SPACELIKE-TIMELIKE BERTRAND EĞRİ ÇİFTİNE AİT $\left(C^*\right)$ SABİT POL EĞRİSİNİN KÜRESEL İNVOLÜTLERİ

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ÖZET

Bu çalışmada $\left(\sim,\sim^*\right)$ spacelike-timelike Bertrand eğri çiftinin Frenet çatıları arasındaki geçiş bağıntısı bulunarak, $\sim^*$ eğrisinin teğetler göstergesi ile binormaller göstergesinin, $\left(C^*\right)$ sabit pol eğrisinin birer küresel involütleri olduğu gösterildi.

Anahtar Kelimeler: Lorentz Uzayı, Spacelike-Timelike Bertrand Eğri Çifti, Küresel İnvolütler

Mathematics Subject Classification (2000) : 53A04, 53B30

SPHERICAL INVOLUTES OF THE FIXED POLE CURVE $\left(C^*\right)$ ON THE SPACELIKE-TIMELIKE BERTRAND CURVE COUPLE

ABSTRACT

In this paper, it has been showed that every single indicatrix of tangents and indicatrix of binormals of the curve, $\sim^*$ are spherical involutes of the fixed pole curve, $\left(C^*\right)$ by finding a transition link of the spacelike-timelike Bertrand curve through Frenet frames.

Keywords:Lorentz Space, Spacelike - Timelike Bertrand Curve Couple, Spherical Involutes

Mathematics Subject Classification (2000) : 53A04, 53B30

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1. PRELIMINARIES

For $\forall X, Y \in IR^3$ and let $g : IR^3 \times IR^3 \rightarrow IR$ be a function. In this case, the lorentzian metric is defined by

$$g(X,Y) = x_1 y_1 + x_2 y_2 - x_3 y_3 .$$

Here , the pair (IR^3, g) is called as Lorentzian space and denoted by IL^3. For $\forall X \in IL^3$, the vector X is said to be spacelike, if $g(X,X) > 0$ or $X = 0$, timelike if $g(X,X) < 0$ and lightlike (or null) if $g(X,X) = 0$. The norm of the $X \in IL^3$ is given by, $[2]$

$$\|X\| = \sqrt{|g(X,X)|} .$$

The Frenet formulas of timelike curve, $\alpha : I \rightarrow IL^3$ are as followings;

$$T' = \kappa N \ , N' = \kappa T - \tau B \ , B' = \tau N \ , [3]$$

and the frenet instantaneous rotation vector is defined by, $[5]$

$$W = \tau T - \kappa B, \quad \|W\| = \sqrt{|\kappa^2 - \tau^2|} .$$

Here, $T \times N = -B$, $\mathrm{N} \times B = T$, $B \times T = -N$. Let φ be the angle between W and $-B$ vectors and if W is taken as spacelike, then the unit Darboux vector can be stated by

$$\begin{cases} \kappa = \|W\| \cosh \varphi \ , \ \tau = \|W\| \sinh \varphi \ , \\ \\ C = \sinh \varphi \, T - \cosh \varphi \, B \end{cases} \qquad (1)$$

And if W is taken as timelike, then it is described by

$$\begin{cases} © = \|W\| \sinh ¨ \ , ² = \|W\| \cosh ¨ \\ C = \cosh ¨ \, T - \sinh ¨ \, B. \end{cases} \qquad (2)$$

Th frenet formulas of spacelike with timelike binormal curve, $\sim : I \to IL^3$ are as followings:

$$T' = © N, \quad N' = -© T + ² B, \quad B' = ² N \ [3] \qquad (3)$$

And the frenet instantaneous rotation vector is defined by, [5]

$$W = ² T - © B, \qquad \|W\| = \sqrt{\left| ²^2 - ©^2 \right|} .$$

Here, $T \times N = B$, $N \times B = -T$, $B \times T = -N$ .Let ¨ be the angle between W and $-B$ vectors and if W is taken as spacelike, then the unit Darboux vector can be stated by

$$\begin{cases} © = \|W\| \cosh ¨ , \ ² = \|W\| \sinh ¨ , \\ C = \sinh ¨ \, T - \cosh ¨ \, B \end{cases}$$

and if W is taken as timelike, then it is describedby

$$\begin{cases} © = \|W\| \sinh ¨ , \ ² = \|W\| \cosh ¨ , \\ C = \cosh ¨ \, T - \sinh ¨ \, B. \end{cases}$$

The vector product of X , $Y \in IL^3$ is calculated by, $[1]$

$$X \wedge Y = \left(x_3 y_2 - x_2 y_3 , x_1 y_3 - x_3 y_1 , x_1 y_2 - x_2 y_1 \right).$$

The curvatures drawn by unit speednon-nullcurve, $\sim : I \to IL^3$ at the point $\sim(s)$ with the frenet vectors T, N, B and the unit Darboux vector, C over the Lorentzian unit sphere $S_1^{\ 2}$ or Hyperbolic unit sphere $H_0^{\ 2}$ are named respectively as indicatrix of tangents, indicatrix of principal normals, indicatrix of binormals and fixed pole curve. These curvatures are indicated in order as $(T),(N),(B)$ and (C), $[4]$.

Let $\sim : I \to IL^3$ be spacelike with timelike binormal curve and $\sim^* : I \to IL^3$ be timelike curve and if the tangent of $\sim$, $\sim(s)$ posses through the point $\sim^*(s)$ and $\left\langle T^*(s), T(s) \right\rangle = 0$, then the curve $\sim^*$ is said to be the involute of $\sim$, $[6]$.

