Eurasian Journal of Science Engineering and Technology

Research

ON THE STABILITY OF A PARTIALLY IONIZED PLASMA

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ABSTRACT

The Rayleigh-Taylor instability of an infinitely conducting plasma of variable density in the presence of a horizontal magnetic field is considered when the effects of finite ion Larmor radius (FLR) and collisions with neutral atoms simultaneously present. Here we considered the perturbations propagating along the ambient magnetic field. It is observed that, real part of n is negative, where n is the growth rate of disturbance, so that instability does not arise in the form of increasing amplitude, i.e. overstability. To obtain an approximate solution of the problem, a variational principle is used. The case of two semi-infinitely extending plasmas of constant densities separated by a horizontal interface is also considered, where it is found that the system is stable (for some wave numbers) for potentially stable configuration and unstable (for other wave numbers) for potentially unstable configuration even if there are collisions with dust particles. Also it is observed that the criteria determining stability and instability are independent of FLR effects.

Keywords: Rayleigh-Taylor instability, Conducting plasma, Horizontal magnetic field

1. INTRODUCTION

The instability of the plane interface separating two fluids when one is accelerated towards the other or when one is superposed over the other has been studied by several authors and Chandrasekhar [1] has given a detailed account of these investigations together with the possible extensions in various domains of interest. The stabilizing influence of finite Larmor radius effects has individually been shown on thermal instability, thermosolutal instability, gravitational instability and Rayleigh-Taylor instability, by several authors [2-6].

Quite often the plasma is not fully ionized and is, instead, partially ionized. Partially ionized plasma represents a state which often exists in the Universe and there are several situations when the interaction between the ionized and neutral gas components becomes important in cosmic physics. The study of partially ionized plasmas has become a hot topic because solar structures such as spicules, prominences, as well as layers of the solar atmosphere (photosphere and chromosphere), are made of partially ionized plasmas. On the other hand, considerable developments have taken place in the study of partially ionized plasmas applied to the physics of the interstellar medium, molecular clouds, the formation of protostellar discs, planetary magnetospheres/ionospheres, exoplanets atmospheres, etc. For instance, molecular clouds are mainly made up of neutral material which does not interact with magnetic fields. However, neutrals are not the only constituent of molecular clouds since there are also several types of charged species which do interact with magnetic fields. Furthermore, the charged fraction also interacts with the neutral material through collisions. These multiple interactions produce many different physical effects which may have a strong influence on star formation and molecular cloud turbulence. A further example can be found in the formation of dense cores in molecular clouds induced by MHD waves. Because of the low ionization fraction, neutrals and charged particles are weakly coupled and ambipolar diffusion plays an important role in the formation process. Even in the primeval universe, during the recombination era, when the plasma, from which all the matter of the universe was formed, evolved from fully ionized to neutral, it went through a phase of partial ionization. Partially ionized plasmas introduce physical effects which are not considered in fully ionized plasmas, for instance, Cowling's resistivity, isotropic thermal conduction by neutrals, heating due to ion/neutral friction, heat transfer due to collisions, charge exchange, ionization energy, etc., which are crucial to fully understand the behaviour of astrophysical plasmas in different environments. Stromgren [7] has reported that ionized hydrogen is limited to certain rather sharply bounded regions in space surrounding, for example, O-type stars and clusters of such stars and that the gas outside these regions is essentially non-ionized. Other examples of the existence of such situations are given by Alfven's [8] theory on the origin of the planetary system, in which a high ionization rate is suggested to appear from collisions between a plasma and a neutral-gas cloud and by the absorption of plasma waves due to ion-neutral collisions such as in the solar photosphere and chromosphere and in cool interstellar clouds [9, 10]. Lehnert [11] has found that both ion viscosity and neutral gas friction have a stabilizing influence on cosmical plasma interacting with a neutral gas. According to Hans [12] and Bhatia [13], the medium may be idealized as a composite mixture of a hydromagnetic (ionized) component and a neutral component,

