Advances in the Theory of Nonlinear Analysis and its Applications **5** (2021) No. 1, 39–48.
https://doi.org/10.31197/atnaa. 749630
Available online at www.atnaa.org
Research Article

Advances in the Theory of Nonlinear Analysis and its Applications

ISSN: 2587-2648

Peer-Reviewed Scientific Journal

Exact solution for heat transport of Newtonian fluid with quadratic order thermal slip in a porous medium

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Abstract

In this communication, an analytical solution for the thermal transfer of Newtonian fluid flow with quadratic order thermal and velocity slips is presented for the first time. The flow of a Newtonian fluid over a stretching sheet which is embedded in a porous medium is considered. Karniadakis and Beskok's quadratic order slip boundary conditions are taking into account. A closed form of analytical solution of momentum equation is used to derive the analytical solution of heat transfer equation in terms of confluent hyper-geometric function with quadratic order thermal slip boundary condition. Accuracy of present results is assured with the numerical solution obtained by Iterative Power Series method with shooting technique. The impacts of porous medium parameter, tangential momentum accommodation coefficient, energy accommodation coefficient on velocity and temperature profiles, skin friction coefficient and reduced Nusselt number are discussed. The Nusselt number increases with the higher estimations of tangential momentum and energy accommodation coefficients.

Keywords: Analytical solution, Newtonian fluid, Quadratic order thermal slip, Porous medium, Hyper-geometric functions.
2010 MSC: 33C15, 35Q79, 65N99, 76D05.

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Received June 14, 2020; Accepted: December 26, 2020; Online: December 29, 2020.

1. Introduction

The investigation of fluid flow in the presence of slip boundary conditions has received considerable interest due its accuracy of predicting the realistic behaviour in many engineering processes. For example, the fluid flow in micro pumps, micro nozzles, micro vales and hard disk experiences slip at wall. The use of no-slip condition in the above cases does not predict the actual physical situation. The consideration of velocity slip and temperature jump in this type of flow regime is very important to determine the velocity and temperature, respectively. The investigation of heat transfer in a fluid flow induced by a moving surface is very important in the processes of glass blowing, continuous casting, cool oil slurries, metal spinning and plastic films etc. A primary investigation on this type of problem was done by Sakiadis [1,2] and Crane [3]. Much attention has been given to this type of fluid flow problem with various physical effects via both analytical and numerical techniques [4-12].

Karniadakis and Beskok [13] proposed a quadratic order slip boundary condition. Xiou et al. [14] studied the gas flow in microtube with quadratic order slip conditions. Hamdan et al. [15] modelled the micro gas flow with quadratic order slip conditions. Fang et al. [16] considered the Wu's [17] quadratic order velocity slip condition in the problem of viscous fluid flow over a shrinking sheet. Fang et al. [16] derived closed form analytical solutions for momentum equation. Nandeppanavar et al. [18] studied the quadratic order velocity slip with heat transfer in a viscous fluid. A closed form analytical solution was presented for the momentum equation and the energy equation was solved numerically in [18]. Turkyilmazoglu [19] derived the analytical solution for MHD viscous fluid flow using Laguerre polynomials with quadratic order velocity slip. The quadratic order velocity slip effects on nanofluid flow over a stretching/shrinking sheet were investigated both numerically and analytically in the articles [20,21]. In the above articles, a closed form solution for momentum equation was presented and the energy equation was solved using confluent hyper-geometric function. Numerous numerical investigations on the fluid flow with quadratic order velocity slip can be found in the literature [22-32]. In all the above studies [18-35], the effects of quadratic order temperature boundary conditions were omitted and Wu's quadratic order velocity boundary condition was considered. Arikoglu et al. [33] used differential transform method (DTM) to analyse the Karniadakis and Beskok's quadratic order velocity slip and thermal jump impacts in a rotating disk problem. Recently, Ganesh et al. [34] have carried out a numerical investigation on the Newtonian fluid over a vertical stretching sheet with quadratic order velocity and thermal slip boundary conditions of Karniadakis and Beskok.

