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A MATLAB Supported learning and Students' Conceptual Understanding of Domain and Range of a Function of Two Variables: Wolkite University, Ethiopia

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Abstract: A case study design was conducted at Wolkite University to investigate MATLAB supported learning and students' conceptual understanding in learning Applied Mathematics II using four different comparative instructional approaches: MATLAB supported traditional lecture method, MATLAB supported collaborative method, only collaborative method and only traditional lecture method. Four intact classes Mechanical Engineering groups 1 and 2, Garment Engineering and Textile Engineering students were selected by simple random sampling out of eight departments. The first three departments were considered as treatment groups and the fourth one "Textile Engineering" was assigned as a comparison group, randomly. Qualitative data were collected through reasoning part of the multiple choice items of pre-test and interview items of the post-test were analyzed using APOS analysis based on proposed genetic decompositions. The results of the data show that the majority of the students' conceptual understanding lies in action conception. Students' conceptual understanding on domain and range is a straight forward as that of a function of a single variable which reveals that students haven't developed new schemata for a function of two variables, as different from a function of a single variable. Majority of the respondents were poor on extending a previous concepts to the new concept and had difficulty to represent domain and range using graph. The results also show that there is no difference between students learning through MATLAB supported in combination with collaborative approach and other instructional approaches like MATLAB supported learning in combination with traditional lecture method, traditional lecture method and collaborative method on conceptual understanding. This might be due to lack of students' experience on technology supported learning in such advanced courses. Thus, this study recommends further study on software supported learning in combination with collaborative method for betterment of conceptual understanding.

Keywords: MATLAB supported learning, Collaborative method, Conceptual understanding, Domain and range, Functions of two variables

Introduction

The concepts of domain and range of a function are ideas that help an individual's understanding of the relationships in a function. Every function relies on a specific domain and range that helps to apply to a real world situation (Bennett & Briggs, 2007). Domain of a function is defined as the set of valid or meaningful input x whereas range is the set of coordinating outputs y (Adams, 2003; Stewart, 2008).

According to Rockswold (2010) domain of a function is represented using the concept of interval notation instead of drawing a number line graphs as (,), (,], [,), or [,]. Moreover, Bittinger, Ellenbogen and Johnson (2010) describe domain and range of a given function using an ordered pair like for instance given that {(2,3),

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(4,5), (6,7), (8,9)} the set of the first entries of the given ordered pair is called domain i.e. {2,4,6,8} whereas the set of the second entries is called range of the function i.e. {3,5,7,9}. On top of these, Bittinge et al. (2010) indicate that students could use graphical representation in order to determine domain and range of a given set of ordered pairs. These describe the domain of a function as the set of all x-values that fulfill the curve whereas range is the set of all y-values that are results of the function, whose coordinate point lies on the curve.

Functions of several variables are extensions of functions of single variables. It is a real valued function of n-real variables that take as input (represented by the variables $x_1, x_2, x_3, \ldots, x_n$) to produce another real number, commonly denoted by $f(x_1, x_2, x_3, \ldots, x_n)$. From this one would see that the domain of functions of several variables is a region on which the function is defined. The range is the set of values that f takes. Whereas, in case of a function of a single variable domain of the function is the subset of real numbers that make the given function defined and the range is the set of values that f takes (Adams, 2003; Stewart, 2008). Here, a function of a single variable is considered as a pre-requisite for a function of several variables.

So, in order to tackle poor conceptual understanding of the students, there are researches that highly recommend to use instructional method that enables the students to discuss with one another and let them construct their own understanding in general and collaborative method in particular (Wong, 2001). There are also some other researches that recommend the utilization of different mathematical software to develop students’ understanding (Mulugeta, Zelalem & Kassa, 2015).

According to Al-Ammary (2013) technology supported learning can be considered as a solution to instructional problems that improve the effectiveness and efficiency of learning within education context. It lets learners to be motivated, have clear mental pictures about the content, enhance instructional methods, increase productivity, and equip with up-to-date information. That is why the National Council of Mathematics Teachers (NCTM) (2002) included technology as one principle of mathematics education since it influences content to be taught and enhances students' learning.

