An evaluation of ten estimators for fitting two-parameter weibull function to Nigerian forest stands

Friday Nwabueze Ogana

Department of Social and Environmental Forestry, University of Ibadan, Nigeria
Corresponding author: ogana_fry@yahoo.com

Abstract
The quality fit produce by distribution function such as the Weibull depends to an extent the type of estimator used to derive its parameters. Inappropriate choice of estimator could affect management decision. Though several estimators have been developed for the Weibull function, their application to forestry have been relatively few. Therefore, this study evaluated ten estimators of the Weibull parameters using tree diameter data from five production forest plantations in Nigeria. The estimators were generalized least type I and type II, L-moment, moments, maximum likelihood, percentiles, rank correlation, least squares, U-statistics and weighted least squares. The quality of fits of the Weibull function were evaluated with Kolmogorov-Smirnov, Anderson-Darling, Cramer-von Mises, Akaike information criterion and Bayesian information criterion. Relative rank sum from the evaluation statistics of the methods was analysed using One-way analysis of variance. The results showed that weighted least square had the smallest statistics and relative rank, but not significantly different from L-moment, moments and maximum likelihood (p > 0.05). The performances of least squares, generalized least type I and type II, percentiles and U-statistics were relatively poor. Thus, either the weighted least squares, moments-based or MLE could be used for the Weibull function in the diameter distribution of forest stands in Nigeria.
Keywords: Moment-based methods; weighted least squares, maximum likelihood; Eucalyptus camaldulensis, Gmelina arborea; Tectona grandis

Introduction

The roles of diameter distribution in forest management and planning cannot be overemphasized. It provides information on product specification, value and volume production of forest stand (Gorgoso et al. 2012). It is also a useful tool for planning silvicultural treatments, determining age distribution and stand stability (Carretero and Álvarez 2013). Yield estimates are often derived from diameter distribution; and a common practice is to apply statistical function such as the Weibull to characterise the number of trees per ha into diameter classes. Class volume is then obtained by substituting the diameter class midpoint and mean height mean derived from height-diameter model into appropriate volume function (Burkhart and Tomé, 2012).

The accuracy and precision in characterising tree diameter using statistical function depends on the estimator used to derive the distribution (Zhang et al. 2003, Ogana and Gorgoso-Varela 2015). The Weibull function is the commonly used statistical function to characterise diameter distribution because of its relative flexibility, simplicity, ease of computing relative frequency of trees in diameter class and

has different parameter estimators (Rubin et al. 2006, Carretero and Álvarez 2013). Several estimators including generalized least type I and type II, L-moment, logarithmic moment, maximum likelihood estimator, method of moments, percentiles, rank correlation, least squares, U-statistics and weighted least squares, etc. have been developed for the Weibull function (Sadani et al. 2019). However, not all the methods have been evaluated in forestry, especially in Nigeria.

In Nigeria, most of the production forest plantations are predominantly of *Eucalyptus camaldulensis* Dehn, *Gmelina arborea* Roxb and *Tectona grandis* Linn. Large investments in the plantations have been made in the country to meet the demand of wood and wood products (Ogana, 2019). The diameter distributions of these stands have been described using the Weibull function fitted with maximum likelihood and percentiles estimators (e.g., Ajayi 2013, Ekpa et al. 2014, Saka 2014, Ogana et al. 2020, Ogana and Ekpa 2020). Other studies that have applied the moments, least squares, percentiles, maximum likelihood methods for the Weibull function outside Nigeria include Poudel and Cao (2013), Gorgoso-Varela and Rojo-Alboreca (2014), Sun et al. (2019), etc. Inappropriate choice of estimator to derive the diameter distribution could affect the quality of fit and overall management decision. Therefore, the aim of this study was to evaluate different estimators of the Weibull distribution to characterise tree diameters in five production forest plantations in Nigeria.

