}, \quad u_3(s) = u_1(s) \wedge u_2(s)$$

*ssenyurt@odu.edu.tr

are tangent vector, normal vector and binormal vector of α at the points $\alpha(s)$, respectively and $\{u_1, u_2, u_3\}$ is the Frenet frame of α. The Frenet formulas of α are

(1.2) $u_1' = k_1 u_2 \ , \quad u_2' = -k_1 u_1 + k_2 u_3, \quad u_3' = -k_2 u_2$

where k_1, k_2 are the curvature and the torsion tensor of α, respectively.

Let φ be a ruled surfaces with the leading curve $\alpha = \alpha(s)$ and the generator vector $x(s)$. Then φ has the following parameter representation:

(1.3) $\varphi : I \times IR \to E^3 \quad \varphi(s,\ v) = \alpha(s) + v x(s)$

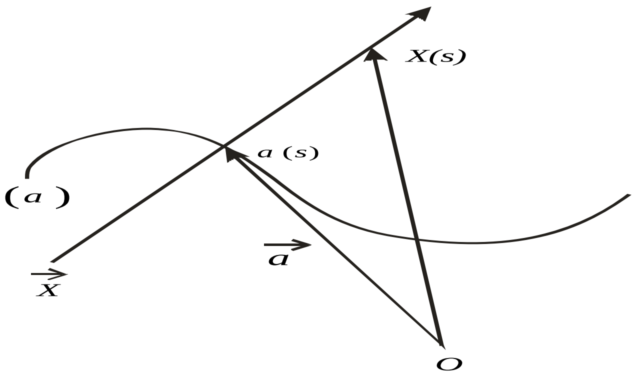

Figure 1.1: Ruled Surface

If γ is the striction curve of $\varphi(s,\ v)$, then

(1.4) $\gamma(s) = \alpha(s) - \dfrac{\langle \alpha'(s), x'(s) \rangle}{\|x'(s)\|^2} x(s)$

Moreover the apex angle, pitch and drall of $\varphi(s,\ v)$ are

(1.5) $\lambda_x = \langle d, x \rangle,\ l_x = \langle V, x \rangle\ ,\ P_x = \dfrac{\det(\alpha', x, x')}{\|x'\|^2},$

respectively. On the other hand the apex angle, pitch and drall of closed ruled surfaces generated by Frenet vectors fiel $u_1(s)$, $u_2(s)$ and $u_3(s)$ are

$$\textbf{(1.6)}\quad \begin{cases} \lambda_{u_1}=\oint\limits_{(\alpha)} k_2 ds \\ \lambda_{u_2}=0 \\ \lambda_{u_3}=\oint\limits_{(\alpha)} k_1 ds \end{cases},\ \begin{cases} l_{u_1}=\oint\limits_{(\alpha)} ds \\ l_{u_2}=0 \\ l_{u_3}=0 \end{cases},\ \begin{cases} P_{u_1}=0 \\ P_{u_2}=\dfrac{k_2}{k_1^2+k_2^2} \\ P_{u_3}=\dfrac{1}{k_2} \end{cases},\ [2].$$

The planes which correspond to the subspaces $sp\{u_1,u_2\}$, $sp\{u_2,u_3\}$ and $sp\{u_3,u_1\}$ are called asymptotic plane, polar plane and central plane [1,4].

Definition

Let $\varphi(s,v)=\alpha(s)+vu_1(s)$ and $\overline{\varphi}(s,v)=\overline{\alpha}(s)+vv_1(s)$ be two ruled surfaces in E^3 with the generators u_1 and v_1, tangent vectors of α and $\overline{\alpha}$, respectively. If

i) The generator vectors of φ and $\overline{\varphi}$ vectors are parallel,

ii) The distance between the polar planes at the corresponding points is constant, then the pair of ruled surfaces φ and $\overline{\varphi}$ are called the parallel p-equidistant [4].

If the striction curve of φ and $\overline{\varphi}$ are the leading curves and the distance between central planes asymptotic planes and polar planes of φ and $\overline{\varphi}$ are $|z|,|q|$ and $|p|$ respectively, it is written that

$$\textbf{(1.7)}\qquad \overline{\alpha}=\gamma+pu_1+zu_2+qu_3$$

Also the following equations are satisfild

(1.8) $\bar{k}_1 = k_1 \dfrac{ds}{d\bar{s}}$ and $\bar{k}_2 = k_2 \dfrac{ds}{d\bar{s}}$, [4]

Where $k_1, \overline{k_1}$ are natural curvatures and $k_2, \overline{k_2}$ are natural torsion of φ and $\bar{\varphi}$, respectively.