Technology has a significant impact on classroom and make student beneficiary. Stoops (2010), claims that technology integrated learning promotes positive attitudes toward learning and encourages low achievers to succeed. It is true that technology, particularly software integrated learning makes the classroom more interactive, and encourage the students to construct their own understanding rather than passive receivers. There is a positive relationship between technology and students' motivation, and also there is a direct association between students' motivation and learning mathematics (Shin & Mills, 2011). MacLuckie (2010) also shows that using technology increases motivation and retention of subject matters. Many scholars recommended that the use of educational technology in the classroom as one way of strategies to enhance student's conceptual understanding and problem solving skill (Al-Ammary, 2013; NCMT, 2000; Jaun, Huertas, Cuypers & Loch, 2012; Majid, 2014).

There is widely available software used for the purpose of teaching. For instance, hand held tools like calculators and mind tools like MATLAB, Mathematica, Maple, Fortran, C++ and so forth (Andreatos & Zagorianos, 2009; Charles-ogan, 2015; Eyasu, Kassa & Mulugeta, 2013; Ogunkunle & Charles-Ogan, 2013; Mulugeta, Zelalem & Kassa, 2015). Specially, MATLAB is used to visualize and plot different 2D and 3D graphs for better understanding and imagination of the problem (Charles-Ogan, 2015; Furner & Marinas, 2013), analyze data, develop algorithm, computation, modeling and simulation. It is also simple to use when compared with other software. A lot of research is done on mathematics software integrated learning and had positive result on students' motivation to learn mathematics (Furner & Marinas, 2013).

MATLAB supported learning was chosen because of its applicability in wide areas of discipline like electrical engineering, mechanical engineering, computer science and so forth for the purpose of simulation work and program writing. On top of this, it is used as a teaching and learning aid for mathematics students specially on sketching graphs of 2D and 3D. Thus, in this study MATLAB supported learning in combination with collaborative and traditional lecture method of teaching is used to find out its effect on student's conceptual understanding.

Method

Research design

This study was conducted to explore MATLAB supported learning and students' conceptual understanding of a domain and range of a function of two variables. Though conceptual understanding could be treated in a number of ways, the research method that fits for this study was a case study research design. It gives more emphasis on understanding the phenomenon under investigation through bringing a word or picture data for thick description and interpretation (Tewksbury, 2009).

Sample and sampling techniques

The study involves the use of four intact groups (Mechanical engineering group 1 and group 2, Garment engineering and Textile engineering) and assigns three of them as treatment and the remaining one as a comparison group using simple random sampling. The intact classes were assigned randomly to comparison and treatment group. All groups were exposed to different learning approaches in order to identify which learning approach is more effective to foster students' conceptual understanding. So, Mechanical Engineering group 1 students learnt though MATLAB supported learning in combination with the traditional lecture method, Mechanical Engineering group 2 students learnt though MATLAB supported learning in combination with collaborative learning method, Garment Engineering students learnt through collaborative method only and Textile Engineering students learnt through the traditional lecture method only. To help substantiate the students' levels of understanding and the nature of their schemata development qualitative approach was employed. The number of students involved in this study from each department was 30, 29, 35 and 32 respectively. Two intact classes (Mechanical engineering group 1 and group 2) were from the main campus of Wolkite University whereas the other two groups were from the cluster campus of the same batch of 2016/17 academic year. All groups have almost equally likely the same when looking at their Applied Mathematics I performance. This indicates that almost the groups are homogenous.

Purposive sampling method was employed to select a sample of students for interview from each of the selected classes. The students selected for interview were from the lower, medium and higher achievers in each group. The cutoff for achiever levels were done based on the students' previous Applied Mathematics I grade report. Those students who scored A- , A and A+ were categorized under higher achievers, C+ , B-, B and B+ were grouped in the medium achievers and those whose grade is below C+ were grouped under low achievers. Totally, there were 12 students selected for interview.