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Recieved: 18.06.2020 Accepted: 07.09.2020

the two interacting through mutual collisional (frictional) effects. A stabilizing effect of collisionals on Rayleigh-Taylor configuration has been shown by [12] and [13]. But the collisional effects are found to be destabilizing for a sufficiently large collisional frequency on Kelvin-Helmholtz configuration by Rao and Kalra [14] and [12]. Chhajlani et. al [15] considered the hydromagnetic Rayleigh-Taylor instability of a composite medium in the presence of suspended particles for an exponentially varying density distribution. The Rayleigh-Taylor instability of a partially ionized plasma in a porous medium in the presence of magnetic field perpendicular to gravity has been considered by Sharma and Sunil [16]. The gravitational instability of a rotating Walters B' viscoelastic partially ionized plasma permeated by an oblique magnetic field in the presence of the effects of Hall currents, electrical resistivity and ion viscosity has been considered by El-Sayed and Mohamed [17]. Hoshoudy [18] has investigated the Rayleigh-Taylor instability in stratified plasma in the presence of combined effect of horizontal and vertical magnetic field. Sharma et. al [19] have investigated the effect of surface tension on hydromagnetic Rayleigh-Taylor instability of two incompressible superimposed fluids in a medium with suspended dust particles in a uniform horizontal magnetic field.

In the present work, we study the simultaneous effects of ion Larmor radius and collisions with neutral atoms on the stability of well-known Rayleigh-Taylor configuration in hydromagnetics. We regard the medium as being a mixture of an infinitely conducting component a neutral component interacting through mutual collisions. We make the assumptions that the individual components by themselves, behave like continuum plasmas and that the effects on the neutral component resulting from magnetic field, pressure and gravity are negligible. The case of a uniform horizontal field and longitudinal perturbations is considered. Next a variational principle is developed to obtain the approximate solutions.

2. FORMULATION OF THE PROBLEM

Here we consider two inviscid, homogeneous, semi-infinitely extending plasmas separated by a plane interface at $z = 0$, each region being permeated with a neutral component of the same density. Initially the configuration is at rest. We give a small disturbance to the system. The linearized perturbation equations for the mixture of the hydromagnetic plasma and a neutral gas moving together in a uniform horizontal magnetic field $\vec{H}(H, 0,0)$ and downward gravitational field $\vec{g}(0, 0, -g)$ are

$$\rho \frac{\partial \vec{q}}{\partial t} = -\nabla \delta \overleftrightarrow{P} + \frac{1}{4\pi}\left(\nabla \times \vec{h}\right) \times \vec{H} + \vec{g}(\delta\rho) + \rho_d \nu_c (\overrightarrow{q_d} - \vec{q}), \tag{1}$$

$$\frac{\partial \overrightarrow{q_d}}{\partial t} = -\nu_c (\overrightarrow{q_d} - \vec{q}), \tag{2}$$

$$\frac{\partial}{\partial t}(\delta\rho) = (\vec{q}.\nabla)\rho, \tag{3}$$

$$\frac{\partial \vec{h}}{\partial t} = \left(\vec{H}.\nabla\right)\vec{q}, \tag{4}$$

$$\nabla.\vec{q} = 0 \qquad \text{and} \qquad \nabla.\vec{h} = 0, \tag{5}$$

where ρ and ρ_d are the unperturbed densities for the hydromagnetics and the neutral component, respectively. ν_c denotes the collisional frequency between the two components and $\overleftrightarrow{P}$ denotes the plasma pressure rendered tensorial due to finite ion Larmor radius effect. Here $\delta\rho, \delta\overleftrightarrow{P}, \vec{q}(u, v, w), \overrightarrow{q_d}(l, r, s), \vec{h}\left(h_x, h_y, h_z\right)$ denote, respectively, the perturbations in density ρ, stress tensor $\overleftrightarrow{P}$, hydromagnetic plasma velocity (initially zero), neutral component velocity (initially zero) and magnetic field $\vec{H}$. Magnetic permeability of the medium is assumed to be unity.

For the magnetic field along x-axis, $\delta\overleftrightarrow{P}$ taking into account the FLR effects has the following components

$$\left.\begin{aligned} P_{xx} = p, \quad P_{xy} = P_{yx} = -2\rho\nu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right), \\ P_{xz} = P_{zx} = 2\rho\nu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right), \\ P_{yz} = P_{zy} = \rho\nu\left(\frac{\partial v}{\partial y}-\frac{\partial w}{\partial z}\right), \\ P_{yy} = p-\rho\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \\ P_{zz} = p+\rho\nu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right), \end{aligned}\right\} \tag{6}$$

where p is the scalar part of the pressure and $\rho\nu = \frac{NT}{4\omega_H}$; ω_H is the ion-gyration frequency, while N and T denote, respectively, the number density and temperature of ions and K^* is the Boltzmann constant.

