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Technological Pedagogical Content Knowledge: A Review of Mathematics Education Literature

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Abstract: In teacher education literature, Technological Pedagogical Content Knowledge (TPACK) was defined as the teacher knowledge required for successful technology integration. It is based on the notion of Pedagogical Content Knowledge, which elaborated the teacher knowledge in terms of content-specific pedagogy. Along with other areas, TPACK studies in mathematics education have gained momentum in recent years. This study aims to review the peer-reviewed articles published between 2005-2019, which examined mathematics and pre-service mathematics teachers' TPACK under three main themes: an examination of TPACK studies based on its components, the development of TPACK and the strategies for developing TPACK. The findings indicated that the studies on TPACK mostly focused on general technological pedagogical knowledge without considering the content dimension. Another noteworthy issue is the large number of recent studies that have examined teacher and student beliefs as a component of TPACK. In contrast, assessment has been a neglected issue in TPACK studies. Finally, our investigation indicated a gap in the literature concerning strategies for developing TPACK.

Keywords: Technological pedagogical content knowledge, TPACK, Mathematics teachers, Pre-service mathematics teachers

Introduction

Digital technologies have been used in the teaching of mathematics especially in the last three decades. Researchers had investigated the effects of tools such as educational software (e.g. dynamic geometry systems) and graphical calculators on students' learning in the context of mathematics. Studies found that use these tools promoted a conceptual understanding of mathematics (Habre & Abboud, 2006). However, the success of using these tools depends on teacher expertise and knowledge. They should have adequate knowledge of using technology in their classrooms effectively. The knowledge required for successful technology integration had been defined as Technological Pedagogical Content Knowledge (TPCK) by Pierson (1999), Niess (2005), and Mishra and Koehler (2006). Later, Koehler and Mishra (2009) referred to the framework as TPACK. In this section, we will elaborate on the Technological Pedagogical Content Knowledge (TPACK) framework further.

TPACK framework is based on Shulman's (1986, 1987) notion of Pedagogical Content Knowledge (PCK) which he defined as an important domain of teachers' knowledge and an amalgam of teachers' content and pedagogical knowledge. Shulman (1987) emphasizes that "pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue" (p. 8). Pierson (1999), Niess (2005) and Mishra and Koehler (2006) conceptualized the TPACK framework as the intersection of three knowledge domains: content knowledge, pedagogical knowledge, and technological knowledge (See Figure 1). Technological Pedagogical Content Knowledge is different from and more powerful than technological, pedagogical, and content knowledge as separate knowledge domains (Akkoç, 2013) just like pedagogical content knowledge is a different domain than pedagogical and content knowledge. Mishra and Koehler (2006) defined the intersections of different knowledge domains in Figure 1. We have already defined pedagogical content knowledge (PCK) as the pedagogical knowledge specific to a particular subject.

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Technological content knowledge (TCK) is concerned with the way technological tools represent a particular subject matter. Technological pedagogical knowledge (TPK) is the general pedagogical knowledge (not specific to a particular subject) for integrating technology into instruction e.g. using the opportunities of technological tools such as getting instant feedback from the computer or the knowledge of the classroom management in a computer lab.