Theorem

There are following relation between angle of the pitches, pitches and dralls of $\varphi(s,v)$ and $\bar{\varphi}(s,v)$, which main lines are Frenet vectors, closed parallel p-equidistant ruled surface couple,

(1.9)
$$\begin{cases} \lambda_{v_1} = \lambda_{u_1} + \oint\limits_{(pu_1+zu_2+qu_3)} k_2 ds \\ \lambda_{v_2} = \lambda_{u_2} = 0 \\ \lambda_{v_3} = \lambda_{u_3} + \oint\limits_{(pu_1+zu_2+qu_3)} k_1 ds \end{cases}$$

(1.10)
$$\bar{k}_1 l_{v_1} = k_1 \, l_{u_1} + k_1 \oint\limits_{(pu_1+zu_2+qu_3)} ds,$$

(1.11)
$$\begin{cases} P_{v_1} = P_{u_1} = 0 \\ P_{v_2} = P_{u_2} \dfrac{d\bar{s}}{ds} \text{ , [3]} \\ P_{v_3} = P_{u_3} \dfrac{d\bar{s}}{ds} \end{cases}$$

2. INTEGRAL INVARIANTS OF PARALLEL P-EQUIDISTANT RULED SURFACES WHICH ARE GENERATED BY INSTANTANEOUS PFAFF VECTOR

Let w is instantaneos pfaff vector of α curve. If the angle between w and u_3 is $\beta = \beta(s)$, from fig. 2.1, it is obtained that

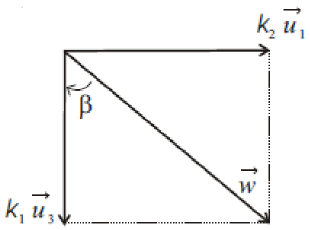

Figure 2.1. Pfaff vector

$$\cos\beta = \frac{k_1}{\|w\|} = \frac{k_1}{\sqrt{k_1^2+k_2^2}} \quad , \sin\beta = \frac{k_2}{\|w\|} = \frac{k_2}{\sqrt{k_1^2+k_2^2}}$$

where $\|w\| = \sqrt{k_1^2+k_2^2} > 0$. If c is the unit vector in the diection of w, we can write

$$c = \frac{w}{\|w\|} = \frac{k_2}{\sqrt{k_1^2+k_2^2}}u_1 + \frac{k_1}{\sqrt{k_1^2+k_2^2}}u_3 \text{ or}$$

(2.1) $$c = \sin\beta\; u_1 + \cos\beta\; u_3$$

From (1.5) the pitch of the closed surface generated by c is

$$\lambda_c = \sin\beta \oint_{(\alpha)} k_2 ds + \cos\beta \oint_{(\alpha)} k_1 ds\ .$$

From here using (1.6), we get

(2.2) $$\lambda_c = \sin\beta\; \lambda_{u_1} + \cos\beta\; \lambda_{u_3}$$

Also from (1.5) the apex angle of the closed ruled surface is

$$l_c = \sin\beta \oint_{(\alpha)} ds \text{ or}$$

(2.3) $$l_c = \sin\beta\; l_{u_1}$$

Moreover the drall of the closed ruled surface is

$$P_c = \frac{\det\left(\dfrac{d\alpha}{ds}\ ,\ c\ ,\ \dfrac{dc}{ds}\right)}{\left\|\dfrac{dc}{ds}\right\|^2},$$

(2.4) $$P_c = \frac{-\cos\beta(\sin\beta k_1 - \cos\beta k_2)}{\beta'^2 + (\sin\beta k_1 - \cos\beta k_2)^2}.$$

Theorem 2.1.

Let $\varphi(s,v)$ and $\bar{\varphi}(\bar{s},v)$ be parallel p-equidistant closed ruled surfaces. Then, there are following relationships, between the apex angles, pitches and dralls of ruled surfaces generated by moving c depending on $\{u_1,u_2,u_3\}$.

$$\lambda_c = \sin\beta\ \lambda_{u_1} + \cos\beta\ \lambda_{u_3},$$

$$l_c = \sin\beta\ l_{u_1},$$

$$P_c = \frac{-\cos\beta(\sin\beta k_1 - \cos\beta k_2)}{\beta'^2 + (\sin\beta k_1 - \cos\beta k_2)^2}.$$

Let $\bar{\alpha} : I \to E^3$ be a differentiable curve with arc-lenght parameter and $\{v_1,v_2,v_3\}$ be its Frenet frame. If the angle between the instantaneous pfaff vector $\bar{w}$ of $\bar{\alpha}$ and v_3 is $\bar{\beta} = \bar{\beta}(\bar{s})$, from the fig. (2.2), we have

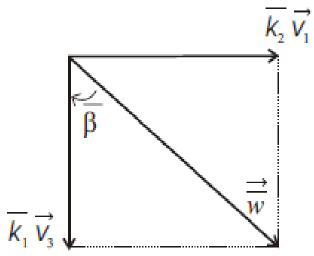

Figure 2.2. Pfaff vector

$$\cos\bar{\beta}=\frac{\bar{k}_1}{\left\|\bar{w}\right\|}=\frac{k_1}{\sqrt{k_1^2+k_2^2}}\quad,\ \sin\bar{\beta}=\frac{\bar{k}_2}{\left\|\bar{w}\right\|}=\frac{k_2}{\sqrt{k_1^2+k_2^2}}$$