Methodology

Data

The data for this study were collected from five different production forest plantations of *Gmelina arborea*, *Tectona grandis* and *Eucalyptus camaldulensis* in Nigeria. Two plantations of *G. arborea* and *T. grandis* are situated in Omo Forest Reserve (FR), Ogun State. The reserve lies between Latitude 6°35′ – 7°05′ N and Longitude 4°10′ – 4°19′ E. The second *G. arborea* and *T. grandis* plantations are in Oluwa FR and Gambari FR, respectively. Oluwa FR is in Ondo State of Nigeria and lies between Latitude 6°55′ – 7°20′ N and Longitude 3°45′ – 4°32′ E (Onyekwelu 2001). While Gambari FR is in Oyo State of Nigeria and lies between Latitude 7°21′ – 7°55′N and Longitude 3°53′ – 3°9′E (Adedeji et al. 2015). The *E. camaldulensis* plantation is in Afaka FR situated between Latitude 10.58° – 10.68°N and Longitude 7.35° – 7.37°E of Kaduna State, Nigeria. Diameter at breast height (1.3m above the ground, dbh in cm) data of 1,052, 1,079, 1,370, 1,916 and 3,988 trees from *G. arborea* in Oluwa FR, *G. arborea* in Omo FR, *T. grandis* in Gambari FR, *T. grandis* in Omo FR and *E. camaldulensis*, respectively were available for this study. The descriptive statistics of the data are presented in Table 1.

Table 1: Descriptive statistics of the data set

Species	Mean	Max	Min	SD	N trees
G. arborea in Oluwa FR	23.0	54.5	3.0	10.4	1052
G. arborea in Omo FR	19.5	49.6	4.6	8.9	1079
T. grands in Gambari FR	19.6	39.2	5.8	6.2	1370
T. grands in Omo FR	17.9	37.9	6.0	5.3	1916
E. camaldulensis	10.5	47.4	2.0	6.3	3988
All species	15.8	54.5	2.0	8.4	9405

Two-Parameter Weibull Function

The probability density function (pdf) and cumulative distribution function (cdf) of the commonly used two-parameter Weibull function (Weibull 1951) are expressed as:

$$f(x) = \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} exp\left(-\left(\frac{x}{b}\right)^{c}\right) \qquad \text{Eq. [1]}$$

$$F(x) = 1 - exp\left(-\left(\frac{x}{b}\right)^{c}\right) \qquad \text{Eq. [2]}$$

Where: $f(x)$ = pdf; $F(x)$ = cdf; c = shape parameter ($c > 0$); b = scale parameter ($b > 0$).

Estimation methods

Ten estimation methods of the Weibull function were evaluated in this study. These include: generalized least type I (GLS1) and type II (GLS2), L-moment, maximum likelihood estimator (MLE), method of moments, percentiles, rank correlation (Rank), least squares (LS), U-statistics (U-stat) and weighted least squares (WLS). Some these estimators are presented in Appendix. Detailed information including the derivations on the various estimators of the Weibull distribution can be found in Teimouri et al. (2013) and Sadani et al. (2019). Each method was used to fit the Weibull distribution to the diameter data from the five forest plantations and for all species combined. The 'ForestFit' package (Teimouri, 2020) implemented in R (R Core Team, 2017) was used for the analysis.

Evaluation statistics

Five evaluation statistics were used to assess the ten estimation methods of the Weibull distribution. For each estimation method, the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Anderson-Darling (AD), Cramer-von Mises (CvM) and Kolmogorov-Smirnov (KS) statistics were computed. The smaller the statistics are, the better the method.

Akaike Information Criterion (AIC):

$$AIC = 2\left(\sum_{i=1}^{n}\left[ln(b) - ln(c) + (1-c)ln\left(\frac{x_i}{b}\right) + \left(\frac{x_i}{b}\right)^{c}\right]\right) + 2p \qquad Eq.\,[3]$$

Bayesian Information Criterion (BIC):

$$BIC = 2\left(\sum_{i=1}^{n}\left[ln(b) - ln(c) + (1-c)ln\left(\frac{x_i}{b}\right) + \left(\frac{x_i}{b}\right)^{c}\right]\right) + pln(n) \qquad Eq.\,[4]$$

Kolmogorov-Smirnov (KS) *statistics*:

$$KS = max\{max_{1 \le i \le n_i}[F_n(x_i) - F_0(x_j)], max_{1 \le i \le n_i}[F_o(x_j) - F_n(x_{i-1})]\} \qquad \text{Eq. [5]}$$