Analyzing the disturbances in terms of longitudinal modes, we seek the solutions of equations (1) - (5) in which $x-t$ dependence is given by

$$exp(ikz+nt), \tag{7}$$

where k denotes the wave number of disturbance and n is the growth rate of disturbance.

Eliminating q_d between equations (1) and (2), and using (6) and (7), equations (1)-(5) can be written as

$$\left[n\rho+\frac{\rho_d\nu_c}{n+\nu_c}\right]u = -ik\delta p - 2ik\nu D(\rho\nu), \tag{8}$$

$$\left[n\rho+\frac{\rho_d\nu_c}{n+\nu_c}\right]v = -2\rho\nu(D^2+k^2)w + \nu D(\rho Dw) + \frac{ikHh_y}{4\pi}, \tag{9}$$

$$\left[n\rho+\frac{\rho_d\nu_c}{n+\nu_c}\right]w = -D(\delta p) + 2\rho\nu k^2 v - \nu D(\rho Dv) + \frac{gw}{n}(DP) + \frac{H}{4\pi}(ikh_z - Dh_x), \tag{10}$$

$$n\delta\rho = -w(D\rho), \tag{11}$$

$$n\vec{h} = ikH\vec{q}, \tag{12}$$

$$iku + Dw = 0, \tag{13}$$

and

$$ikh_z + Dh_x = 0, \tag{14}$$

Where

$$D = \frac{d}{dz}.$$

If we eliminate δp from equations (8) and (10), and use equations (11)-(14), we obtain the following pair of equations in w and v

$$\begin{aligned} n^2\big(\rho k^2 w - D(\rho Dw)\big) - gk^2(D\rho)w - \frac{H^2k^2}{4\pi}(D^2-k^2)w - \nu nk^2[2(D^2+k^2)(\rho\nu) - D(\rho Dv)] \\ + \frac{\nu_c}{n+\nu_c}n^2[\rho_d k^2 w - D(\rho_d Dw)] = 0, \end{aligned} \tag{15}$$

and

$$\left[n\rho+\frac{\rho_d \nu_c}{n+\nu_c}n+\frac{H^2k^2}{4\pi n}\right]v=-\nu[2\rho(D^2+k^2)w-D(\rho Dw)]. \tag{16}$$

Boundary conditions

On a boundary, vertical motion is not possible, thus

$$w=0 \tag{17}$$

on a boundary free or rigid.

If the plasma is bounded by two rigid boundaries which are both ideally conducting, no disturbance within it can change the electromagnetic quantities outside. This merely leads to the boundary condition (17). A boundary condition on v can be prescribed by preluding the presence of surface charge or surface current at the rigid boundaries which are perfectly conducting. Thus we choose

$$v=0, \tag{18}$$

at a surface bounded by an ideal conduction.

If the plasma is confined between two free boundaries, the tangential stresses

$$P_{xx}=2\rho\nu ikv+\frac{ikH^2w}{4\pi n}$$

and $P_{yz}=-\rho\nu Dw$ vanish. Hence

$$v=Dw=0, \tag{19}$$

at a free boundary. Should there be discontinuities in the density as in the case of two superposed layers of different densities, we require the continuity of the vertical component of velocity, tangential stresses and pressure at interface. Thus

$$w,\ \rho Dw,\ \rho v,\ \rho_d Dw,\ \rho_d v \tag{20}$$

and the total pressure must be continuous as at the interface.

3. DISCUSSION

Theorem I: A necessary and sufficient condition for δn^2 to be zero to the first order for all small arbitrary variations δw and δv (connected by equation (38)) in w and v which is compatible with the boundary conditions is that w and v should be the solutions of the eigenvalue problem governed by equations (15) and (16).

