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A Model Of The Laser Printing

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Abstract: In a laser printer system a thin layer, sandwiched between two plastic sheets, is heated by a laser. The dye melts and diffuses into the sheets, the diffusion coefficient being a sensitive function of temperature. The problem is to determine the amount of dye which has diffused into the sheets at time t. The axisymmetric unsteady temperature field produced by the laser is determined and then the dye field is found by solving a diffusion equation using a temperature sensitive diffusion coefficient. The model is used to predict the fraction of dye transferred to the receiver sheet at each radial position after a time which is long compared with the heating time.

Key words: Laser printing, temperature field, diffusion coefficient

Bir Laser Yazıcının Modeli

Özet: Laser yazıcılar sisteminde ince bir tabaka, sandviç şeklindeki iki plastik tabaka arasında bir laser ile ısıtılır. Boya tabakalar arasında erir ve dağılır, yayılma katsayısı sıcaklık fonksiyonuna bağlıdır. Problem t zamanında tabakalar arasında yayılmış boya miktarını hesaplamaktır. Laser ile üretilen eksensel sabit sıcaklık durumu hesaplanır, daha sonra sıcaklığa bağlı yayılma katsayısı kullanılarak yayılma denklemi çözülür ve boya miktarı bulunur. Model, alıcı tabakaya transfer olan boya oranını tahmin etmek ve daha sonra sıcaklık zamanı ile açısal durumu karşılaştırmaktır.

Anahtar Kelimeler: Laser yazıcı, sıcaklık durumu, yayılma katsayısı

AMS Mathematics Subject Classification (2000): 76R50, 76R99

1. Introduction

High quality hard copy of electronically stored images can be produced by thermal dye diffusion printing Hann and Beck [1]. Such copies are of near photographic quality and have been used in such diverse areas as medical imaging and credit card personalisation. In this paper we shall study laser induced thermal dye diffusion, a process which might replace the conventional thermal head process in some applications. We shall consider in particular the ICI Imagedata L2T2 system as it is described by Hutt [2].

Before considering the L2T2 system we shall describe briefly the conventional D2T2 thermal head system of ICI Imagedata. In this geometrically similar system a thin 2 µm dye layer is sandwiched between a 6 µm carrier layer of PET and a relatively thick receiver sheet of thickness 150 µm. Heat is supplied to this layered system by a thermal printer head which is pressed against the top surface of the carrier PET layer.

When the thermal head is switched on heat conducts across the layered system. Once the dye layer has reached a sufficiently high temperature the dye molecules diffuse into the receiver sheet. Experimental evidence shows that this is a diffusion process rather than a sublimation process Hann [3]. The diffusion coefficient is a sensitive function of temperature. After about 6 ms the thermal head is switched off and the system cools leaving a ‘dot’ of dye in the receiver sheet. In colour printing this process is repeated three times with three different ribbons(cyan, yellow and magenta). The final image in the receiver sheet is formed by a large number of dots, typically 50-300 dots per inch. Each dot is produced independently by a single printer element in a long row of printer elements. The amount of dye transferred to the receiver sheet in each dot is controlled by altering the heating time, the wattage being the same for all the heater elements.

Although this process successfully produces high quality hard copies, it does have its disadvantages. Firstly the overall speed of the process is controlled by the time it takes for heat to conduct across the PET layer. Secondly, in order to produce high temperatures in the dye layer, the thermal heat and the PET layer have to withstand even higher temperatures (350 °C typically).

In the ICI Imagedata L2T2 system heat is supplied directly to the dye-coat layer by a laser (see Figure 1). A suitable infrared absorbing material such as copper phthalocyanine is incorporated into the dye coat layer which absorbs radiation strongly at about 800nm. The dye therefore diffuses rapidly into the receiver sheet. The total amount of dye transferred depends on the temperature(via the diffusion coefficient).