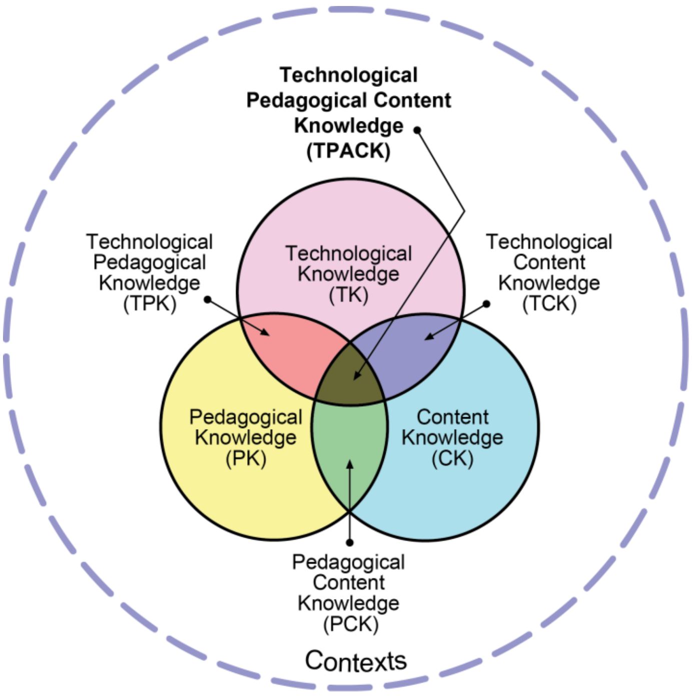

Figure 1. TPACK Framework (Reproduced by permission of the publisher, © 2012 by tpack.org)

Technological pedagogical content knowledge (TPACK), as represented as the intersection of three sets in Figure 1, is the knowledge required for teaching a particular subject matter using technology effectively e.g. how to choose suitable software and use it with appropriate pedagogy to introduce derivative at a point. TPACK builds on the PCK construct and is "achieved when a teacher knows how technological tools transform pedagogical strategies and content representations for teaching specific topics" (Jang, 2010, p. 1744).

After the emergence of the TPACK framework, researchers elaborate on the framework in terms of its development process and its components. Niess (2005) adopted Grossman's (1990) study on PCK components and defined the components of TPACK as the knowledge concerning: (1) what it means to teach a particular subject integrating technology in the learning, (2) instructional strategies and representation for teaching particular topics with technology, (3) students' understanding, thinking, and learning with technology in a particular subject, (4) curriculum and curriculum materials that integrate technology with learning in the subject area.

This study aims to explore how mathematics education literature (2005-2019) conceptualized and investigated the TPACK framework.

Method

The review focused on the peer-reviewed articles published between 2005-2019, which examined mathematics and pre-service mathematics teachers' TPACK. Articles were searched in December 2019 by exploring Google Scholar. The keyword employed was "technological pedagogical content knowledge" and "TPACK" OR "TPCK". We excluded book reviews, conference proceedings, and Ph.D. dissertations. We reached 12 articles all of which are empirical studies. Seven of them focused on pre-service mathematics teachers and five of them on mathematics teachers. An in-depth analysis of these studies revealed three main themes: an examination of TPACK based on its components, the development of TPACK, and the strategies for developing TPACK. Below, we present findings for each theme in detail.

An examination of TPACK studies based on TPACK's components

In this section, we will provide a review of the literature based on the TPACK framework's components. The determined by adapting the components determined for PCK in Depaepe et al. (2013). We also added the assessment and evaluation component which emerged from our review.

Tablo 1. Studies in mathematics education based on TPACK's components

Authors	Students' (mis)conceptions and difficulties	Representations and instructional materials	Mathematical tasks and cognitive demands	Educational ends	Curriculum and media	Technological knowledge	Content knowledge	Pedagogical knowledge	Teachers' and students' beliefs	Assessment and evaluation
Lee ve Hollebrands (2008)						x		x		
Niess et. al.. (2009)						x		x		
Ozmantar, Akkoc, Bingölbali, Demir and Ergene (2010)		x								
Bowers and Stephens (2011)		x				x				
Haciomeroglu, Bu, Schoen, Hohenwarter (2011)						x		x		
Larkin, Jamieson-Proctor, and Finger (2012)						x		x		
Doğan (2012)					x				x	
Akkoç (2015)		x			x	x				x
Hansen, Mavrikis, Geraniou (2016)		x				x				
Psycharis and Kalogeria (2017)		x			x		x	x		
Young et al. 2019						x				
De Freitas and Spangenberg (2019)						x	x		x	

As can be seen in Table 1, the component, "representations and instructional materials", is the most frequent one. This is followed by "curriculum and media" and "technological knowledge". Opposite to PCK studies, we did not find any study regarding the component, "students' (mis)conceptions and difficulties". The component, "assessment and evaluation", was only studied by Akkoç (2015) in the context of formative assessment in a Geogebra environment. It is also a neglected component in PCK literature. TPACK Studies focused on the nature of TPACK in a general sense rather than students' difficulties with a specific topic. Another remarkable finding is that student and teacher belief is a focus of attention in TPACK studies as oppose to PCK studies in the context of mathematics education.