Proof: Let n_i and n_j denote the two characteristic values, and let the solutions belonging to these characteristics values be distinguished by the subscripts i and j. Multiplying equation (15) for i by w_j and integrating with respect to z over the whole vertical extent of the plasma (denoted by $\int_L$), we obtain with the help of equation (16) and boundary conditions,

$$n_i^2\int_L \rho\left(w_iw_j+\frac{1}{k^2}Dw_iDw_j\right)dz+\frac{\nu_c}{n_i+\nu_c}n_i^2\int_L \rho_d\left(w_iw_j+\frac{1}{k^2}Dw_iDw_j\right)-g\int_L (D\rho)w_iw_jdz$$
$$+\frac{H^2k^2}{4\pi}\int_L\left(w_iw_j+\frac{1}{k^2}Dw_iDw_j\right)dz+n_in_j\int_L \rho v_ivdz+\frac{\nu_cn_in_j}{n_j+\nu_c}\int_L \rho_dv_iv_jdz+\frac{H^2k^2n_i}{4\pi n_j}\int_L v_iv_jdz$$
$$=0. \tag{21}$$

Taking $i=j$ and suppressing the subscripts, we obtain the following variational formulation of the problem

$$n^2[I_1+I_4+I_6+I_7]-gI_2+I_3+I_5=0, \tag{22}$$

where

$$I_1 = \int_L \rho \left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \tag{23}$$

$$I_2 = \int_L (D\rho) w^2 dz, \tag{24}$$

$$I_3 = \frac{H^2k^2}{4\pi} \int_L \left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \tag{25}$$

$$I_4 = \int_L \rho v^2 dz, \tag{26}$$

$$I_5 = \frac{H^2k^2}{4\pi} \int_L v^2 dz, \tag{27}$$

$$I_6 = \frac{\nu_c}{n+\nu_c} \int_L \rho_d \left[w^2 + \frac{1}{k^2}(Dw)^2\right] dz, \tag{28}$$

$$I_7 = \frac{\nu_c}{n+\nu_c} \int_L \rho_d v^2 dz. \tag{29}$$

Consider a change δn^2 on n^2 of an arbitrary variation δw and δv in w and , respectively to satisfy the boundary conditions (17) and (18) of the eigen-value problem, we have to the first order, from 22

$$\delta n^2(I_1 + I_4 + I_6 + I_7) + n^2(\delta I_1 + \delta I_4 + \delta I_6 + \delta I_7) - g\delta I_2 + \delta I_3 + \delta I_5 = 0, \tag{30}$$

where $\delta I_s (s = 1\ to\ 7)$ are the corresponding variations in $I_s (s = 1\ to\ 7)$. After one or more integrations by parts, we find that these latter variations are given by

$$\frac{1}{2}\delta I_1 = \int_L \left[\rho w - \frac{1}{k^2} D(\rho Dw)\right] \delta w dz, \tag{31}$$

$$\frac{1}{2}\delta I_2 = \int_L (D\rho) w \delta w dz, \tag{32}$$

$$\frac{1}{2}\delta I_3 = \frac{H^2k^2}{4\pi} \int_L \left(w - \frac{1}{k^2} D^2 w\right) \delta w dz, \tag{33}$$

$$\frac{1}{2}\delta I_4 = \int_L \rho v \delta v dz, \tag{34}$$

$$\frac{1}{2}\delta I_5 = \frac{H^2k^2}{4\pi} \int_L v \delta v\, dz, \tag{35}$$

$$\frac{1}{2}\delta I_6 = \frac{\nu_c}{n+\nu_c} \int_L \left[\rho_d w - \frac{1}{k^2} D(\rho_d Dw)\right] \delta w\, dz \tag{36}$$

and

$$\frac{1}{2}\delta I_7 = \frac{\nu_c}{n+\nu_c} \int_L \rho_d \delta v\, dz. \tag{37}$$

Furthermore, δw and δv are connected by the relation

$$\delta n \left[\rho - \frac{H^2k^2}{4\pi n^2} + \frac{\rho_d \nu_c^2}{(n+\nu_c)^2}\right] v + n\left[\rho + \frac{H^2k^2}{4\pi n^2} + \frac{\rho_d \nu_c}{n+\nu_c}\right] \delta v = -\nu[2\rho(D^2+k^2)\delta w - D(\rho Dw)]. \tag{38}$$