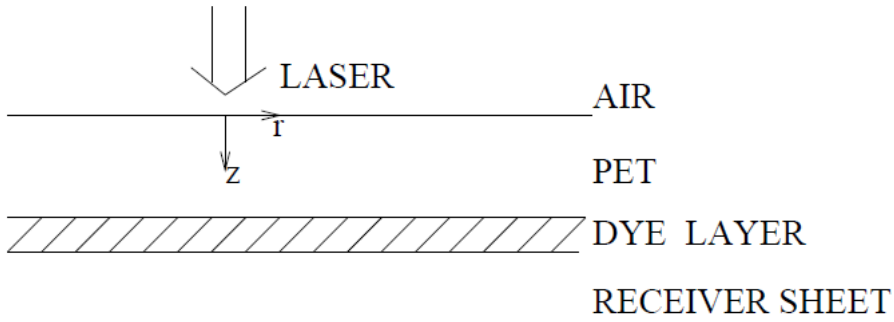

Figure 1: Typical laser system. Heat is generated in the dye layer.

As heat is supplied directly to the dye layer the transfer of dye is much more rapid than in the D2T2 system leading to more rapid and efficient printing. Typically laser heating for only 100μs is required. Another benefit of the L2T2 system is that higher resolution can be obtained since smaller dots can be produced by using lasers with a diameter of 30μm say. The diameter of a typical thermal head in the D2T2 system is 120μm.

The number of theoretical studies in the open literature of dye diffusion via laser heating is very small. Egashira et al have published a paper in which they propose a matematical model of the process and compare it with experiments [4]. However their treatment of the diffusion equation appears to be incorrect. Nevertheless the structure of their model has similarities with that presented here, temperature field due to a moving laser has been published by Irie et al for a similar problem, but they did not discuss the dye diffusion problem [5].

The purpose of this paper is to formulate a model which allows us to predict the fraction of dye which eventually diffuses in the receiver sheet at a given radial location. We first formulate the axi-symmetric temperature field produced by this heat source is then determined on the assumption that the thermal properties of the plastic layers are constants. The mass of dye which diffuses into the receiver sheet when there is a temperature dependent diffusion coefficient is then determined numerically. It can be shown that, because the diffusion coefficient is a sensitive function of temperature, the dye concentration field exhibits a sharp front.

2. Mathematical Model

A laser with a constant wattage w_0 is normally incident on the layered system illustrated in Fig. 1 for a time t_h, typically 100μs. The z- coordinate is measured into the layered system with the air interface situated at $z = 0$. The top (PET) layer has thickness h_1 and the dye –coat layer has thickness h_2. The receiver sheet can be regarded as infinitely thick in practice. The radial coordinate r is measured from the axis of the laser.

Consider first the absorption of laser radiation by the infra-red absorber in the dye-coat layer. Assuming that the distribution of the absorber is uniform, then the laser radiation flux per unit area, $I(r,z,t)$, satisfies the differential equation

$$\frac{\partial I}{\partial z} = -\frac{\lambda}{h_2} I, \tag{1}$$

for $h_1 \le z \le h_1 + h_2$ where λ is a constant which is known experimentally. Assuming that the radial dependence of the radiation flux on $z = h_1$ is gaussian

$$I(r,z,t) = I_0 \exp\left(-\frac{\lambda(z-h_1)}{h_2} - \frac{r^2}{r_0^{\,2}} \right), \tag{2}$$

Where I_0 is a constant and r_0 is a characteristic radius for the beam. The wattage of the beam W_0 is found integrating $2\pi r I(r,z,t)$ radially over the beam at $z = h_1$ and we find that $W_0 = \pi r_0^{\,2} I_0$. If the laser is switched on for a time t_h then the heat released per unit volume per unit time in the dye layer for $0 < t < t_h$ is

$$q(r,z,t) = -\frac{\partial I}{\partial z} = \frac{W_0 \lambda}{\pi r_0^{\,2} h_2} \exp\left(-\frac{\lambda(z-h_1)}{h_2} - \frac{r^2}{r_0^{\,2}} \right), \quad h_1 < z < h_1 + h_2. \tag{3}$$

This is of course largest at the top of the dye layer, $z = h_1$. No laser radiation is absorbed elswhere, so that $q(r,z,t) \equiv 0$ for $t \ge t_h$
where the heat source term is given by (3), α is the thermal diffusivity, ρ is the density and c_p is the specific heat.

