ON ϕ-RECURRENT LORENTZIAN β-KENMOTSU MANIFOLDS

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Abstract: In this paper, we study Lorentzian β-Kenmotsu manifold and we shown that ϕ-recurrent Lorentzian β-Kenmotsu manifold is an Einstein manifold and a pseudo-projective ϕ-recurrent Lorentzian β-Kenmotsu manifold is an η - Einstein manifold. And also we get the expression for 1-form A in a ϕ-recurrent Lorentzian β-Kenmotsu manifold.

Key words: β-Kenmotsu manifold, locally pseudo-projective ϕ -symmetric manifold, ϕ-recurrent Lorentzian β-Kenmotsu manifold, Einstein manifold, η-Einstein manifold

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ϕ-TEKRARLI (RECURRENT) LORENTZ β-KENMOTSU MANİFOLDLARI ÜZERİNE

Özet: Bu çalışmada Lorentz β-Kenmotsu manifoldları çalışıldı. ϕ-tekrarlı (recurrent) Lorentz β-Kenmotsu manifoldunun bir Einstein manifoldu olduğu, bir yarı projektif ϕ-tekrarlı Lorentz β-Kenmotsu manifoldunun da bir η - Einstein manifoldu olduğu gösterildi. Aynı zamanda bir ϕ-tekrarlı Lorentz β-Kenmotsu manifoldunda A 1-formunun ifadesi elde edildi.

Anahtar kelimeler: β-Kenmotsu manifoldu, local yarı projektif ϕ -simetrik manifoldu, ϕ-tekrarlı Lorentz β-Kenmotsu manifoldu, Einstein manifoldu, η-Einstein manifoldu

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1. INTRODUCTION

In the paper TAKAHASHI (1977) introduced the notion of locally ϕ-symmetric Sasakian manifold and obtained few interesting properties. Many authors like DE & PATHAK (2004), VENKATESHA & BAGEWADI (2006) and SHAIKH & DE (2000)

have extended this notion to 3-dimensional Kenmotsu and trans-Sasakian and LP-Sasakian manifolds respectively.
Motivated by the above studies, in this paper, we define ϕ-recurrent Lorentzian β-Kenmotsu manifold and pseudo-projectively ϕ-recurrent Lorentzian β-Kenmotsu manifold (M^{2n+1}, g) and obtain some interesting results.

The paper is organized as follows. In preliminaries, we give a brief account of Lorentzian β-Kenmotsu manifolds. In section 3, we shown that ϕ-recurrent Lorentzian β-Kenmotsu manifold is an Einstein manifold and pseudo-projectively ϕ-recurrent Lorentzian β-Kenmotsu manifold is an η-Einstein manifold and also find the value of associated 1-form A for a ϕ-recurrent Lorentzian β-Kenmotsu manifold.

2. PRELIMINARIES

An (2n +1) dimensional differentiable manifold M^{2n+1} is called an Lorentzian β-Kenmotsu manifold where β is a smooth function on M (BAGEWADI et al. 2008a, BAGEWADI et al. 2008b) if it admits a (1,1) tensor field ϕ, a contravariant vector field ξ, a 1-form η and a Lorentzian metric g which satisfy

$$\phi^2 X = X + \eta(X)\xi, \tag{2.1}$$

$$\eta(\xi) = -1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$\eta(\phi X) = 0, \tag{2.5}$$

$$\nabla_X \xi = \beta[X - \eta(X)\xi], \tag{2.6}$$

$$(\nabla_X \eta)(Y) = \beta[g(X,Y) - \eta(X)\eta(Y)], \tag{2.7}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.
It can be easily seen that in an Lorentzian β-Kenmotsu manifold, the following relation holds:

$$Q\xi = -2n\beta^2\xi. \tag{2.8}$$

Also in an Lorentzian β-Kenmotsu manifold, the following relations hold

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = \beta^2[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)], \tag{2.9}$$

$$R(X,Y)\xi = \beta^2[\eta(X)Y - \eta(Y)X], \tag{2.10}$$

$$S(X,\xi) = -2n\beta^2\eta(X), \tag{2.11}$$

$$S(\xi,\xi) = 2n\beta^2, \tag{2.12}$$

$$S(\phi X, \phi Y) = S(X,Y) - 2n\beta^2\eta(X)\eta(Y), \tag{2.13}$$

for any vector fields X, Y, Z, where R(X, Y)Z is the Riemannian curvature tensor, and S is the Ricci tensor (BAGEWADI et al. 2008a , MATSUMOTO & MIHAI 1988).