Anderson-Darling (AD) statistic:

$$AD = -n_i - \sum_{j=1}^{n_i}(2j-1)\left[ln\left(F_o(x_j)\right) + ln\left(1 - F_n(x_{i-1})\right)\right]\Big/n_i \qquad Eq.\,[6]$$

Cramer-von Mises (W^2) statistic:

$$W^2 = \sum_{i=1}^{n}\left\{\hat{F}(x_i) - \frac{(i - 0.5)}{n}\right\}^2 + \frac{1}{12n} \qquad Eq.\,[7]$$

Where $F(x_i)$ is the observed cumulative frequency distribution for x_i (i ranged from 1 to n); $F_0(x_i)$ is the theoretical cumulative frequency distribution; b and c are the scale and shape parameters of the Weibull distribution; p is the number of parameter; ln is the natural logarithm

Ranking of Methods

Relative rank introduced by Poudel and Cao (2013) was used in this study. It is given by:

$$R_i = 1 + \frac{(m-1)(S_i - S_{min})}{S_{max} - S_{min}} \qquad \text{Eq. [8]}$$

where R_i = relative rank of method i (i = 1, 2, …, m); m = number of methods evaluated (10 estimation methods), S_i = evaluation statistics value of method i; S_{max} and S_{min} = maximum and minimum values of S_i, respectively. The relative rank is real number between 1 (best) and 10 (worst). For each estimation method, the relative ranks were summed across the five evaluation statistics, analysed and plotted. One-way analysis of variance (ANOVA) was then used to analysed the relative rank sum of the ten methods at 5% level of significant. Methods that were significantly different were separated with Duncan multiple range test (DMRT).

Results

The estimates of the parameters of the Weibull distribution from the ten fitting methods are presented in Table 2. The estimated Weibull shape and scale parameters from the ten methods ranged from 2.1418 to 2.4344 and 25.8840 to 26.50, respectively in the *G. arborea* stand in Oluwa FR. For *G. arborea* stand in Omo FR, the shape and scale parameters, respectively, ranged from 1.9808 to 2.5347 and 21.8721 to 22.3649. The values of the parameters ranged from 3.4436 to 3.7408 and 21.6373 to 21.8576, and 3.5456 to 4.0992 and 19.40 to 20.0427, respectively in the *T. grandis* stands in Gambari FR and Omo FR, respectively. In the case of the *E. camaldulensis* stand, the shape and scale parameters, respectively, ranged from 1.6306 to 2.2042 and 11.4237 to 12.0086. For the pooled data i.e., all species, the estimates of the shape and scale parameters ranged from 1.8245 to 2.1318 and 17.6678 to 17.8696, respectively. There was lack of fit for the Weibull distribution fitted with weighted least squares (WLS) to the pooled data.

The evaluation statistics of the ten methods for fitting the Weibull distribution by species and all species showed that the WLS and L-moment had in most cases smallest values (Table 3). However, L-moment, moment and Rank correlation (Rank) had the best evaluation statistics for all species combined. Larger AD, CvM and KS were observed in the least squares (LS), generalized least squares both type 1 (GLS1) and type 2 (GLS2) for most of the stands.

The plot of the relative rank sum (mean ± standard errors) for the ten estimation methods are presented in Figure 1. A boundary line was used to demarcate the methods with relative rank sum < 20 from those > 20. The L-moments, MLE, moment and WLS methods were within the lower region of 0 – 20. The method of WLS had the smallest value. Though Rank correlation had average value that was < 20, its upper limit was slightly beyond the boundary line. The GLS1, GLS2, LS, Percentiles and U-statistics (U-stat) methods were those above the boundary line. Further result from the analysis of variance (ANOVA) showed a significant difference in the relative rank sum of the ten methods. Methods within the same region were not significantly different; whereas methods between regions were significant (Figure 1). That is, no significant difference between L-moments, MLE, moment, Rank and WLS methods; but they differed significantly from those in the upper region, i.e., GLS1, GLS2, LS, Percentiles and U-stat methods.

The graph of the observed and fitted Weibull function with three best methods in the individual stand and all species combined are presented in Figure 2a to f. In all cases, the fitted Weibull function approximated the observed diameter distribution of the stands. Only in diameter class of 10 cm in the *E. camaldulensis* stand that the Weibull function underestimated the relative frequencies of trees.