Data collection tools

Eight two tiered conceptual test of reasoning part as a pre-test and eight semi-structured interview questions as a post-test were designed by the researchers to collect qualitative data in order to get in-depth understanding on students' conceptual understanding. All questions are open ended items and the entire respondents were asked the same questions. Semi-structured interview allows the researchers to capture a deeper understanding of the topic to develop relevant and meaningful results i.e. to understand students' mental construction as per the genetic decomposition proposed after interventions were administered, hereunder. Each questions of the semi-structured interview derived from each questions of the conceptual test and needs further clarifications and discussions about the concept.

Data Collection Procedure

The data collection procedure emanates from the designed genetic decomposition that included classifications of concept categories. In this study the concepts under investigation were classified into four categories. Those were: definition, extending definition, algebraic/symbolic representation and graphic representation of a domain and a range of a function of several variables. In order to investigate the students' conception, the researchers used a genetic decomposition predicted beforehand based on their experience and relating these with available literature.

The genetic decomposition proposed for the concepts of a function of two variables had four different activities. Those activities were set to help students make constructions predicted. In these activities, students were asked about domain and range of a function of a single variable and required to move those concepts to a function of two variables so that they can be able to interiorize those actions into a process. Then, they were asked to find domain and range of a function of two variables, and they were also asked to show those concepts using graphical representations. Lastly, students were probed to thematize different concepts of a function of two variables.

The initial genetic decomposition proposed for the concepts of functions of several variables backed by the four different activities were set to help students make constructions predicted. These include the task for which students were asked to 1) define, 2) determine, 3) algebraically and graphically represent the domain and range of functions of single variables and 4) extend the concept of a function of single variable into a function of two variables.

Students were asked the above four questions before they deal with domain and range of a function of two variables. Besides, the following activities were designed.

1. An action conception which enables to define, extend definitions, and state rules and principles of domain and range of a function of two variables, whose rules are given in the algebraic/symbolic form.
2. A process conception which enables to determine domain and range of a function of several variables. This could involve studying the structure of the function, detecting whether a rule could be applied or whether the function should be written in a standard form which enables the application of the appropriate rules to solve a given problem.
3. An object conception which enables the seeing of strings of processes as a totality and performing mental or written actions on the internal structure of the given functions of several variables which enables to convert algebraic representations to graphical representation and vice versa.
4. Organizing the action, process, and object related to the concept of a function of two variables and linking them into a coherent framework. This framework includes various interpretations of functions of several variables in different contexts, and possible techniques for finding domain and range, and applying rules in finding domain and range.

Trustworthy of the Genetic Decomposition

Initially genetic decompositions were proposed by the researchers and then the proposed genetic decompositions were tested through pilot study. The hypotheses of the proposed genetic decompositions were checked and refined to better describe what students do and to design activities that help students to construct their understanding and exhibit difficulties (Martinez-Planell & Gaisman, 2013; Martinez-Planell, Gaisman, & McGee, 2015). Based on the results of the pilot study, the genetic decompositions were refined and made ready for the actual study.

Method of data analysis

The qualitative data collected were analyzed through thematizing students' reasoning into four different areas related to definition, extending definition to the new concept, algebraic/symbolic representation, and graph representation of a domain and range of a function of two variables. These were analyzed base on the APOS Theory framework.

Results and Discussion

A function of two variables is a function whose domain is a subset of the plane $\mathbb{R}^2$ and whose range is a subset of $\mathbb{R}$. If the domain is denoted by D, then a function f is a rule that assigns every point $(x, y) \in D$ to a unique real number $f(x, y) \in \mathbb{R}$. For functions of three variables, every point $(x, y, z) \in B$ where $B \subseteq \mathbb{R}^3$ is assigned to a unique real number$f(x, y, z) \in \mathbb{R}$ (Adams, 2003; Stewart, 2008).