Having all the above literature in mind, we focused in this communication on the problem of Newtonian fluid flow over a stretching sheet immersed in a porous medium with suction. we aimed to derive an analytical solution in terms of confluent hyper-geometric function for energy equation in the presence of the following quadratic order temperature and velocity slip boundary conditions [13]:

$$\beta_{v1}\left(\frac{\partial u_s}{\partial y}\right)+\beta_{v2}\left(\frac{\partial^2 u_s}{\partial y^2}\right)=-\left(u_w-u_s\right),\quad \text{(Quadratic order Velocity slip)},$$
$$\beta_{t1}\left(\frac{\partial T}{\partial y}\right)+\beta_{t2}\left(\frac{\partial^2 T}{\partial y^2}\right)=-\left(T_w-T_s\right),\quad \text{(Quadratic order Thermal slip)}.$$

It is worth mentioning herein that -to achieve this target-, we firstly derived a closed form analytical solution of momentum equation following the footsteps in [16, 18-21].

2. Mathematical formulation

We consider the steady, 2D laminar flow of a Newtonian fluid over a stretching sheet in a Darcian porous medium with suction effects. The sheet stretching velocity $u_w = dax$ with constant 'a' and stretching parameter 'd' are assumed. The x-axis runs along the stretching surface with velocity 'u'. The y-axis runs perpendicular to the sheet with velocity 'v'. The temperature at the stretching surface takes the constant value T_w, while the ambient value, attained as y tends to infinity, takes the constant value T_∞. It is assumed that the fluid experiences both second order velocity and thermal slips. The governing equations of this problem can be expressed as:

$$\frac{\partial v}{\partial y}+\frac{\partial u}{\partial x}=0, \tag{1}$$

$$\frac{\partial u}{\partial x}u+\frac{\partial u}{\partial y}v+\frac{\mu}{\rho}\frac{u}{K}-\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}=0, \tag{2}$$

$$\frac{\partial T}{\partial x}u+\frac{\partial T}{\partial y}v-\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}=0, \tag{3}$$

where $\mu,\ \rho,\ K,\ k$ and C_p are representing the viscosity, density, permeability of the porous medium, thermal conductivity, and specific heat capacity, respectively. The quadratic order velocity and temperature boundary conditions of Eqs. (1)-(3) are considered as

$$\begin{aligned}
&u=dax+\beta_{v1}\frac{\partial u}{\partial y}+\beta_{v2}\frac{\partial^2 u}{\partial y^2},\\
&v=v_w,\\
&T=T_w+\beta_{t1}\frac{\partial T}{\partial y}+\beta_{t2}\frac{\partial^2 T}{\partial y^2},\quad \left(T_w=T_\infty+bx\right),\quad \text{at}\quad y=0,\\
&u\to 0, T\to T_\infty \text{ as } \ y\to\infty,
\end{aligned} \tag{4}$$

where $\beta_{v1}=\dfrac{(2-\sigma_v)\lambda}{\sigma_v}$ is the first order velocity factor, $\beta_{v2}=\dfrac{(\sigma_v)\beta_{v1}^2}{2(2-\sigma_v)}$ is the quadratic order velocity slip factor, $\beta_{t1}=\dfrac{2\beta(2-\sigma_t)\lambda}{\sigma_t Pr(\beta+1)}$ represents the first-order thermal jump factor, $\beta_{t2}=\dfrac{\beta_{t1}^2(\sigma_t)(\beta+1)Pr}{4\beta(2-\sigma_t)}$ represents the quadratic-order thermal slip factor. In addition, λ is the mean free path and β is specific heat ratio.

3. Solution of Momentum equation

The method of solution is based on the similarity transformations; given by:

$$\frac{u}{a}=xg'(\eta),\quad \frac{v}{\sqrt{av_f}}=-g(\eta),\quad \frac{\eta}{y}=\sqrt{\frac{a}{v_f}},\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}. \tag{5}$$

Plugging the similarity transformations (5) into Equations (2) and (4); one obtains

$$g'''+gg''-g'^2=k_1 g', \tag{6}$$

$$\begin{aligned}
&g(\eta)=s,\quad g'(\eta)=d+\gamma_v g''(\eta)+\delta_v g'''(\eta)\quad \text{as}\quad \eta\to 0,\\
&g'(\eta)\to 0 \text{ as } \ \eta\to\infty,
\end{aligned} \tag{7}$$

where $k_1=\dfrac{\mu}{\rho ak}$ is porous medium parameter, $\gamma_v=\beta_{v1}\sqrt{\dfrac{a}{v_f}}$ is first order velocity slip parameter, $\delta_v=\beta_{v2}\dfrac{a}{v_f}$ is quadratic order velocity slip parameter and s is suction/injection parameter. An analytical solution of Eq. (6) subject to boundary conditions (7), is obtained following similar procedure given in [16, 18-21]; hence, we obtain

$$g(\eta)=s+\frac{d\left(1-e^{-\alpha\eta}\right)}{\alpha\left(1+\gamma_v\alpha-\delta_v\alpha^2\right)}\quad \text{and}\quad g'(\eta)=\frac{de^{-\alpha\eta}}{\left(1+\gamma_v\alpha-\delta_v\alpha^2\right)}. \tag{8}$$