Table 2: Estimated parameters from the fitting methods in the different forest stands

Methods	*G. arborea* in Oluwa FR		*G. arborea* in Omo FR		*T. grandis* in Gambari FR		*T. grandis* in Omo FR		*E. camaldulensis*		All species	
	Shape	**Scale**	**Shape**	**Scale**	**Shape**	**Scale**	**Shape**	**Scale**	**Shape**	**Scale**	**Shape**	**Scale**
Greg1	2.4344	25.9581	2.5347	21.9922	3.6167	21.8199	3.9603	19.9357	2.1847	12.0081	2.1318	17.8682
Greg2	2.4005	25.9667	2.5009	21.9986	3.5805	21.8227	3.9341	19.9368	2.1777	12.0086	2.1269	17.8696
L-moment	2.3308	25.9835	2.3064	21.9751	3.5413	21.7553	3.7872	19.8221	1.8560	11.8403	1.9562	17.7859
MLE	2.3957	26.0239	2.3729	22.0366	3.4436	21.7791	3.5456	19.8623	1.8145	11.9133	1.9813	17.8520
Moment	2.3640	25.9781	2.3320	21.9719	3.5128	21.7647	3.7346	19.8375	1.7267	11.7973	1.9472	17.7839
Percentiles	2.1418	26.5000	1.9808	22.3649	3.7408	21.6373	3.8542	19.4000	1.6306	11.4592	1.8245	17.7000
Rank	2.3294	25.8967	2.3051	21.9260	3.5398	21.8576	3.7860	20.0427	1.8557	11.9879	1.9561	17.7998
LS	2.4318	25.8840	2.4907	21.8721	3.6816	21.6825	4.0992	19.6996	2.1836	11.7236	2.1278	17.6678
U-stat	2.3792	26.0340	2.4373	21.9945	3.6348	21.7324	4.0082	19.7663	2.2042	11.6967	2.0771	17.7867
WLS	2.2765	26.0916	2.2185	21.9719	3.5336	21.6450	3.6885	19.6116	1.8468	11.4237		

Table 3: Evaluation statistics of the different methods in the forest stands

| Methods	G. arborea in Oluwa FR					G. arborea in Omo FR				
	AIC	BIC	AD	CvM	KS	AIC	BIC	AD	CvM	KS
GLS1	7839	7849	2.6348	0.4243	0.0405	7689	7699	9.1294	1.3084	0.0738
GLS2	7838	7848	2.1462	0.3403	0.0382	7686	7696	8.0777	1.1563	0.0703
L-moment	7839	7849	1.5073	0.2088	0.0336	7681	7691	4.3173	0.5020	0.0480
MLE	7838	7848	2.0551	0.3267	0.0362	7680	7690	5.1622	0.6833	0.0570
Moment	7838	7848	1.7483	0.2642	0.0357	7680	7690	4.5788	0.5622	0.0509
Percentiles	7864	7874	2.7141	0.1839	0.0319	7741	7751	7.9843	0.5831	0.0642
Rank	7839	7849	1.5849	0.2217	0.0361	7681	7691	4.3238	0.5008	0.0468
LS	7839	7849	2.6767	0.4258	0.0426	7686	7696	7.8353	1.0976	0.0665
U-stat	7838	7848	1.8668	0.2908	0.0349	7681	7691	6.4073	0.8962	0.0633
WLS	7843	7853	1.3275	0.1386	0.0272	7688	7698	3.9870	0.3621	0.0485
	T. grandis in Gambari FR					T. grandis in Omo FR				
GLS1	8882	8892	2.2988	0.3040	0.0333	11943	11954	12.6178	2.1889	0.0697
GLS2	8879	8890	2.1600	0.2933	0.0320	11936	11947	12.0843	2.0950	0.0683
L-moment	8878	8888	1.7886	0.2166	0.0288	11912	11923	8.1486	1.1846	0.0554
MLE	8875	8886	2.0496	0.2794	0.0348	11892	11903	8.6319	1.1061	0.0498
Moment	8877	8887	1.8190	0.2307	0.0307	11904	11915	7.9756	1.1456	0.0546
Percentiles	8898	8909	2.7256	0.2082	0.0275	11955	11966	8.6557	0.6535	0.0388
Rank	8877	8888	2.2677	0.3333	0.0339	11908	11919	12.0068	2.1737	0.0698
LS	8890	8900	2.2199	0.1939	0.0279	12001	12012	12.8293	1.7148	0.0627
U-stat	8884	8895	2.0201	0.2111	0.0290	11965	11976	11.0646	1.6048	0.0621
WLS	8879	8889	1.6047	0.1567	0.0284	11907	11918	6.5598	0.6024	0.0383
	E. camaldulensis					All species				
GLS1	25147	25160	113.1166	20.0076	0.1395	65458	65472	46.9262	5.6606	0.0575
GLS2	25134	25147	111.5718	19.7491	0.1387	65451	65465	45.7733	5.5162	0.0569
L-moment	24818	24830	64.8736	9.6506	0.0923	65362	65376	22.0706	1.9715	0.0341
MLE	24812	24824	66.4468	9.8241	0.0909	65359	65373	23.8078	2.3434	0.0380
Moment	24830	24843	67.1428	8.8588	0.1009	65364	65378	21.8793	1.9121	0.0348
Percentiles	24895	24907	77.3515	9.3111	0.1221	65466	65480	30.3126	2.5521	0.0459
Rank	24815	24828	69.3386	10.9219	0.0991	65362	65376	22.1679	1.9914	0.0340
LS	25202	25214	100.8920	16.2469	0.1257	65473	65487	44.1083	4.8872	0.0533
U-stat	25252	25265	104.6804	16.6574	0.1267	65403	65418	34.3245	3.8464	0.0492
WLS	24845	24857	61.0646	7.5250	0.0918	nf	nf	nf	nf	nf