Students were asked to choose the correct answer and justify their responses for conceptual tests. Their responses were categorized into different mental constructions as per APOS theory and based on the proposed genetic decompositions. Questions were given to the students to determine their understanding on domain and range of a function of two variables. These questions were given to assess the way students' defined domain and range, extend concepts of domain and range of functions of a single variable to that of several variables, algebraic representation/ symbolic representation of domain and range, and graphical representation of domain and range of functions of several variables. Thus, data gathered through reasoning part of the pre-test of conceptual understanding items and interview items of the post-test were analyzed using APOS theory to determine the level of students' conceptions before and after the interventions.

Before giving any treatments, students' conceptions were probed by the following table that defines a function f whose domain is represented by the variables x, y and whose range is represented by the variable $z: z = f(x, y)$. The values for x are in the first column of the table, the ones corresponding to y are in the first row of the table below (Table 1).

Table 1. Domain and range

x/y	**2**	**3**	**4**	**5**
0	3	4	5	6
1	5	2	4	3
2	6	2	5	5
3	7	3	3	4

Students' reasoning and interview data on definition of domain and range, extending definition of domain and range to functions of several variables, algebraic/ symbolic representations and graphical representation of each group under investigation have been extracted and the discussion for each is provided below.

In line with definition of a domain and a range of a function of several variables, students were given the above table of values so that they can define a domain and a range of a function of two variables. Even though the question given to the students was clearly indicating that domain is represented by the variables x and y and range as the value of $f(x, y)$, respondents did not show proper understanding to define domain and range of a function of several variables in general and a function of two variables in particular. For instance, the data reveal that before intervention was given students were defining domain as:

- *a set containing all elements in the first column of the given table* (respondents M1S3, M2S9, and GS13)
- *the first entry of the function* (respondent M1S2)
- *a value of* x (respondents M1S8, M2S1, GS1, & TS3)

Moreover, respondents were defining range as:

- *a set containing all elements in the first row of the given table*(respondents M1S10, M2S25, & TS29)
- *a value of* y (respondents M2S9 & GS13)

Definitions of domain and range as given above are basically correct only if the given function is a function of a single variable, but these do not serve a function of two variables. They were symbolically representing domain and range as $Df = \{x/x = 0,1,2,3\}$ (for instance M2S1, M2S2 and M2S5) and $Rf = \{y/y = 2,3,4,5\}$ (M2S4 and M2S11) respectively which is not correct. Such misconceptions were demonstrated because of lack of understanding the nature of the given function and extending some basic concepts of a function of a single variable to a function of two variables. Schwarzenegger (1980) and Tall (1992) state that if students had difficulty in understanding concepts of a function of a single variable, then it will cause difficulty to understand concepts of a function of two variables. Thus, no matter how the students have difficulties of conceptualizing prior knowledge or have better understanding, then they could be challenged to extend the prior knowledge to the new concepts.

In Regards to extending definition of domain and range of a function of a single variable to a function of two variables, students were given that $z = f(x, y)$ where both x and y are considered as independent variables whereas $f(x, y)$ is a dependent variable. It was conceptually wrong to think that a variable x as an independent variable and y as a dependent variable in a function of two variables. But, students were defining domain as "*the value of x and y in ordered pair*" (M1S10, GS3,) and "*a subset of a set of ordered pairs*" (M1S5, M2S16, TS1) whereas range as "*an output of f*" (for instance M2S16) and "*the value of y*"(for instance M1S10). Basically, some of the respondents seem to have acceptable conception, but the justification that consider the subset of a set of an ordered pairs as element of real number represented as $Df = \{(x, y) \in \mathbb{R}\}$ and the value of y i.e. $y = \{2,3,4,5\}$ as a range of a function of two variables is not correct. Such misconceptions might be demonstrated due to lack of understanding the nature of the dimension (space) on which the domain of a given function of two variables is defined and how to determine range of a given function.

Thus, the entire students' conception presented above on definition of domain and range of a function of two variables can be categorized under action conception as per the proposed genetic decomposition. This result is in favor of the research result done by Martinez-Planell & Trigueros (2009) whose research shows that, students' understanding on domain and range of a function of two variables is not different from that of a function of single variable.