If we substitute for I_s and $\delta I_s (s = 1\ to\ 7)$ in equation (30) and make use of equation (38), we obtain after some further integrations by parts,

$$\delta n^2 \left[I_1 + \frac{1}{n^2} I_5 + I_6 \right] + \frac{2}{k^2} \int_L \left[n^2\{\rho k^2 w - D(\rho D w)\} - g k^2 (D\rho) w + \frac{\nu_c}{n + \nu_c} n^2 \{\rho_d k^2 w - D(\rho_d D w)\} - \frac{H^2 k^2}{4\pi}(D^2 - k^2) w - \nu k^2 n \{2(D^2 + k^2)(\rho v) - D(\rho D v)\} \right] \delta w \, dz = 0. \quad (39)$$

We observe that the quantity occurring as a factor of δw under the integral sign vanishes if and only if equation (15) is satisfied. Thus a necessary and sufficient condition for δn^2 to be zero to the first order for all small arbitrary variations δw and δv (connected by equation (38)) in w and v which is compatible with the boundary conditions is that w and v should be the solutions of the eigenvalue problem governed by equations (15) and (16). A variational procedure of solving for the characteristic values is, therefore, possible.

Theorem II: If oscillatory modes exist they should be stable.

Proof: From equation (21), we have

$$n_i \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j \right) dz - \frac{g}{n_i} \int_L (D\rho) w_i w_j \, dz + \frac{H^2 k^2}{4\pi n_i} \int_L \left(w_i w_j + \frac{1}{k^2} D w_i D w_j \right) dz + n_j \int_L \rho v_i v_j \, dz + \frac{H^2 k^2}{4\pi n_j} \int_L v_i v_j \, dz + \frac{\nu_c n_i}{n_i + \nu_c} \int_L \rho_d \left(w_i w_j + \frac{1}{k^2} D w_i D w_j \right) dz + \frac{\nu_c n_i}{n_j + \nu_c} \int_L \rho_d v_i v_j \, dz = 0. \quad (40)$$

Interchanging i and j and noting that the above integrals are symmetric in i and j, we obtain

$$n_j \int_L \rho \left(w_i w_j + \frac{1}{k^2} D w_i D w_j \right) dz - \frac{g}{n_j} \int_L (D\rho) w_i w_j \, dz + \frac{H^2 k^2}{4\pi n_j} \int_L \left(w_i w_j + \frac{1}{k^2} D w_i w_j \right) dz + n_i \int_L \rho v_i v_j \, dz + \frac{H^2 k^2}{4\pi n_i} \int_L v_i v_j \, dz + \frac{\nu_c n_j}{n_j + \nu_c} \int_L \rho_d \left(w_i w_j + \frac{1}{k^2} D w_i D w_j \right) dz + \frac{\nu_c n_i}{n_i + \nu_c} \int_L \rho_d v_i v_j \, dz = 0. \quad (41)$$

Let us consider two solutions characterized by n and n^*, the complex conjugate of n. We expect that the corresponding solutions will also be the complex conjugates of each other. Hence if $n_i = n, n_i = n^*$, then $w_i = w, w_j = w^*, v_i = v\ and\ v_j = v^*$.

Then, from (40) and (41) by addition and subtraction, we have

$$Re(n) \left[\overline{I_1} + \overline{I_5} - \frac{g}{|n|^2} \overline{I_2} + \frac{H^2 k^2}{4\pi |n|^2} \overline{I_3} + \frac{H^2 k^2}{4\pi |n|^2} \overline{I_4} + \frac{\nu_c^2 (\overline{I_6} + \overline{I_7})}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2} \right] = \frac{-\nu_c |n|^2 (\overline{I_6} + \overline{I_7})}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2}, \quad (42)$$

and

$$Im(n) \left[\overline{I_1} - \overline{I_5} + \frac{g}{|n|^2} \overline{I_2} - \frac{H^2 k^2}{4\pi |n|^2} (\overline{I_3} - \overline{I_4}) + \frac{\nu_c^2 (\overline{I_6} - \overline{I_7})}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2} \right] = 0, \quad (43)$$

where

$$\left. \begin{aligned} \overline{I_1} &= \int_L \rho \left[|w|^2 + \tfrac{1}{k^2} |Dw|^2 \right] dz, \quad \overline{I_2} = \int_L (D\rho) |w|^2 dz, \quad \overline{I_3} = \int_L \left[|w|^2 + \tfrac{1}{k^2} |Dw|^2 \right] dz, \\ \overline{I_4} &= \int_L |v|^2 dz, \quad \overline{I_5} = \int_L \rho |v|^2 dz, \quad \overline{I_6} = \int_L \rho_d \left[|w|^2 + \tfrac{1}{k^2} |Dw|^2 \right] dz, \\ \overline{I_7} &= \int_L \rho |v|^2 dz. \end{aligned} \right\} \quad (44)$$