3. ϕ-RECURRENT LORENTZIAN β-KENMOTSU MANI FOLDS

Definition 3.1. A Lorentzian β-Kenmotsu manifold is said to be locally ϕ-symmetric if

$$\phi^2((\nabla_W R)(X,Y)Z) = 0, \tag{3.1}$$

for all vector fields X, Y, Z, W orthogonal to ξ (SHAIKH & DE 2000).

Definition 3.2. A Lorentzian β-Kenmotsu manifold is said to be ϕ-recurrent manifold if there exists a non-zero 1-form A such that

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z, \tag{3.2}$$

for arbitrary vector fields X, Y, Z, W.

If the 1-form A vanishes, then the manifold reduces to a ϕ-symmetric manifold.
Let us consider a ϕ-recurrent Lorentzian β-Kenmotsu manifold. Then by virtue of (2.1) and (3.2) we have

$$(\nabla_W R)(X,Y)Z + \eta((\nabla_W R)(X,Y)Z)\xi = A(W)R(X,Y)Z, \tag{3.3}$$

from which it follows that

$$g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) = A(W)g(R(X,Y)Z,U). \tag{3.4}$$

Let $\{e_i\}$, i = 1, 2, . . . , 2n+1, be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (3.4) and taking summation over i, $1 \le i \le 2n+1$, we get

$$(\nabla_W S)(Y,Z) = A(W)S(Y,Z). \tag{3.5}$$

Replacing Z by ξ in (3.5), we have

$$(\nabla_W S)(Y,\xi) = -2n\beta^2 A(W)\eta(Y). \tag{3.6}$$

Now we have

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi).$$

Using (2.6), (2.7) and (2.11) in the above relation, it follows that

$$(\nabla_W S)(Y,\xi) = -2n\beta^3 g(Y,W) - \beta S(Y,W). \tag{3.7}$$

In view of (3.6) and (3.7) we have

$$S(Y,W) = -2n\beta^2 g(Y,W) + 2n\beta A(W)\eta(Y). \tag{3.8}$$

Replacing Y by ϕY and W by ϕW in (3.8), we have

$$S(\phi Y,\phi W) = -2n\beta^2 g(\phi Y,\phi W). \tag{3.9}$$

Using (2.3), (2.13) in (3.9), then we obtain

$$S(Y, W) = -2n\beta^2 g(X, W). \tag{3.10}$$

This leads to the following theorem:

Theorem 3.1. *A ϕ-recurrent Lorentzian β-Kenmotsu manifold (M, g) is an Einstein manifold.*

Now from (3.3) we have

$$(\nabla_W R)(X,Y)Z = -\eta((\nabla_W R)(X,Y)Z)\xi + A(W)\, R(X,Y)Z\,. \quad (3.11)$$

From (2.7) and the Bianchi identity we get

$$A(W)\,\eta(R(X,Y)Z) + A(X)\,\eta(R(Y,W)Z) + A(Y)\,\eta(R(W,X)Z) = 0\,. \quad (3.12)$$

By virtue of (2.9) we obtain from (3.12) that

$$A(W)\beta^2\left[g(X,Z)\eta(Y) - g(Y,Z)\eta(X)\right] + A(X)\beta^2\left[g(Y,Z)\eta(W) - g(Z,W)\eta(Y)\right] \quad (3.13)$$
$$+\ A(Y)\beta^2\left[g(Z,W)\eta(X) - g(X,W)\eta(Z)\right] = 0.$$