Substituting Eq. (8) in Eq. (6) gives the following algebraic equation

$$d=-\left(1+\gamma_v\alpha-\delta_v\alpha^2\right)\left[k_1+\alpha(s-\alpha)\right],$$

which has four distinct roots expressed as

$$\begin{aligned}
\alpha &= -\frac{-\gamma_v - s\delta_v}{4\delta_v} - \frac{\beta_4}{2} - \frac{\sqrt{\beta_6 - \beta_7}}{2}, \qquad (I)\\
\alpha &= -\frac{-\gamma_v - s\delta_v}{4\delta_v} - \frac{\beta_4}{2} + \frac{\sqrt{\beta_6 - \beta_7}}{2}, \qquad (II)\\
\alpha &= -\frac{-\gamma_v - s\delta_v}{4\delta_v} + \frac{\beta_4}{2} - \frac{\sqrt{\beta_6 + \beta_7}}{2}, \qquad (III)\\
\alpha &= -\frac{-\gamma_v - s\delta_v}{4\delta_v} + \frac{\beta_4}{2} + \frac{\sqrt{\beta_6 + \beta_7}}{2}, \qquad (IV)
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
\beta_1 &= 2^{1/3}\left(12(d+k_1)\delta_v + (-1+s\gamma_v - k_1\delta_v)^2 - 3(s+k_1\gamma_v)(-\gamma_v - s\delta_v)\right),\\
\beta_2 &= 27(s+k_1\gamma_v)^2\delta - 72(d+k_1)\delta_v(-1+s\gamma_v-k_1\delta_v) + 2(-1+s\gamma_v-k_1\delta_v)^3\\
&\quad -9(s+k_1\gamma_v)(-1+s\gamma_v-k_1\delta_v)(-\gamma_v-s\delta_v) + 27(d+k_1)(-\gamma_v-s\delta_v)^2,\\
\beta_3 &= \left(\beta_2 + \sqrt{-4\left(\frac{\beta_1}{2^{1/3}}\right)^3 + \beta_2^2}\right)^{1/3},\\
\beta_4 &= \sqrt{\frac{-2(-1+s\gamma_v-k_1\delta_v)}{3\delta_v} + \frac{(-\gamma_v-s\delta_v)^2}{4\delta_v^2} + \frac{\beta_1}{3\delta_v\beta_3} + \frac{\beta_3}{3\delta_v 2^{1/3}}},\\
\beta_5 &= \frac{-8(s+k_1\gamma_v)}{\delta_v} + \frac{4(-1+s\gamma_v-k_1\delta_v)(-\gamma_v-s\delta_v)}{\delta_v^2} - \frac{(-\gamma_v-s\delta_v)^3}{\delta_v^3},\\
\beta_6 &= \frac{-4(-1+s\gamma_v-k_1\delta_v)}{3\delta_v} + \frac{(-\gamma_v-s\delta_v)^2}{2\delta_v^2} - \frac{\beta_1}{3\delta_v\beta_3} - \frac{\beta_3}{3\delta_v 2^{1/3}},\\
\beta_7 &= \frac{\beta_5}{4\beta_4}.
\end{aligned}$$

A physically meaningful solution is obtained using the IVth of Eq. (9). Notice that the non-dimensional form of local skin friction coefficient is obtained as follows:

$$Re_x^{1/2}C_f = -g''(0) = \frac{\alpha d}{(1+\alpha\gamma_v - \delta_v\alpha^2)}.$$

4. solution of Energy Equation

Plugging similarity transformations (5) into both Eq. (3) and the temperature boundary conditions in (4), we obtain

$$\theta'' + Prg\theta' = Prg'\theta, \tag{10}$$

$$\theta(\eta) = 1 + \gamma_t\theta'(\eta) + \delta_t\theta''(\eta), \text{ at } \eta = 0 \text{ and } \theta(\eta) \to 0 \text{ as } \eta \to \infty, \tag{11}$$

where $\gamma_t = \beta_{t1}\sqrt{\frac{a}{v_f}}$ is first order thermal slip parameter, $\delta_t = \beta_{t2}\frac{a}{v_f}$ is quadratic order slip parameter and Pr is the Prandtl number.