nf = no fit
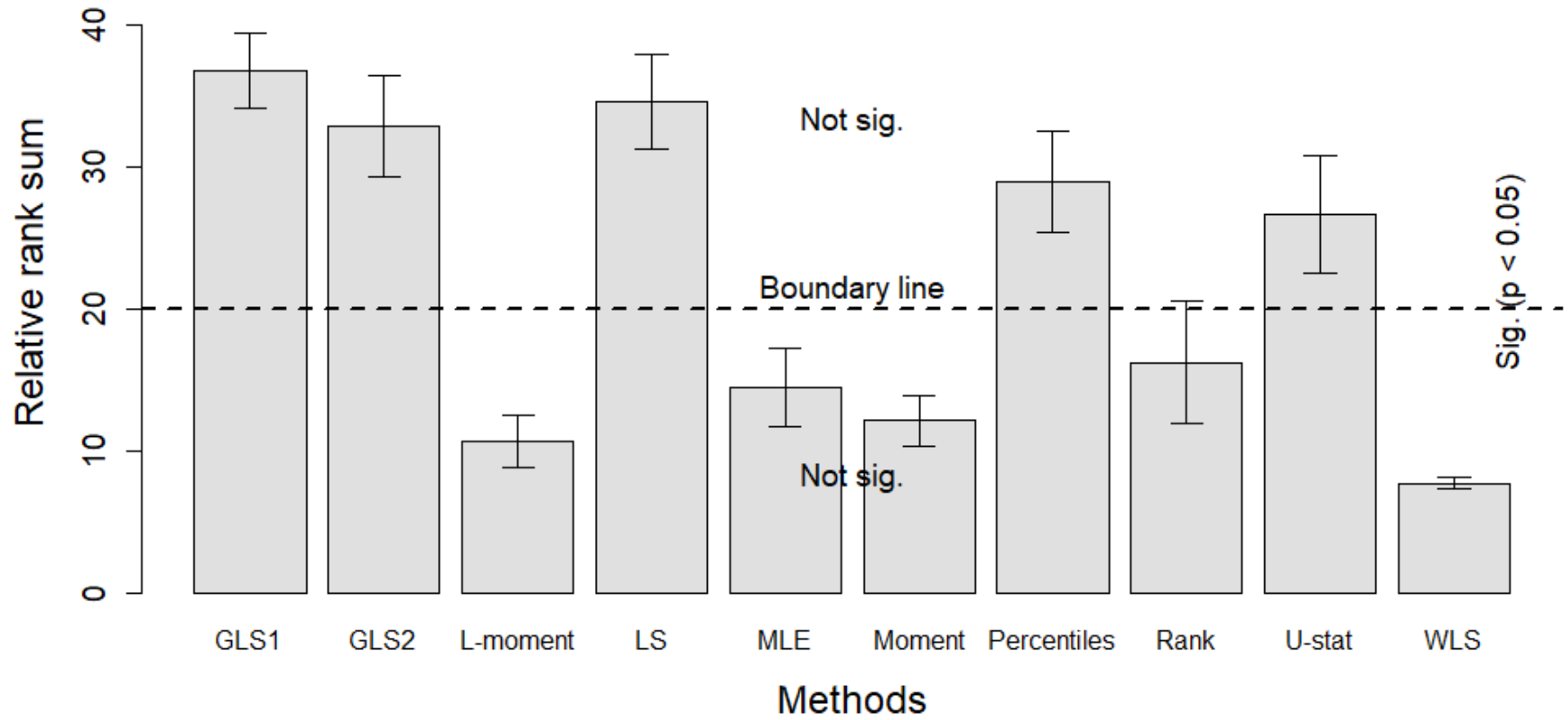
Figure 1: Bar graph of relative rank sum of ten Weibull parameter estimation methods. Methods in the same region are not