Integrals $\overline{I_s} (s = 1\ to\ 7)$ are all positive.

If n is complex, $Im(n) \neq 0$, hence (43) gives

$$\overline{I_1} - \overline{I_5} + \frac{g}{|n|^2}\overline{I_2} - \frac{H^2k^2}{4\pi|n|^2}(\overline{I_3} - \overline{I_4}) + \frac{\nu_c^2(\overline{I_6} - \overline{I_7})}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2} = 0, \tag{45}$$

so that (42) gives

$$2Re(n)\left[\overline{I_1} + \frac{H^2k^2}{4\pi|n|^2}\overline{I_4} + \frac{\nu_c^2\overline{I_6}}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2}\right] = -\frac{\nu_c|n|^2(\overline{I_6} + \overline{I_7})}{|n|^2 + 2\nu_c R_e(n) + \nu_c^2}. \tag{46}$$

From equation (46) it follows that $R_e(n)$ is negative, which implies that if oscillatory modes exist they should be stable, thus ruling out possibility of overstability.

4. DISCUSSION ON THE CASE OF TWO SEMI-INFINITELY EXTENDING PLASMAS OF CONSTANT DENSITIESSEPARATED BY A HORIZONTAL PLANE

We consider the case when two semi-infinitely extending plasma layers of constant densities ρ_1 and ρ_2, and dust particle densities ρ_{d_1} and ρ_{d_2} are separated by a horizontal boundary at $z = 0$. The subscripts 1 and 2 distinguish the lower and upper plasma layers, respectively.

We choose the following trial function for $w(z)$,

$$w(z) = \begin{cases} Ae^{+kz} & z < 0; \\ Ae^{-kz} & z > 0\,, \end{cases} \tag{47}$$

which is consistent with the boundary conditions (17) - (19). Here the same constant has been chosen to ensure the continuity of w at $z = 0$.

The value of v in the two regions can be calculated from equation (16) and noting that ρ is constant, we have

$$v(z) = \begin{cases} Z_1e^{+kz} & z < 0; \\ Z_2e^{-kz} & z > 0\,, \end{cases} \tag{48}$$

where

$$Z_{1\,,2} = \frac{-3\nu k^2 nA}{n^2\left[1 + \frac{\alpha_0\nu_c}{n+\nu_c}\right] + k^2V_{1\,,2}^2}, \tag{49}$$

$$V_1^2 = \frac{H^2}{4\pi\rho_1} \; and \; V_2^2 = \frac{H^2}{4\pi\rho_2}. \tag{50}$$

We assume that $\frac{\rho_{d_1}}{\rho_1} = \frac{\rho_{d_2}}{\rho_2} = \alpha_0$ as the simplifying assumption does not obscure any of the essential features of the problem.

To evaluate the integrals $I_s(s = 1 \; to \; 7)$ in equation (22), we divide the region of integration into three parts $(i) - \infty < z < -\varepsilon$ (ii) $\varepsilon < z < \infty$ $(iii) - \varepsilon < z < \varepsilon$ and then pass it over to the limit $\varepsilon \to 0$. On substituting their values in equation (22), we obtain the following dispersion relation between h and k,

$$n^2 - gk(\alpha_2 - \alpha_1) + k^2V_A^2 + \frac{\alpha_0\nu_c}{n+\nu_c}n^2(\alpha_1 + \alpha_2) + \frac{g}{2}\nu^2k^4n^2\left\{\frac{\alpha_1}{n^2\left[1 + \frac{\alpha_0\nu_c}{n+\nu_c}\right] + k^2V_1^2} + \frac{\alpha_2}{n^2\left[1 + \frac{\alpha_0\nu_c}{n+\nu_c}\right] + k^2V_2^2}\right\} = 0, \tag{51}$$