Substituting $\xi = \frac{-B}{\alpha}e^{-\alpha\eta}$ and Eqs. (8) into equations (10) & (11); we can derive the solution for the energy equation with quadratic order thermal slip boundary condition, in terms of confluent hyper-geometric function which is given by

$$\theta(\eta) = \frac{\left(\frac{-\alpha}{B}\right)^p}{C - D\gamma_t - E\delta_t}\left(\frac{-B}{\alpha}e^{-\alpha\eta}\right)^p M\left(p-1, 1+p, \frac{-B}{\alpha}e^{-\alpha\eta}\right), \tag{12}$$

where

$$B = \frac{Pr}{\alpha(1+\gamma_v\alpha-\delta_v\alpha^2)},$$
$$C = M\left(p-1, 1+p, \frac{-B}{\alpha}\right)$$
$$D = \alpha p, M\left(p-1, 1+p, \frac{-B}{\alpha}\right) + \frac{B(p-1)}{1+p}M\left(p, 2+p, \frac{-B}{\alpha}\right),$$
$$E = (p\alpha)^2 M\left(p-1, 1+p, \frac{-B}{\alpha}\right) - \frac{Bp\alpha(p-1)}{(1+p)}M\left(p, 2+p, \frac{-B}{\alpha}\right),$$
$$-B\alpha(p-1)M\left(p, 2+p, \frac{-B}{\alpha}\right) + \frac{B^2p(p-1)}{(1+p)(2+p)}M\left(1+p, 3+p, \frac{-B}{\alpha}\right),$$
$$p = \frac{Pr}{\alpha}\left(s + \frac{1}{\alpha(1+\gamma_v\alpha-\delta_v\alpha^2)}\right).$$

Here, M is the confluent hyper-geometric function defined in [20,21]

$$M\left([c], [d], z\right) = 1 + \frac{c}{d}z + \frac{c(c+1)z^2}{d(d+1)2!} + \cdots = \sum_{i=0}^{\infty}\frac{(c)_i}{(d)_i}\frac{z^i}{i!}.$$

The non-dimensional form of reduced Nusselt number is derived as

$$Re_x^{-1/2}Nu_x = -\theta'(0),$$

where

$$\theta'(0) = \left(\frac{(\frac{-\alpha}{B})^p}{C - D\gamma_t - E\delta_t}\right)\left(\frac{-B}{\alpha}\right)^p$$
$$\left[-p\alpha M\left(p-1, 1+p, \frac{-B}{\alpha}\right) + \frac{B(p-1)}{(1+p)}M\left(p, 2+p, \frac{-B}{\alpha}\right)\right].$$

5. Results and Discussion

The derived analytical solutions are verified with numerical solutions of governing equations by Iterative Power Series method with shooting method [35]. A comparison with Turkyilmazoglu [19] has been included in Table 1 for $-g''(0)$ which gives confident on the accuracy of present results.

Table 1

γ_v	δ_v	Present Results: Analytical solution	Present Results: Numerical solution	Turkyilmazoglu [19] with Mn=0
0	-1	0.38942825653	0.3894282565	0.38942826
3	-3	0.10449186634	0.1044918663	0.10449187
5	-5	0.06420511134	0.0642051113	0.06420511

Comparison results of $-g''(0)$

Table 2 Numerical values of local skin friction coefficient and reduced Nusselt number with d=1, s=1, Pr = 0.71 and β =0.5.

Parameters	Values	$-g''(0)$ Analytical	$-g''(0)$ Numerical	$-\theta'(0)$ Analytical	$-\theta'(0)$ Numerical
σ_v and σ_t	0.4	2.000000000	2.000000000	0.4681211168	0.46812112
	0.6	2.645816798	2.64581680	0.7021552540	0.70215525
	0.8	4.373238534	4.37323853	1.0676155810	1.06761558
k_1	0.0	5.199261870	5.19926187	0.9515851852	0.95158518
	0.5	3.869356743	3.86935674	0.7016313171	0.70163131
	1.0	2.000000000	2.000000000	0.4681211168	0.46812112