significant, while between regions are significant.
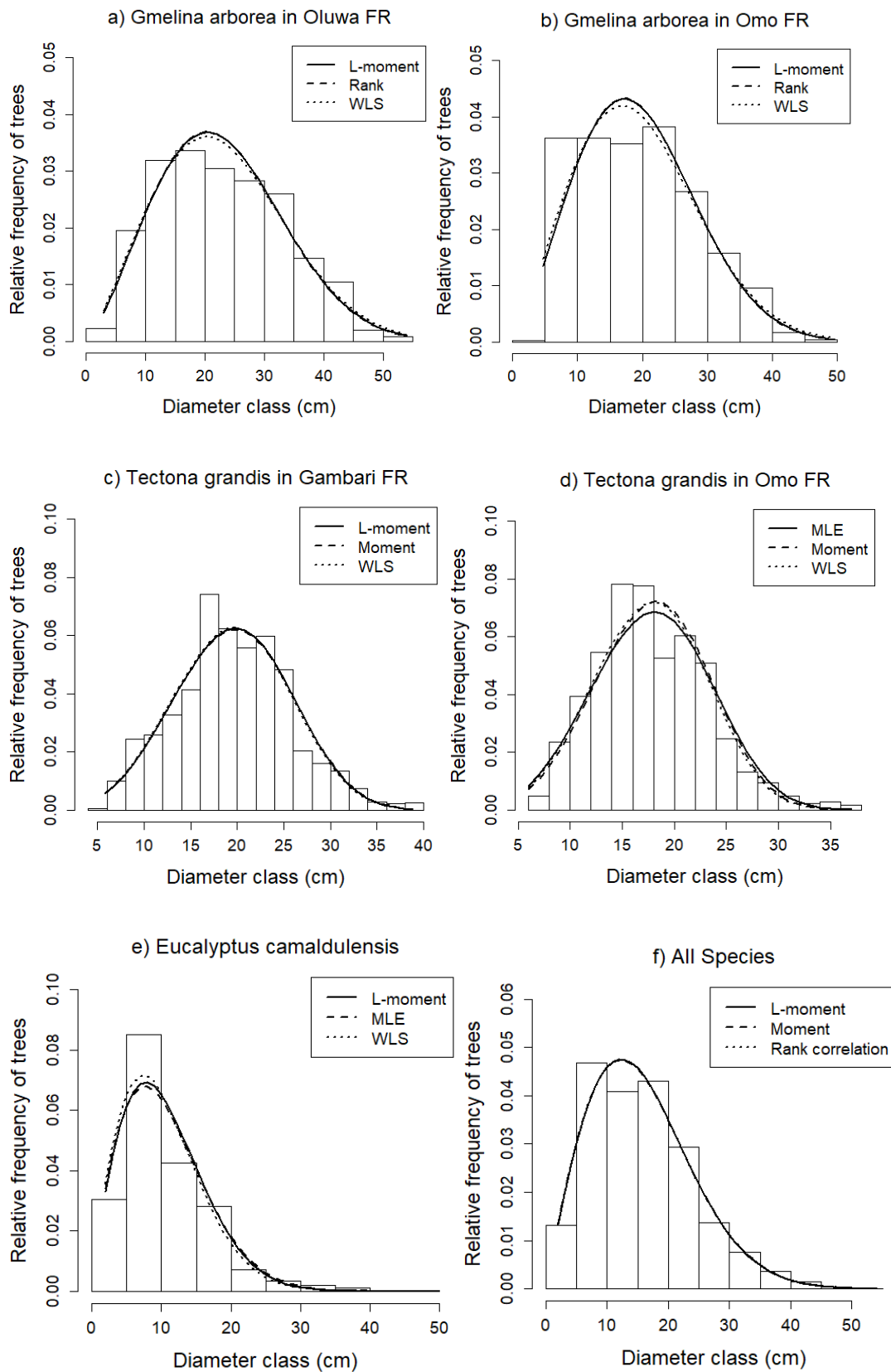
Figure 2: Observed and fitted Weibull function with three best methods across the stands.