Where $\|w\|=\sqrt{k_1^2+k_2^2}>0$. If $\bar{c}$ is the unit vector in the direction of w, we can write

$$\bar{c}=\frac{k_1}{\sqrt{k_1^2+k_2^2}}v_1+\frac{k_2}{\sqrt{k_1^2+k_2^2}}v_3,$$ or

(2.5) $$\bar{c}=\sin\beta\ v_1+\cos\beta\ v_3$$

From (1.5), the pitch of the closed ruled surface generated by $\bar{c}$ is

$$\lambda_{\bar{c}}=\sin\beta\oint_{(\bar{\alpha})}k_2ds+\cos\beta\oint_{(\bar{\alpha})}k_1ds$$

From here using (1.7), we get

$$\lambda_{\bar{c}}=\sin\beta\oint_{(\gamma)}k_2ds+\cos\beta\oint_{(\gamma)}k_1ds+\sin\beta\oint_{(pu_1+zu_2+qu_3)}k_2ds+\cos\beta\oint_{(pu_1+zu_2+qu_3)}k_1ds$$

Assuming that γ is the leading curve and using (2.2) in the last equation, we have

(2.6) $$\lambda_{\bar{c}} = \lambda_c + \sin\beta \oint_{(pu_1+zu_2+qu_3)} k_2 ds + \cos\beta \oint_{(pu_1+zu_2+qu_3)} k_1 ds$$

Thus, we can give the following theorem:

Theorem 2.2.

Let $\varphi(s,v)$ and $\bar{\varphi}(\bar{s},v)$ be parallel p-equidistant closed ruled surfaces. Then, there is following relationship, between the apex angles of closed ruled surfaces generated by c depending on $\{u_1,u_2,u_3\}$ and $\bar{c}$ depending on $\{v_1,v_2,v_3\}$:

$$\lambda_{\bar{c}} = \lambda_c + \sin\beta \oint_{(pu_1+zu_2+qu_3)} k_2 ds + \cos\beta \oint_{(pu_1+zu_2+qu_3)} k_1 ds$$

Also, from (1.5) the pitch of the ruled surface generated by $\bar{c}$ is

$$l_{\bar{c}} = \sin\beta \oint_{(\bar{\alpha})} ds$$

From here using (1.7), we get

$$l_{\bar{c}} = \sin\beta \oint_{\gamma} ds + \sin\beta \oint_{(pu_1+zu_2+qu_3)} ds$$

Assuming that γ is the leading curve and using (2.2) in the last equation, we have

(2.7) $$l_{\bar{c}} = \sin\beta\, l_c + \sin\beta \oint_{(pu_1+zu_2+qu_3)} ds$$

Theorem 2.3.

Let $\varphi(s,v)$ and $\bar{\varphi}(\bar{s},v)$ be parallel p-equidistant closed ruled surfaces. Then, there is the following relationship between the pitches closed ruled surface generated by c depending on $\{u_1,u_2,u_3\}$ and $\bar{c}$ depending on $\{v_1,v_2,v_3\}$:

$$l_{\bar{c}} = \sin\beta l_c + \sin\beta \oint_{(pu_1+zu_2+qu_3)} ds.$$

On the other hand, from (1.5), the drall of the ruled surface generated by $\bar{c}$ is

$$P_{\bar{c}} = \frac{-\cos\beta(\sin\beta k_1 - \cos\beta k_2)}{(\beta')^2 + (\sin\beta k_1 - \cos\beta k_2)^2} \frac{d\bar{s}}{ds}$$

By substituting (2.4) in the last equation, we obtain

(2.8) $$P_{\bar{c}} = P_c \frac{d\bar{s}}{ds}$$

Lastly, we can give the following theorem:

Theorem 2.4.

Let $\varphi(s,v)$ and $\bar{\varphi}(\bar{s},v)$ be parallel p-equidistant closed ruled surfaces. Then, there is the following relationship between the dralls closed ruled surface generated by c unit depending on $\{u_1,u_2,u_3\}$ and $\bar{c}$ depending on $\{v_1,v_2,v_3\}$: $P_{\bar{c}} = P_c \frac{d\bar{s}}{ds}$

REFERENCES

[1] Blaschke W., *Istanbul University Publication*, **(1949)**, Translate from Turkish by Ord. Prof. Dr. K. Erim, Math No 433, p.399.
[2] Hacısalioğlu H.H., *Inönü University Science-Arts Faculty Publication*, **(1983)**, *Math. No.7*, Malatya.
[3] Masal M., Kuruoğlu, N., *Pure and Applied Mathematika Sciences*, **(1999)**, 50(1-2), p.35-42.
[4] Valenontis I., *Manuscripta Math*. **(1986)**, 54, p.391-404.