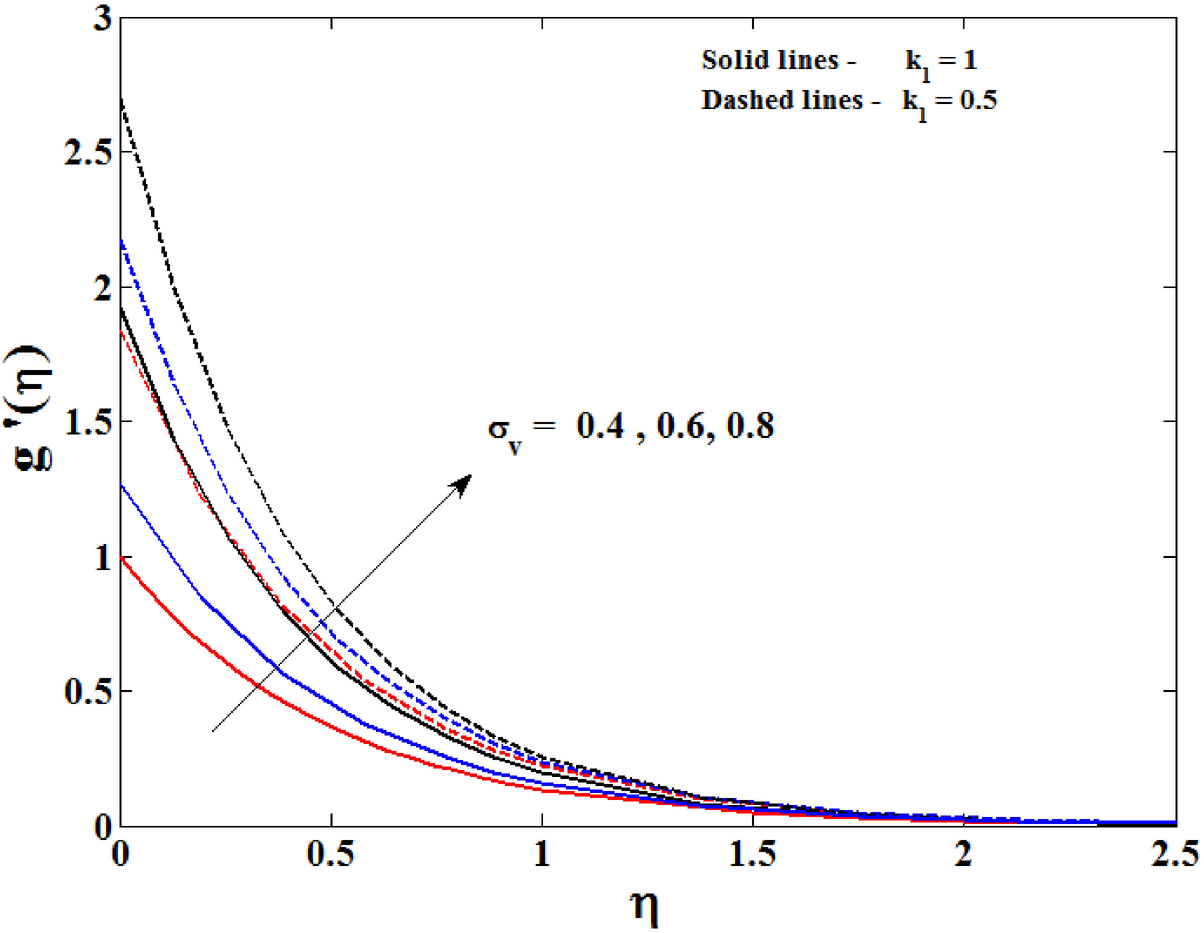

Fig. 1. Impacts of tangential momentum accommodation coefficient (σ_v) and porous medium parameter (k_1) on velocity profile with d=1 and s=1.

The results are discussed through graphical representations with following fixed values of parameters: d=1, s=1, Pr = 0.71, β =0.5 and k_1 = 0.5. The influences of porous medium parameter (k_1), tangential momentum accommodation coefficient (σ_v) and energy accommodation coefficient ((σ_t) on velocity profile, temperature profile, skin friction and reduced Nusselt number are discussed through analytical solutions. Both σ_v and σ_t are varied from 0.2 to 0.8. The first and quadratic order velocity and thermal slip parameters are calculated using σ_v and σ_t. The numerical values of $-g''(0)$ are calculated by both numerical and analytical methods and presented in Table. 2.

The behaviour of velocity profile with σ_v and k_1 is shown in Fig.1. For higher estimations of σ_v (0.4, 0.6, 0.8), an increasing behaviour in velocity profile has been noted. The tangential momentum accommodation coefficient has a significant effect on the velocity profile for a certain rage of η. This is due the fact that, σ_v is inversely proportional of the first and quadratic order velocity slip parameters. Increase in σ_v leads to decrease the velocity slip parameters. Hence the velocity profile increases. The velocity profile reduces with porous medium parameter due to Darcian resistant force in the flow region.