where

$$V_A = \left\{\frac{H^2}{2\pi(\rho_1 + \rho_2)}\right\}^{1/2} \tag{52}$$

can be termed as mean Alfven velocity and

$$\alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}. \tag{53}$$

Letting

$$n = \frac{g}{V_A} n^*, \qquad k = \frac{g}{V_A^2} k^*$$

and omitting the asterisks for simplicity, so that the equation (51) takes the following dimensionless form

$$A_9 n^9 + A_8 n^8 + A_7 n^7 + A_6 n^6 + A_5 n^5 + A_4 n^4 + A_3 n^3 + A_2 n^2 + A_1 n + A_0 = 0, \tag{54}$$

$$A_9 = 4,\ \ A_8 = 12A, A_7 = 4A^2 + \frac{2k^2 B}{\alpha_1 + \alpha_2} + 4\{k^2 - k(\alpha_2 - \alpha_1)\} + 2Lk^4 B,$$
$$A_6 = 2k^2 B\left\{\frac{1}{\alpha_1\alpha_2} + Lk^2\right\}(A + \nu_c') + 4A^3 + \frac{2k^2 AB}{\alpha_1\alpha_2} + (8A + 4\nu_c')\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_5 = \frac{2ABk^2}{\alpha_1\alpha_2}(A + 2\nu_c') + \frac{k^4}{\alpha_1\alpha_2} + 2Lk^4\left(k^2 + B\nu_c'^2\right) + \left(4A^2 + \frac{2k^2 B}{\alpha_1\alpha_2} + 8A\nu_c'\right)\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_4 = \frac{k^4}{\alpha_1\alpha_2}(A + 2\nu_c') + 2Ak^2\nu_c'\left(\frac{AB}{\alpha_1\alpha_2} + Lk^2\nu_c' B + 3Lk^4\right) + \left\{\frac{2Bk^2}{\alpha_1\alpha_2}(A + 2\nu_c') + 4A^2\nu_c'\right\}\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_3 = \frac{k^4\nu_c'}{\alpha_1\alpha_2}(1 + 2A) + 6Lk^6\nu_c'^2 + \left\{\frac{2Bk^2\nu_c'}{\alpha_1\alpha_2}(2A + \nu_c') + \frac{k^4}{\alpha_1\alpha_2}\right\}\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_2 = \frac{k^4\nu_c' A}{\alpha_1\alpha_2} + 2Lk^6\nu_c'^3 + \frac{k^4\nu_c'}{\alpha_1\alpha_2}(3 + 2AB\nu_c')\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_1 = \frac{k^4\nu_c'}{\alpha_1\alpha_2}(1 + 2\nu_c')\{k^2 - k(\alpha_2 - \alpha_1)\},$$
$$A_0 = \frac{k^4\nu_c'^2}{\alpha_1\alpha_2}\{k^2 - k(\alpha_2 - \alpha_1)\},$$

$$A = (1 + \alpha_0)\nu_c', \qquad B = \alpha_1 + \alpha_2, \qquad \nu_c' = \nu_c\frac{V_A}{g}, \tag{55}$$

and

$$L = \frac{\nu^2 g^2}{V_A^6}$$

is a non-dimensional number measuring the relative importance of FLR effects and magnetic field.

For the potentially stable configuration $(\alpha_2 < \alpha_1)$, all the coefficients of equation (54) are positive, if

$$k > k^*, \tag{56}$$

where

$$k^* = \alpha_2 - \alpha_1. \tag{57}$$

So no positive real root or complex root with negative real part exists. **Therefore, the medium is stable even in the presence of collisions for disturbances of all wave numbers as it is if there are none.**

For the potentially unstable configuration $(\alpha_2 > \alpha_1)$, the absolute term in equation (54) is negative, if

$$0 < k < k^*. \tag{58}$$

Therefore (54) possesses at least one real root which is positive leading to an instability of the configuration even if there are collisions with dust particles.

Also we see that k^* is independent of *L*, a measure of FLR effect. Hence we conclude that for longitudinal perturbations, the stability criterion is independent of magnetic viscosity.

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