Regarding representation, this study reveals that majority of the students have difficulty of interpreting the given function in tabular form and then define domain and range of the function. Even though the question given in the tabular form clearly indicates x and y as independent variables and z as a dependent variable in the given function, majority of the students could not understand the given questions, and were not able to define domain and range properly. Moreover, those students who answered the question properly have difficulties of justifying their answers. This difficulty might be due to lack of understanding of the tabular representation of the given function. Sajika (2003) states that if students are always introduced to a function as equation they have a difficulty to understand a function given in tabular or graphical form. Similarly, Metcalf (2007) suggests it is better to let students use various representations of function so that they can understand the concept.

Moreover, Carlson (1997) states that if students can be able to interpret features of a function from different representations and understand formal definitions, then they have deep conceptual understanding. This study reveals that students have difficulty to understand the given function in tabular form and poor conceptual understanding on concepts of domain and range of a function of two variables. Thus, majority of them could be categorized under action conception as per proposed genetic decomposition. The gap in using various representations might roll from the way they studies prior courses which needs further investigation.

Only few students were able to extend concepts of a function of a single variable to a function of two variables and chose a correct answer from the given alternatives. Regardless of their correct answer, except few students representing domain as $Df = \{(x,y):(x,y) \in \mathbb{R}^2\}$ and range as $Rf \subseteq \mathbb{R}$, majority of them were not able to justify their answer. As excerpt, the researchers presented some of the responses to define domain as:

- *a subset of a set ordered pairs* (respondents M1S1, M2S3, GS4 and TS5)
- *a point at which the given function is defined* (respondents M2S8)
- *a set represented by variables x and y* (respondents M1S10, and GS11).

And, range as

- a *value represented by the variable* $z = f(x,y)$ (respondents M1S14, M2S3 and GS11)
- *the value of f* (respondents TS29)

Literally, all the definitions given seem to be correct. But, students were not able to represent their justification in various ways. The representations could be table, diagram, equation, and verbal description (Metcalf, 2007; Rockswold, 2010). The data reveal that students were at mild stage to represent a given function in different forms and also were weak to represent domain and range of the given function in an appropriate form using either algebraically or graphically. Thus, this study shows that students were demonstrating difficulties on using appropriate representation for different functions and transfer between the representations with relative ease (Dubinsky & Harel, 1992; Eisenberg & Dreyfus, 1994; Metcalf, 2007).

The reasoning in the pre-test in general shows that, students had difficulty in defining domain and range of a function of two variables, extending concept of a domain and a range of a function of a single variable to a function of several variables, and symbolically representing domain and range of a function of two variables. These results go in line with studies conducted by Akkus, Hand & Seymour (2008); Carlson, Oehrtman & Engelke (2010), and Davis (2007) that reveal that students had great difficulty of connecting various representations of functions including equations graphs, tables and word forms.

After intervention was given to the students, interviews were conducted to collect a thick data from the respondents. The researchers observed that there were students who continued to define a domain and a range of a function of two variables either in the same way as before or in another form, but wrongly. When they were provided the function $f(x,y) = \frac{xy}{x+y}$ and asked to define domain and range some students were defining domain in the same way as that of a function of a single variable. For instance, M1S1, and TS8 defined domain as "*the set of all real numbers except at a point* $x = -y$". M1S3, M2S4, GS9 & TS12 defined domain as "*the set of points at which the given function is defined"*. Here, the students' conceptions were either wrong or not far from that of a function of single variable. They tried to define domain of a function of two variables within the set of real number which indicates that there is a misconception on concepts of domain of the given function, at least were not able to differentiate a subset of the set of real numbers and that of a region in the Cartesian plane. For example, conceptions of M1S1 and TS8 on domain are associated with a set of real number rather than a set of ordered pairs in the given surface. On top of this, others tried to define domain as a set of points which makes the given function defined but they failed to prove understanding when they justify it symbolically as if the ordered pairs are element of real number.