Features of σ_v, σ_t and k_1 on the temperature profile portrayed in Fig 2. For higher estimations of σ_v and σ_t (0.4, 0.6, 0.8), an augmentation in the temperature profile has been observed. This is because, an increase in σ_v and σ_t causes the first and quadratic order velocity and thermal slip parameter to decrease. A notable effect has been seen via k_1. The temperature profile is decreased with k_1 near the wall. Thermal boundary layer thickness is enhanced with k_1.

The variation of skin friction coefficient and reduced Nusselt number with σ_v, σ_t and k_1 is shown in Figs.

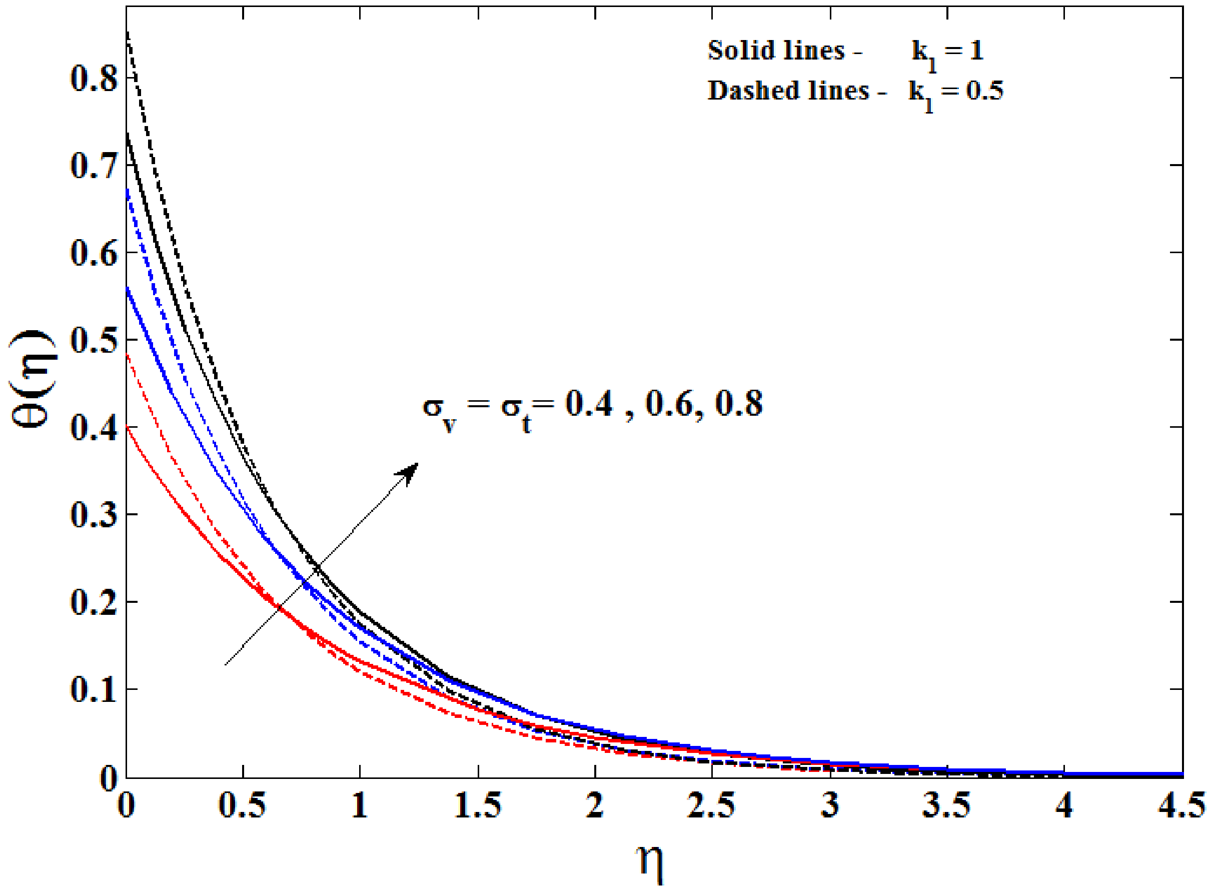

Fig. 2 . Impacts of tangential momentum accommodation coefficient (σ_v), energy accommodation coefficient (σ_t) and porous medium parameter (k_1) on temperature profile with d=1, Pr= 0.71, β=0.5 and s=1.