We assume that there is no heat lost at the air/PET interface so that

$$\frac{\partial T}{\partial z} = 0, \text{ on } z = 0, \tag{4}$$

and that the temperature tends to the constant ambient temperature T_A for large value of z and r. Initially $T = T_A$ everywhere in the system.

It is possible to use the heat source method to obtain on exact expression for $T(r, z, t)$ as a double integral (Uyhan [6]). Then for 0 p t p t_h

$$T = T_A + \int_0^t \int_{h_1}^{h_1+h_2} \frac{Q\gamma(\tau)\lambda}{2\sqrt{\pi\alpha\tau}} G(z', z, \tau) \exp\left(-\frac{\gamma(\tau) r^2}{r_0^2}\right) dz' dt', \tag{5}$$

where

$$G(z', z, \tau) = \exp\left\{-\frac{(z-z')^2}{4\alpha\tau} - \frac{(z+z')^2}{4\alpha\tau}\right\} \exp\lambda \frac{(h_1 - z')}{h_2}, \tag{6}$$

$Q = W_0 / \pi r_0^2 h_2 \rho c_p$, $\gamma(\tau) = r_0^2 / (r_0^2 + 4\alpha\tau)$ and $\tau = t - t'$. For t f t_h, the upper limit in the integral (6) is replaced by t_h. This expression incorporates an image heat source disribution so that there is no heat flux at the air interface by symmetry.

Typical temperature profiles on the axis of symmetry $r = 0$ are illustrated in Figure 2 and 3 for the heating and cooling phase. Here we have used $t_h = 100\mu s$, $W_0 = 75mW$, $T_A = 25^0 C$, $\alpha = 3.2x10^{-8} m^2 / s$, $\rho = 1.3x10^3 kg / m^3$, $c_p = 1.9x10^3 J / kg.K$, $r_0 = 15\mu m$, $h_1 = 6\mu m$, $h_2 = 2\mu m$ and $\lambda = 1$. It can be seen that maximum temperatures near $1000^0 C$ are achieved in the dye layer just before the laser is turned off. Such high temperatures are an inevitable consequence of the high laser wattage.

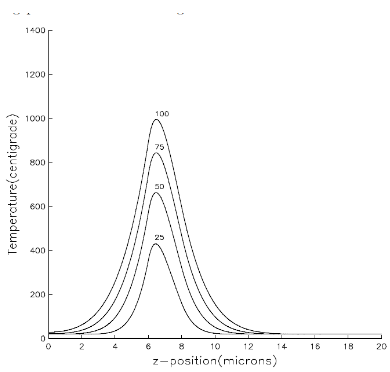

Figure 2: Temperature profiles on $r = 0$, at times $t = 25, 50, 75$ and $100\mu s$ for an uniformly distributed absorber in the dye layer (Heating phase).

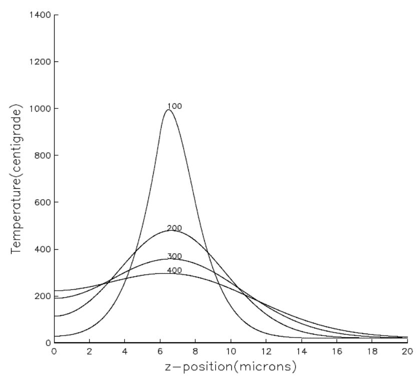

Figure 3: Temperature profiles on $r = 0$, at times t=100, 200, 300 and 400μs for an uniformly distributed absorber in the dye layer (cooling phase).