There were also some students who defined domain as:

- *x-intercept*(respondent, M1S2)
- *a set of ordered pairs*(respondent, GS10)
- *a set containing x and y* (respondent, GS11)
- *a number at which the given function is undefined*(respondent, M2S5) and
- *a subset of a set of an ordered pairs at which the given function is defined* (respondent, M2S6)

Moreover, students were defining range of the given function as:

- *the set of all real numbers* (respondents, M1S1, M1S3, TS8 and TS12)
- *y-intercept*(respondent, M1S2)
- *all values of x that make the function greater than zero*(respondent, M2S4)
- *the value that we get when we substitute domain within the given function*(respondents, M2S5, GS9, GS10)
- *the output of f* (respondents, M2S6, TS7)
- *all values of x that make the function different from zero*(respondent, GS11)

These data clearly show that students were demonstrating difficulty on understanding the nature of a given function even after intervention was given to them. They were defining domain and range of a given function almost in the same way as before. This indicates that there is no change of schema observed on students' conception particularly on Textile Engineering (i.e. students exposed to traditional lecture method) whereas other students from Mechanical Engineering group two (students learnt through MATLAB supported learning in combination with Collaborative method) and Garment Engineering (students learnt through Collaborative method only) were defining domain correctly even those who had a priory misconceptions. The data also reveal that students' difficulty of understanding a given function was not dependent on the way the function was presented to them. Because, in pre-test a function of two variables was given to the students in a tabular form, whereas during interview students were probed using an equation form but still there is a problem of understanding the given function and the respondents were unable to define domain and range of a function of two variables correctly.

It is possible to consider M2S5 who represented domain symbolically as$Df = \{(x,y):(x,y) \in \mathbb{R}^2\}$ which was correct. But, his definition as presented above states that domain is a number at which the given function becomes undefined is not correct. Such numbers could be considered as a restriction that we have to exclude them from the domain. Similarly, M1S2 was defining domain and range of the given function through connecting it with the concept of intercept which could be considered as a wrong conception.

Moreover, out of the 12 students interviewed after intervention only three of them (i.e. M1S3, M2S6 and GS9) showed a process conception on domain and range of a function of two variables. All of them defined and clearly represented domain and range of a function of two variables. Their responses show that they have interiorized the actions described in the genetic decomposition throughout questions given to them. At this level students clearly identify that domain of the given function is the set of ordered pairs written in the form of $Df = \{(x,y):(x,y) \in \mathbb{R}^2\}$at which the given function is defined, whereas range is an output of the given function and represented as $Rf \subseteq \mathbb{R}$. This is the basic difference represented between action conception and process conception. Here, they have interiorized the actions of finding all elements in the domain of the given function. This indicates that the respondents clearly understood the concept of a function of two variables, the nature of the function, and how to write domain and range of the function. Thus, they can be categorized under a process conception. This agrees with the results of Trigueros and Martinez-Planell (2007, 2010, and 2011) who indicate that those of students who had difficulties of representing ordered pairs on two dimensional space, treatment and conversion between different representations can be categorized under process conception.

Gaisman and Martinez-Planell (2010) state that at process level students had difficulty with some coordination of processes which seem to be important in the construction of an object conception. It is true that, all of them could not convert algebraic or symbolic representation in to graphical representation, or coordination of schemata for set, function and $\mathbb{R}^3$. Most of these difficulties showed that students were unable to develop $\mathbb{R}^2$ schema. In spite of the difficulties, all students attempted to determine domain and range of the given function, however. They also indicated that each pair of numbers goes to real numbers which is the range of the given function. This implies that it was expected to construct the processes involved in the conversion. This result is in agreement with the work of Duval (2006) who claims that students who had difficulty of transforming representations that happens within representation register were categorized under the process level of conception. Thus, this result shows that few of the respondents were categorized under the process conception after interventions except in the comparison group. This shows that the intervention has supported students improve their level of conception, but not to the highest level.