3 and 4 respectively. A comparison between analytical and numerical solution of present problem is also highlighted. It is observed from Fig. 3, the magnitude of $g''(0)$, increases with k_1 and reduced with σ_v. The skin friction is lower in the slip flow case compared to no-slip case. The magnitude of $-\theta'(0)$ decreases with k_1 and increases with σ_v and σ_t.

6. Conclusion

An analytical solution for the heat transport of Newtonian fluid flow over a stretching sheet with quadratatic velocity and thermal slips conditions is obtained. The obtained solutions are verified with the numerical solutions through Iterative Power Series method with shooting method. Important findings of present analytical study are listed below:

- The velocity profile increases with tangential momentum accommodation coefficient and decreases with porous medium parameter.
- The thermal boundary layer thickness enhances with tangential momentum accommodation coefficient, energy accommodation coefficient and porous medium parameter.
- The magnitude of skin friction increases via porous medium parameter and decreases via tangential momentum and energy accommodation coefficients.

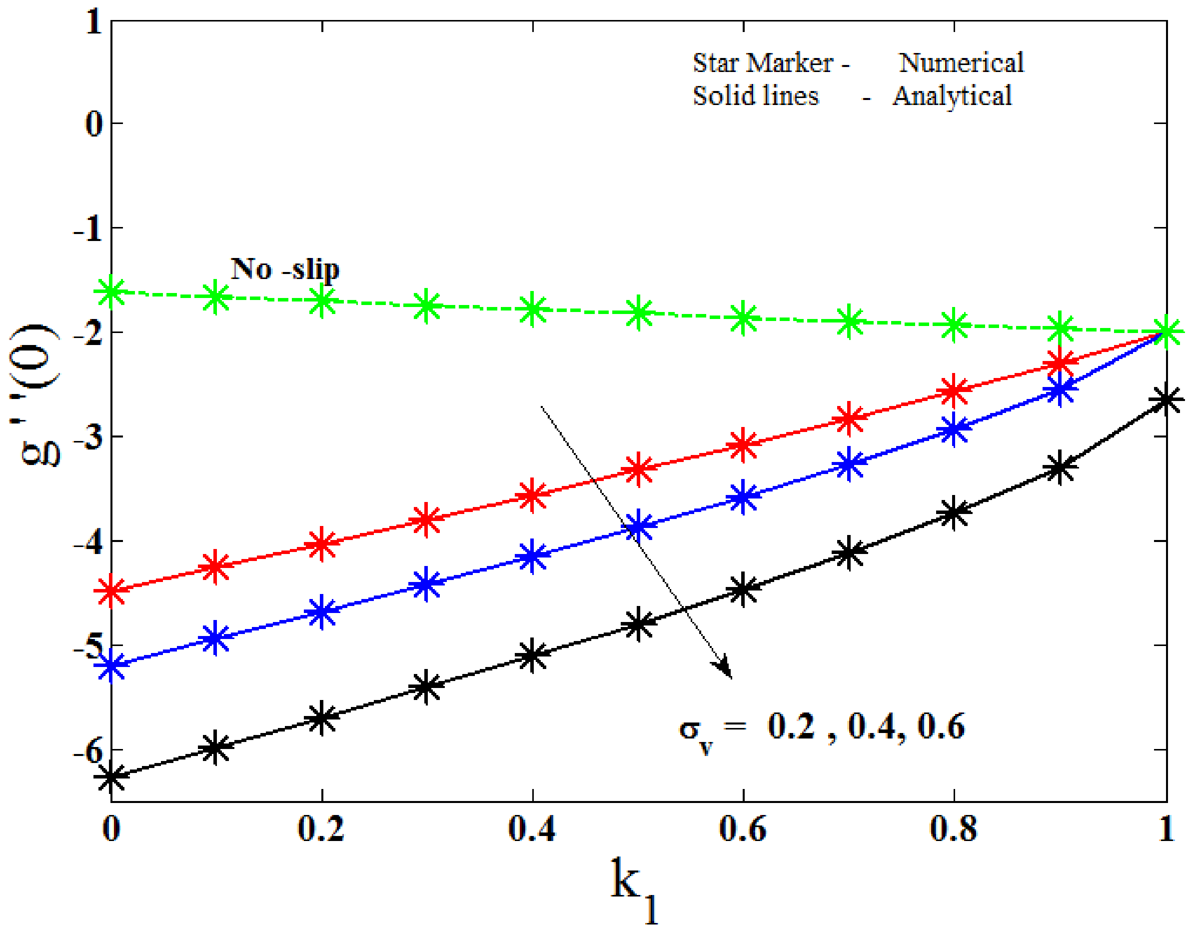

Fig. 3. Impacts of tangential momentum accommodation coefficient (σ_v) and porous medium parameter (k_1) on local skin friction coefficient with d=1 and s=1.