Consider next the temperature field $T(r, z, t)$ produced by the laser. Assuming for simplicity that the thermal properties are constants and equal in all the layers, the unsteady axi-symmetric heat conduction equation is

$$\frac{\partial T}{\partial t} = \alpha \left\{ \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r} \right) \right\} + \frac{q(r,z,t)}{\rho c_p}, \qquad (7)$$

Consider next the diffusion of dye into the receiver sheet. The dye concentration field $c(r,z,t)$ satisfies the linear diffusion equation

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial c}{\partial z} \right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial c}{\partial r} \right), \qquad (8)$$

where $D = D(T)$ has been measured experimentally by Beck et al for the copolymer in the receiver sheet [7]. He finds that below $225^0 C$ it is sensitive functions of temperature which can be fitted to a curve of Arhenius type i.e.

$$D = A\exp\left(-E / T\right), \qquad (9)$$

where $A = 14.0m^2 / \sec$ and $E = 12404K$ and T here is measured on degrees K. The maximum temperature predicted by the thermal model using realistic values fort he laser wattage and thermal properties is about $1000^0 C$ which is well outside the range of validity of the experimental results. Indeed (9) predicts unrealistically large values of D at these high temperatures. We have assumed that the diffusion coefficient D cannot exceed $D_{\max}$, taken to be the thermal diffusion coefficient $\alpha = 3.2x10^{-8} m^2 \sec^{-1}$ here. The graph of $D(T)$ incorporating this cut-off is illustrated in Figure 4.

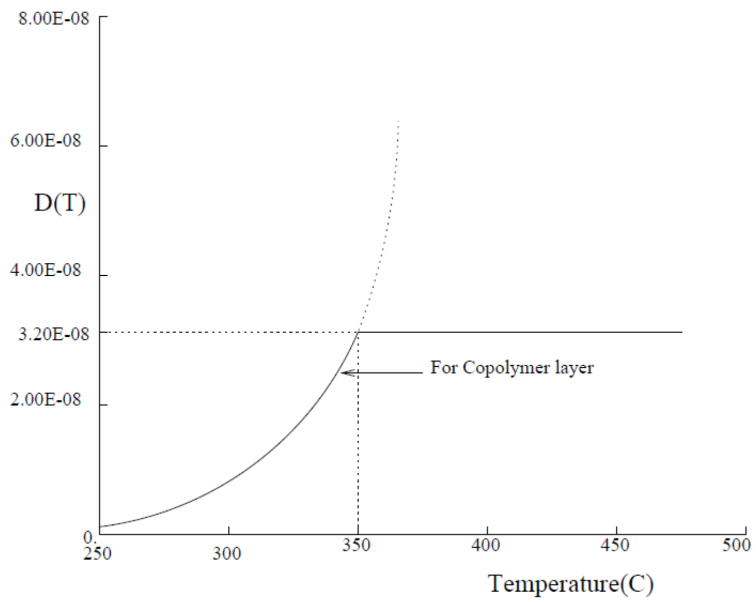

Figure 4: The graph of $D(T)$ showing a cut off where $D = \alpha$.

We assume that at time $t = 0$ the dye is confined to the dye-coat layer and has uniform concentration there i.e.

$$c \equiv \begin{cases} 1, & h_1 \le z \le h_1 + h_2, \\ 0, & elsewhere. \end{cases} \tag{10}$$

For all values of r. To prevent dye diffusion into the PET layer adjacent to the air layer the L2T2 system incorporates a very thin barrier layer at $z = h_1$. As a consequence the flux of dye at $z = h_1$ is zero. i.e.

$$\frac{\partial c}{\partial z} = 0, \quad z = h_1, \quad t \ge 0. \tag{11}$$

Finally at large distances

$$\frac{\partial c}{\partial z} \to 0, \quad z \to \infty, \quad r \to \infty. \tag{12}$$

The diffusion equation (8) has been solved numerically by finite differences. The fraction $f(r)$ of dye which has diffused into the receiver sheet after a long time ($800\mu s$) as a function of r/r_0 is shown in Figure 5, for three heating times $t_h = 75\mu s$, $100\mu s$ and $125\mu s$.