An examination of the mathematics education literature on the development of TPACK

In this section, we will provide a review of mathematics education literature that focused on the developments of mathematics teachers' or pre-service mathematics teachers' TPACK. What is meant by the development of TPACK is the development of the knowledge and skills concerning the use of technology with appropriate pedagogies for teaching mathematics. Studies mostly investigated the TPACK development at the end of a course, a module, or a project. Most of them used the existing TPACK models or components while others (Niess et.al., 2009) re-conceptualized the framework to build TPACK development models.

Among the studies that used existing TPACK components, Lee and Hollebrands (2008) offered an integrated approach to developing technological pedagogical content knowledge to prepare mathematics teachers to teach data analysis and probability topics using specific technology tools. They shared and discussed some examples from materials developed by the Preparing to Teach Mathematics with Technology (PTMT) project. Their integrative approach emphasizes the content dimension when developing mathematics teachers' technological knowledge. In a similar sense, Haciömeroğlu et. al. (2011) focused on a specific technological tool (Geogebra) and investigated pre-service mathematics teachers' TPACK in the context of methods courses. They examined

68 pre-service teachers' TPACK development using their written reflections, lesson plans, and classroom observations.

On the other hand, Niess et. al. (2009) revised the TPACK framework and offered a five-stage development model called the "Mathematics Teacher Development Model". They attempted to answer the question of what knowledge is needed to teach mathematics with digital technologies. To do that, they first defined "Mathematics Teacher TPACK Standards" which offer guidelines for thinking about the framework in the context of mathematics. They emphasized that these standards "may guide teachers, researchers, teacher educators, professional development consultants, and school administrators in the development and evaluation of professional development activities, mathematics education programs, and school mathematics programs" (p.4). They offered a Mathematics Teacher Development Model to describe the development of TPACK toward meeting these standards.

Their five-stage developmental process when learning to integrate a particular technology in teaching and learning mathematics is as follows (Niess, 2009, p. 9):

> 1. Recognizing (knowledge), where teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.
> 2. Accepting (persuasion), where teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology.
> 3. Adapting (decision), where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.
> 4. Exploring (implementation), where teachers actively integrate teaching and learning of mathematics with an appropriate technology.
> 5. Advancing (confirmation), where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology.

As can be seen from the quotation above, the model describes how teachers develop the knowledge for integrating technology rather than the components of TPACK. Although the model emerged from a research study with mathematics teachers, it could be used to investigate the development of TPACK of teachers for other subject domains.

An examination of the mathematics education literature on the strategies for developing TPACK

In this section, we will provide a review of mathematics education literature that focused on the strategies for developing mathematics teachers' or pre-service mathematics teachers' TPACK. We purposefully distinguished this theme from the theme above (development of TPACK) because studies that will be mentioned in this section particularly describe and prescribe the strategies for developing TPACK rather than merely reporting the development of TPACK. In other words, they give a detailed account of the intervention that aimed at the development of TPACK.

Ozmantar, Akkoc, Bingölbali, Demir, and Ergene (2010) conducted an intervention and suggested strategies for developing TPACK is. In their wider project, they developed a course for pre-service teachers to develop their TPACK and specified five content-specific components of the framework. In their study, they focused on one of the components of TPACK (concerning multiple representations of derivative). Teacher preparation course content and method of delivery were based on the defined TPACK framework. They defined content-specific learning gains for TC, TCK, TPK, PCK, and TPCK. They found that the course improved pre-service mathematics teachers' lesson plans and micro-teaching concerning their knowledge of representations, of connections established among the representations, and of the aspects of derivative emphasized by these connections in technology-rich environments.