Discussion

Ten methods have been evaluated for fitting the two-parameter Weibull function to five forest stands in Nigeria. There was little variation in the estimates of the shape and scale parameters from the ten methods. However, the estimate of the shape parameter from Percentiles tend to have lower value for relatively skewed stands e.g., *G. arborea* in Omo FR, *E. camaldulensis* and for all species combine. Much of the variabilities in relative rank sum of the ten methods came from these skewed stands. Whereas only little variations exist in the methods for stands with gaussian shape (symmetric) such as the *T. grandis* stands in Gambari FR and Omo FR. This shows that some of the estimators (e.g., GLS1, GLS2, LS, Percentiles and U-stat) of Weibull parameters are most appropriate for stands with gaussian structure (symmetric). Other methods such as L-moment, MLE, Moment, Rank and WLS estimators can be used to fit Weibull function to both symmetric and asymmetric stands.

Heavily skewed or asymmetric structure for plantations could be due to thinning, poor forest management, illegal exploitation and other forms of disturbances. The *E. camaldulensis* and *G. arborea* stands have been previously reported to have suffered from severe disturbance – both anthropogenic and wind damage (Ogana et al. 2017, 2018). Though silvicultural practices such as selection cutting can be used to convert even-aged stand to uneven-aged (reverse J-shaped), it is not a common practice in Nigeria.

Among the suitable methods identified for fitting the Weibull function, only the MLE and moments have been frequently used to model diameter distribution of forest stands especially in Nigeria (e.g. Ajayi 2013, Ige et al. 2013, Ogana et al. 2015, Ogana and Gorgoso-Varela 2015). In Spain, Carretero and Álvarez (2013) found MLE and moment to be less efficient compared to LS for fitting two-parameter Weibull to Cork oak stands. Gorgoso et al. (2007) reported similar result for beech stand in northwest Spain. Recently, Gorgoso-Varela et al. (2020) observed smallest KS value for MLE in three species - *E. globulus* Labill, *Pinus radiata* D. Don. (temperate forest) and *G. arborea* (tropical forest). Though numerically the L-moment and WLS methods had the best results in all the stands, their relative ranks are not significantly different from MLE and moments. The application of L-moment and WLS to fit the Weibull function has been limited in forestry.

One important factor to consider in the selection of an estimator is relative simplicity (i.e., ease of estimation) without compromising the quality of fits. The estimation procedures of the five suitable methods (estimators below the boundary line) vary in complexity. The moment-based estimators are handier compared to MLE and WLS. In consequence, when complex estimators do not outperform a simpler alternative, the simpler method should be selected (Gorgoso-Varela et al. 2019, 2020).

Conclusion

This study has evaluated the performance of ten estimators for fitting the Weibull function to some production stands in Nigeria. The quality of fit produced by the Weibull function varies with the different estimators. While some estimators such as the least squared, generalized least squared type 1 and 2, percentiles and the U-statistics are more appropriate for stands with gaussian structure; other estimators - the moment-based, MLE, rank and weighted least square can be used to fit Weibull function to stands with either symmetric or asymmetric structure.

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