2. SPHERICAL INVOLUTES OF THE FIXED POLE CURVE $\left(C^*\right)$ ON THE SPACELIKE-TIMELIKE BERTRAND CURVE COUPLE

Definition 2.1: Let $\{T, N, B\}$ and $\{T^*, N^*, B^*\}$ be respectively the frenet frames of the spacelike with timelike binormal curve and timelike curve, $\sim : I \to IL^3$ and $\sim^* : I \to IL^3$ at points $\sim(s)$ and $\sim^*(s)$. If the principal normal vectors N and N^* are linearly dependent, then the pair $\left(\sim, \sim^*\right)$ is said to be spacelike – timelike Bertrand curve couple.

Theorem 2.1: There is a connection between spacelike-timelike Bertrand curve couple and Frenet frames that are written as followings

$$\begin{cases} T^* = \sinh ¯ \, T - \cosh ¯ \, B \\ N^* = N \\ B^* = \cosh ¯ \, T - \sinh ¯ \, B \end{cases}$$

Here, the angle ¯ is the angle between T and T^*.

Proof: By taking the derivative of $\sim^{*}(s)=\sim(s)+\text{ª}N(s)$ with respect to arclenght s and using the equation , we get

$$T^{*}\frac{ds^{*}}{ds}=T\left(1+\text{ª©}\right)-\text{ª}2\,B\,. \qquad (4)$$

The iner products of the above equation with respect to T and B are respectively defined as

$$\begin{cases}\sinh - \dfrac{ds^{*}}{ds}=1-\text{ª©}\\ \cosh - \dfrac{ds^{*}}{ds}=-\text{ª}2\end{cases}$$

and substituting these present equations in (4) we obtain

$$T^{*}=\sinh^{-}T-\cosh^{-}B\,. \qquad (5)$$

Here, by finding the derivative of (5) and using (3) we get

$$N^{*}=N\,.$$

Firstly, we can write

$$B^{*}=\cosh^{-}T-\sinh^{-}B \qquad (6)$$

By availing the equation $B^{*}=-\left(T^{*}\times N^{*}\right)$.

By the derivative of $\sim_{T^{*}}\left(s_{T^{*}}\right)=T^{*}(s)$ with respect to arclenght $s_{T^{*}}$, weget

$$T_{T^{*}}=\frac{dT^{*}}{ds}.\frac{ds}{ds_{T^{*}}}\,.$$

After wards, by some algebraic manipulations and substituting (5) in T_{T^*}, the following result can be achieved

$$T_{T^*} = N \,. \tag{7}$$

Similarly, by taking the derivative of $\sim_{B^*}\left(s_{B^*}\right) = B^*(s)$ with respect to arclenght s_{B^*}, we get

$$T_{B^*} = \frac{dB^*}{ds} \cdot \frac{ds}{ds_{B^*}} \,.$$

By using the equation (6), we write down

$$T_{B^*} = N \,. \tag{8}$$

Lastly, by taking the derivative of $\sim_{C^*}\left(s_{C^*}\right) = C^*(s)$ with respect to arclenght s_{C^*}, we obtain

$$T_{C^*} = \frac{dC^*}{ds} \cdot \frac{ds}{ds_{C^*}}$$

If W^* is spacelike, then by considering (1) and with some algebraic operation we get

$$T_{C^*} = \cosh \ddot{}\,{}^{*}T^{*} - \sinh \ddot{}\,{}^{*}B^{*} \tag{9}$$

İf W^* is timelike, then then by considering (2) and with some algebraic operation we get

$$T_{C^*} = \sinh \ddot{}\,{}^{*}T^{*} - \cosh \ddot{}\,{}^{*}B^{*} \tag{10}$$

Theorem 2.2: Let $\left(\sim,\sim^{*}\right)$ be spacelike-timelike Bertrand curve couple. Tach of the indicatrix of tangents $\left(T^{*}\right)$ and the indicatrix of binormals $\left(B^{*}\right)$ of $\sim^{*}$ curve is a spherical involute of the fixed pole curve $\left(C^{*}\right)$.

Proof: In order to show that $\left(T^{*}\right)$ and $\left(B^{*}\right)$ of the $\sim^{*}$ curve is each a spherical involute of $\left(C^{*}\right)$, we need to prove the following statements $\langle \frac{dT^{*}}{ds}, \frac{dC^{*}}{ds} \rangle = 0$ and

$$\langle \frac{dB^{*}}{ds}, \frac{dC^{*}}{ds} \rangle = 0 .$$

If W^{*} is spacelike, by taking into account (7) and (9) we write down

$$\langle \frac{dT^{*}}{ds}, \frac{dC^{*}}{ds} \rangle = \langle N, \cosh \ddot{}\,^{*} T^{*} - \sinh \ddot{}\,^{*} B^{*} \rangle .$$

Next, by using (5) and (6) and doing required manipulations, the following result can be obtained

$$\langle \frac{dT^{*}}{ds}, \frac{dC^{*}}{ds} \rangle = 0 .$$

Further more, by exploiting thee quations $(5),(6),(8)$ and (9), we do the similar calculations to get

$$\langle \frac{dB^{*}}{ds}, \frac{dC^{*}}{ds} \rangle = 0 .$$

On the other hand, if W^{*} is timelike, we use (7) and (10) to reach

$$\langle \frac{dT^*}{ds}, \frac{dC^*}{ds} \rangle = \langle N, \sinh \ddot{}\,^{*}T^* - \cosh \ddot{}\,^{*}B^* \rangle .$$

Here, by substitution of (5) and (6) in the above the formula, we write

$$\langle \frac{dT^*}{ds}, \frac{dC^*}{ds} \rangle = 0 .$$

Once again, when the given relations $(5),(6),(8)$ and (10) are taken into consideration, we get

$$\langle \frac{dB^*}{ds}, \frac{dC^*}{ds} \rangle = 0 .$$

3.REFERENCES

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