Bowers and Stephens (2011) offered a different TPACK perspective than Ozmantar et. al.'s (2010) study to help pre-service mathematics teachers develop their TPACK. Instead of a conception of TPACK as a subset of knowledge skills, their perspective considers learning as a social process motivated by communication. In their study, preservice teachers engaged in "technology-enhanced mathematical explorations with the explicit goal of discussing how technology enabled them to describe relationships among objects on the screen that could not have been developed without the tools employed" (p. 291). A 6-week course that took place at a large university in the United States. Each pre-service teacher prepared a final project which included choosing a mathematical topic, developing a geometer's sketchpad sketch, and exploring how technology could be used to enhance a textbook-only lesson. Analysis of these projects and discussions of them were used to investigate pre-service teachers' TPACK development. The study concludes that TPACK may better be viewed as an orientation than a set of subskills or knowledge constructs and that this view could guide teacher educators to plan instruction for pre-service teachers. More specifically, the findings shed a light on "various pedagogical moves such as probing questions and unique technological features that support the need for causal explanations to support deeper mathematical understanding" (p. 301). Another strategy that made the course effective was guiding students' metacognitive processes as they reflect on their learning and development efforts.

Young, Young, Hamilton, and Pratt (2019) used meta-analytic thinking which compares their results to prior results from similar studies to evaluate the effects of a technology professional development. Their quantitative study investigated the effects of a three-week professional development for urban mathematics teachers on TPACK and how the effects compare to previous interventions to increase teacher TPACK. Their strategy for the technology professional development was based on drag/drop, hide/reveal, highlighting, movement/animation using interactive whiteboards (as the TK dimension), arithmetic, algebra, statistics, and geometry (as CK dimension) and demonstration, discussion, drill/practice, modelling, simulation (as the PK dimension). Pretest- posttest analysis of a survey illustrated teachers' development on TPACK and Interaction Whiteboard (IWB) use in the classroom.

Results and Discussion

This study aimed at an analysis of TPACK studies in mathematics education literature to reveal the current situation and shed a light on the future direction of research in that area. We reviewed the TPACK studies published between 2005-2019 under three themes: an examination of TPACK based on its components, the development of TPACK, and the strategies for developing TPACK.

Regarding the TPACK components, mathematics education literature focused most on certain components of TPACK: "technological knowledge" and "representations and instructional materials". Technological knowledge is also the main component of the framework in teacher education studies as well as mathematics education (Akkoç, 2013). The notion of multiple representations is an important area of research in mathematics education especially in the context of technology-enhanced environments. Therefore, representations and the way they are used constitute an important component of TPACK. On the other hand, our review also revealed that certain components were neglected in mathematics education literature. "Students' (mis)conceptions and difficulties" and "mathematical tasks and cognitive demands" are two of them. They are both closely related to the subject-matters such as overcoming students' difficulties with functions using technological tools. There is no study on mathematical concepts targeting overcoming student difficulties and misconceptions using technological tools. The content dimension is a neglected issue in TPACK studies in general (Akkoç, 2013). Another neglected component is "assessment and evaluation". We came across only one study focusing on this component (Akkoç, 2015).

Concerning the development of TPACK, we only found three studies two of which used the existing TPACK models or components. Niess et. Al. (2009), on the other hand, put forward a new model of TPACK development which is an important contribution to the field. Although they call it the "Mathematics Teacher Development Model", it has the potential to be used in other fields of teacher education that would focus on TPACK development.

Regarding the strategies for developing TPACK, we found three studies that particularly described their intervention that aimed a TPACK development. We found that strategies differed in the way they conceptualize the TPACK framework. While some of the studies separately used dimensions and/or components of the framework to design their courses on TPACK (Ozmantar et. al., 2010; Young et. al, 2019), Bowers and Stephens (2011) treated the TPACK as an orientation than a set of subskills or knowledge constructs and that this view could guide teacher educators to plan instruction for pre-service teachers.

Conclusions

The findings indicated that the studies on TPACK in mathematics education literature mostly focused on general technological pedagogical knowledge without considering the content dimension. Another noteworthy issue is the large number of recent studies that have examined teacher and student beliefs as a component of TPACK. In contrast, assessment has been a neglected issue in TPACK studies. Finally, our investigation indicated a gap in the literature concerning strategies for developing TPACK.

Recommendations

We suggest mathematics education researchers to focus on TPACK, components "students' (mis)conceptions and difficulties", "mathematical tasks and cognitive demands", and "assessment and evaluation" by bringing the content dimension into play. We also recommend teacher education researchers to conduct intervention studies that would adopt Niess et. al.'s (2009) five-stage TPACK development model to other fields in teacher education. We also recommend future studies that would elaborate the effective strategies to develop teachers' and pre-service teachers' TPACK.

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