Putting Y = Z = e_i in (3.13) and taking summation over i, $1 \le i \le 2n+1$, we get

$$A(W)\ \ \eta(X) = A(X)\ \eta(W). \quad (3.14)$$

for all vector fields X, W. Replacing X by ξ in (3.14), we get

$$A(W) = -\ \eta(W)\ \eta(\rho). \quad (3.15)$$

for any vector field W, where A(ξ) = g(ξ, ρ) = η(ρ), ρ being the vector field associated to the 1-form A i.e.,

$$g(X, \rho) = A(X).$$

From (3.14) and (3.15), we can state the following:

Theorem 3.2. *In a ϕ-recurrent Lorentzian β-Kenmotsu manifold (M^{2n+1},g), the characteristic vector field ξ and the vector field ρ associated to the 1-form A are co-directional and the 1-form A is given by A(W) = − η(W) η(ρ).*

Definition 3.3. A Lorentzian β-Kenmotsu manifold is said to be pseudo-projective ϕ-recurrent manifold if there exists a non-zero 1-form A such that

$$\phi^2((\nabla_W \overline{P})(X,Y)Z) = A(W)\,\overline{P}(X,Y)Z\,, \quad (3.16)$$

for arbitrary vector fields X, Y, Z, W, where $\overline{P}$ is a pseudo-projective curvature tensor given by

$$\overline{P}(X,Y)Z = aR(X,Y)Z + b\left[S(Y,Z)X - S(X,Z)Y\right]$$
$$-\frac{r}{(2n+1)}\left[\frac{a}{2n} + b\right]\{g(Y,Z)X - g(X,Z)Y\}, \quad (3.17)$$

where a and b are constants such that a, b ≠ 0, R is the curvature tensor, S is the Ricci tensor and r is the scalar curvature (BAGEWADI et al. 2008b).

If the 1-form A vanishes, then the manifold reduces to a locally pseudo-projective ϕ-symmetric manifold.

Let us consider a pseudo-projective ϕ-recurrent Lorentzian β-Kenmotsu manifold. Then by virtue of (2.1) and (3.16) we have

$$(\nabla_W \overline{P})(X,Y)Z + \eta((\nabla_W \overline{P})(X,Y)Z)\xi = A(W)\,\overline{P}(X,Y)Z \quad (3.18)$$

from which it follows that

$$g((\nabla_W \overline{P})(X,Y)Z,U) + \eta((\nabla_W \overline{P})(X,Y)Z)\eta(U) = A(W)\,g(\overline{P}(X,Y)Z,U)\,. \quad (3.19)$$

Let $\{e_i\}$, i = 1, 2, . . . , 2n + 1, be an orthonormal basis of the tangent space at any point of the manifold. Then putting X = U = e_i in (3.18) and taking summation over i, $1 \le i \le 2n+1$, we get

$$(a + 2nb)(\nabla_W S)(Y,Z) - b\left[(\nabla_W S)(Y,Z) + (\nabla_W S)(\xi,Z)\eta(Y)\right] = 0\,. \quad (3.20)$$

Replacing Z by ξ in (3.20) we have

$$(\nabla_W S)(Y,\xi) = \left[\frac{8nb\beta^3}{a+(2n-1)b}\right]\eta(Y)\eta(W). \qquad (3.21)$$

Now we have

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi).$$

Using (2.5), (2.7) and (2.11) in the above relation, it follows that

$$(\nabla_W S)(Y,\xi) = -2n\beta^3 g(Y,W) + 2n\beta^2(\beta-1)\eta(Y)\eta(W) - \beta S(Y,W). \qquad (3.22)$$

In view of (3.21) and (3.22) we have

$$S(Y,W) = -2n\beta^2 g(Y,W) + \left[2n\beta(\beta-1) - \frac{8nb\beta^3}{a+(2n-1)b}\right]\eta(Y)\eta(W). \qquad (3.23)$$

This leads to the following theorem:

Theorem 3.3. *A pseudo-projective ϕ-recurrent Lorentzian β-Kenmotsu manifold (M^{2n+1}, g) is an η-Einstein manifold.*

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