- Higher tangential momentum and energy accommodation coefficients increase the Nusselt number.
- The accuracy of analytical results is assured with numerical results.

Acknowledgements

Authors would like to acknowledge and express their gratitude to the United Arab Emirates University, Al Ain, UAE for providing the financial support with Grant No. 31S363-UPAR (4) 2018.

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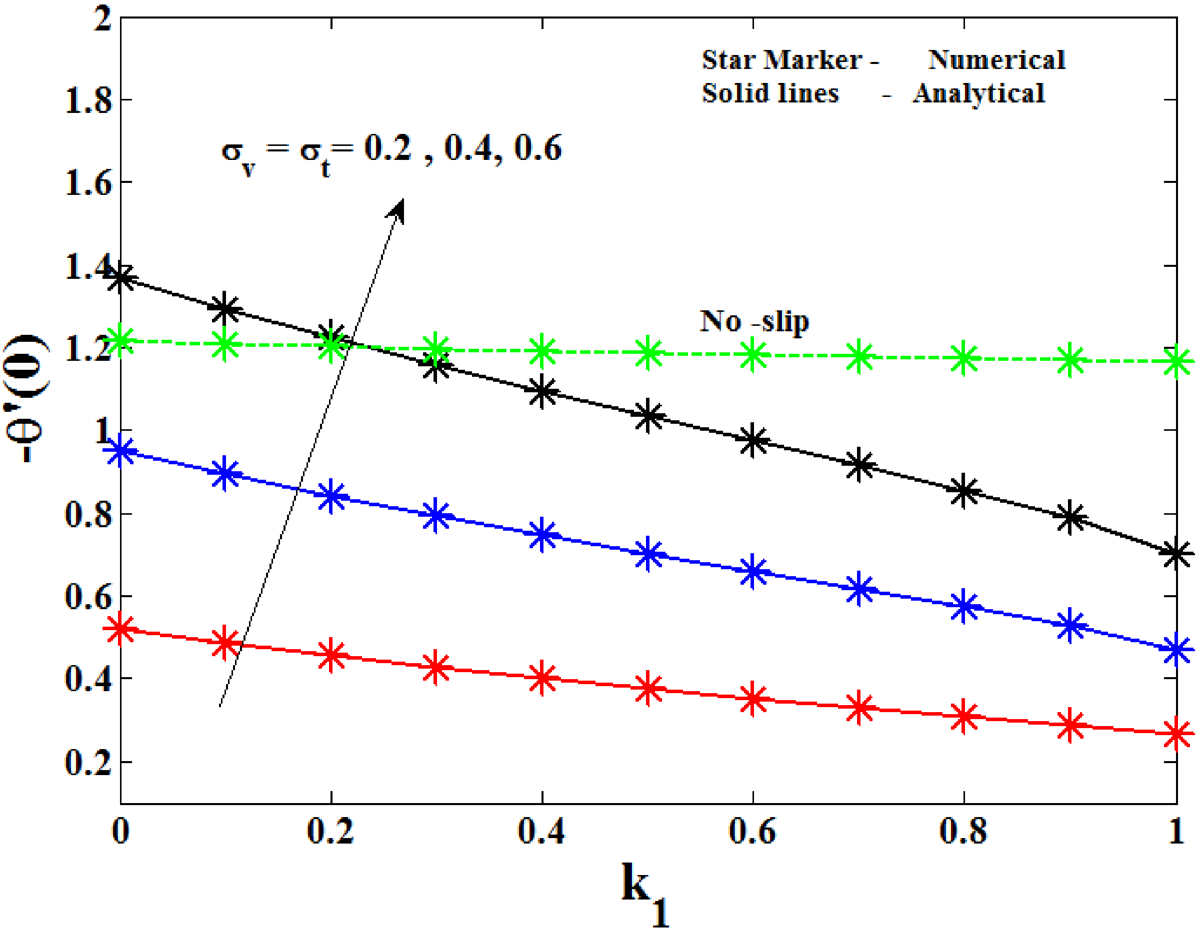

Fig. 4 . Impacts of tangential momentum accommodation coefficient (σ_v), energy accommodation coefficient (σ_t) and porous medium parameter (k_1) on reduced Nusselt number with d=1, Pr= 0.71, β=0.5 and s=1.

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