It can be seen that $f(r)$ is largest on $r = 0$, where the temperature is highest, and varies significantly with the heating time. When the heating time is $100\mu s$, we predict that on $r = 0$ about 52 percent of the dye is in the receiver sheet eventually. Significant dye diffusion occured during the cooling phase. The total amount of dye $m(r)$ (scaled on its initial value) as a function of r/r_0 at $t = 800\mu s$ is shown in Figure 6 for $t_h = 100\mu s$. In the absence of radial diffusion $m(r)$ is a constant, but we see that in fact dye has diffused towards the axis of symmetry because it is hotter there.

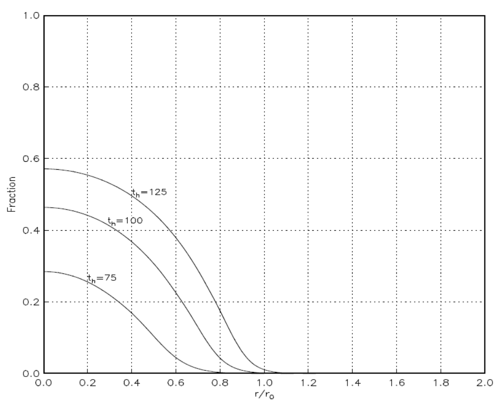

Figure 5: The fraction f of dye in the receiver sheet as a function of r/r_0 at time $800\mu s$ when t_h=75,100 and $125\mu s$.

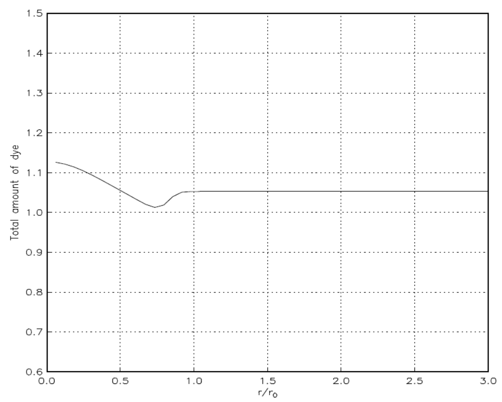

Figure 6: The total amount of dye(divided by the initial amounts of dye), in the dye and receiver sheet as a function of r/r_0 at time $800\mu s$ when $t_h = 100\mu s$.

A significant feature of the solutions obtained is that there is a sharp moving concentration front on which the temperature T_f varies slowly with time and has a low value typically about $260^0 C$. This is a direct consequence of the sensitive this dye front based upon the assumption $T_f / E = 1$ yields a differential equation for the position of this front.

3. Conclusions

In this paper we first formulated a simplified mathematical model which allowed us to determine the temperature field $T(r,z,t)$ produced by an axisymmetric gaussian laser heat source in a dye layer sandwiched between two copolymer layers. When the heating time was $100\mu s$, maximum temperatures of about $1000^0 C$ where found in the dye layer. Subsequently the temperatures fell slowly as heat diffused axially and radially.
The dye concentration field $c(r,z,t)$ was then determined by solving a diffusion equation which incorporates a temperaturesensitive dye diffusion coefficient. The fraction of dye in the receiver sheet as a function of radius after $800\mu s$ was then determined. This is strongly influenced by the fact that there is a sharp moving concentration front on which the temperature T_f is a slowly varying function of time and has a low value typically about $260^0 C$. This is a direct consequence of the sensitive behaviour of $D(T)$ at these temperatures. An asymptotic analysis of the front based upon the assumption $T_f / E = 1$ yields a differential equation for the position of the front.

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