It was also revealed that none of the students arrived at object conception and schema conception. In order to arrive at schema conception students need to demonstrate action, process and object conceptions. This shows that students did not develop a new schema for a function of two variables in general, and a domain and a range of a function of several variables in particular. They were not that strong in algebraic representation and graphical representations, including treatments and conversion. This result agrees with the research result of Kerrigan (2015) and Martinez-Planell&Gaisman (2009) who revealed that majority of the students in their study had difficulty in describing the domain and range of functions of two variables and some of them had no clear idea of elements in the domain and type of function they are doing with. They also added that some of the respondents have no idea about the nature and type of functions they are talking about.

The result also reveals that students who learnt using MATLAB supported learning in combination with collaborative method improved students' level of conceptual understanding than that of other instructional approaches considered in this study. Literature indicates that supporting instructional method with educational software gives a privilege of learning how to learn through constructing their own understanding, and make the classroom environment attractive, interactive, and active as a cosmetic of teaching and learning process (Eyasu, Kassa & Mulugeta, 2013).

The result of this study also agrees with a research conducted by Gaisman and Martinez-Planell (2014) who indicated in their research that students had difficulty in transforming algebraic or symbolic representations to graphical representation, and vice versa. In this study, students were given both pre-test reasoning and post intervention interview form of questions. In both cases, students demonstrated difficulty to transform tabular to graphical and also algebraic representations into graphical forms. On top of these, Dubinsky & Harel (1992), Eisenberg & Dreyfus (1994) and Metcalf (2007) claim that if students understand the given concept, then they can be able to represent it in multiple ways like using appropriate symbolic representation, algebraic expression, graphical representation and transfer between the representations. Similarly, studies conducted by Drlik (2015) and Martinez-Planell&Gaisman (2009) show that students were demonstrating difficulty to connect the information given through table and in equation form. Hence, students had difficulty to represent a given concept in different ways even though they were exposed to different instructional approaches.

All difficulties mentioned in this study are related to the coordination of students' schema for $\mathbb{R}^2$ and that of a function of several variables. They had difficulty to write a domain and a range of a function of several variables analytically or graphically. They cannot consider sets of ordered pairs in the plane as possible domains of functions of several variables. They also had problem ofdetermining range using the domain of the function. In relation to range of a function, most students had difficulty of interiorization of an action needed to find values of functions into the process. Very few of students achieved a process conception and none of the students arrived at an object level. This result was in agreement with the result of Kashefi, Ismail &Yusof (2010) and Martinez-Planell & Gaisman (2009) that shows students had difficulty in understanding domain and range of functions of several variables.

Conclusion

This study shown that students' conception of domain and range of functions of several variables were not different from that of a function of a single variable except some group of the students who learnt through MATLAB supported learning in combination with collaborative method and collaborative learning method only. These groups got a chance to discuss with each other and were able to see the nature of the function through visualization and peer-support. The difficulties of the students in finding and describing domain and range of a function of two variables could be due to weak coordination between the schema of $\mathbb{R}^2$ and that of a function of a single variable. It seems that generalization is straight forward in an incremental learning such as one dimensional into two dimensional schemata, but for most of the students this does not work in the case of a function of two variables. Most of the students were not able to transcend the various forms of representations of functions of one variable into functions of two variables. The result shows that majority of students were not able to interiorize the notion of a function of two variables into a higher schema even those students who learnt through MATLAB supported learning in combination with collaborative learning method. All these convict that the instructional approaches did not help students to reach at the higher schemata, albeit moderate shift from action to process.

Recommendations

This study reveals that there is no as such a significant difference on students' conceptual understanding between students who learnt through anyone of the instructional approaches in learning a domain and a range of a function of two variables. This might be due to lack of students' exposure to such method of instruction or students were novice to instructional technology specifically MATLAB software. Thus, this needs further study.

Acknowledgements or Notes

Please provide acknowledgements or notes in a separate section at the end of the article before the references.

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