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Diffusion Approximation to Neutron Transport Equation with First Kind of Chebyshev Polynomials

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Abstract: The first kind of Chebyshev polynomials are used for the series expansion of the neutron angular flux in neutron transport theory. The first order approximation known as the diffusion approximation is applied to one-dimensional neutron transport equation to determine the diffusion coefficients of one-speed neutrons for selected values of the scattering parameters.

***Key words*: First kind of Chebyshev polynomials, neutron transport equation, diffusion approximation**

Birinci Tip Chebyshev Polinomları ile Nötron Transport Denklemine Difüzyon Yaklaşımı

Özet: Nötron transport teorisinde açısal akının seri açılımı için I. Tip Chebyshev polinomları kullanıldı. Tek hızlı nötronların difüzyon katsayılarını belirlemek için difüzyon yaklaşımı olarak bilinen ilk mertebe yaklaşımı saçılma parametrelerinin seçilen değerleri için tek boyutlu nötron transport denklemine uygulanmıştır.

***Anahtar kelimeler*: I. tip Chebyshev polinomları, nötron transport denklemi, difüzyon Yaklaşımı**

1. Introduction

In nuclear reactor theory, the neutron transport equation is referred to as an integro-differential equation which describes the neutron population and interactions throughout the system. It is not very easy to solve transport equation accurately since the neutrons are neutral particles and thus move in complicated paths in the systems. To get a satisfactory solution, this problem can be solved with diffusion approximation which is also used in other branches of engineering. To do this, one should have knowledge about nuclear cross sections, their energy dependence and also neutron distributions. Diffusion approximation which is widely used for the first estimates of a nuclear system has the great advantage to predict many of the properties of nuclear reactors such as neutron transport and energy spectrum. This theory gives adequate results if the number of secondary neutrons per collision, c, is close to unity [1].

There are many methods developed for the solutions of the problems related with neutron transport theory. The spherical harmonics (P_N) method is one of the most preferred one among the polynomial expansion based techniques for the problems in neutron transport theory. The lowest-order P_1 approximation or more widely known as diffusion approximation can be derived quite simply for arbitrary geometry and its results are compared with rigorous solutions obtained from singular eigenfunction expansions [2,3]. However, the spherical harmonics method is not the unique one valid for all cases such as the computation of the extrapolated end points. Therefore, Aspelund, Conkie and Yabushita used the Chebyshev polynomials instead of Legendre polynomials in series expansion of the angular flux and reported the advantages of their new methods [4,6].

Since then, the first and second kind of Chebyshev polynomials are often used in transport theory calculations. In some of recent studies, the eigenvalues spectrum, the criticality problem and diffusion length calculations are investigated for bare and reflected slabs and spheres using higher order applications of the T_N method, i.e. the first kind of Chebyshev polynomials approximation [7,11]. However, in any of those studies, T_1 approximation is not used for the calculation of the diffusion length. Therefore, it is worth to apply the T_1 approximation to determine the diffusion length of one-speed neutrons in a homogeneous slab. In this study, it is shown that the T_1 approximation can easily be applied to transport equation for the determination of the diffusion length as well as the traditional P_1 approximation. Furthermore, numerical results for the diffusion length of the neutrons are calculated using the present method for various values of the collision parameter c and they are given in the tables with the literature values for comparison.

2. Theory

The neutron transport equation for one-speed neutrons in a homogeneous slab with isotropic scattering can be written as,

$$\mu \frac{\partial \psi(x,\mu)}{\partial x} + \sigma_T \psi(x,\mu) = \frac{c\sigma_T}{2} \int_{-1}^{1} \psi(x,\mu')\,\mathrm{d}\mu' , \tag{1}$$

where $\psi(x,\mu)$ is the neutron angular flux at position x traveling in direction μ, cosine of the scattering angel between the neutron velocity vector and the positive x-axis. σ_T is the total macroscopic cross section [2].

In this work, the neutron angular flux is expanded in terms of the first kind of Chebyshev polynomials as described in Ref. [7] to analyze the diffusion or asymptotic relaxation length for one-speed neutrons in a slab,

$$\psi(x,\mu) = \frac{\Phi_0(x) T_0(x)}{\pi\sqrt{1-\mu^2}} + \frac{2}{\pi\sqrt{1-\mu^2}} \sum_{n=1}^{\infty} \Phi_n(x) T_n(x) , \quad -a \le x \le a , \quad -1 \le \mu \le 1 . \tag{2}$$

The orthogonality and recurrence relations of the Chebyshev polynomials of first kind can be given as [10],

$$\int_{-1}^{1} \frac{T_m(x) T_n(x)}{\sqrt{1-\mu^2}} \mathrm{d}\mu = \begin{cases} 0; & m \ne n \\ \dfrac{\pi}{2}; & m = n \ne 0 \\ \pi; & m = n = 0 \end{cases} \tag{3}$$

$$T_{n+1}(\mu) - 2\mu T_n(\mu) + T_{n-1}(\mu) = 0 . \tag{4}$$

Eq. (2) is inserted into Eq. (1), then multiplying the resultant equation by $T_0(\mu)$ and $T_1(\mu)$ and then integrating over $\mu \in [-1, 1]$ by means of Eq. (3) and (4), one can obtain the equations of moments for $n = 0$ $n = 1$, respectively;

$$\frac{\mathrm{d}\Phi_1(x)}{\mathrm{d}x} + \sigma_T(1-c)\Phi_0(x) = 0 \tag{5}$$

$$\frac{\mathrm{d}\Phi_2(x)}{\mathrm{d}x} + \frac{\mathrm{d}\Phi_0(x)}{\mathrm{d}x} + 2\sigma_T\Phi_1(x) = 0 \tag{6}$$

Eqs. (5) and (6) are T_1 equations for the neutron transport equation and the condition for $n = 1$ stated in Eq. (6) is equivalent to diffusion approximation as in spherical harmonics (P_N) method by setting $\mathrm{d}\Phi_{N+1}(x)/\mathrm{d}x = 0$ [12]. In the case of T_1 approximation, a familiar equation known as Fick's law is obtained by taking $\mathrm{d}\Phi_2(x)/\mathrm{d}x = 0$ in Eq. (6),

$$\Phi_1(x) = -\frac{1}{2\sigma_T}\frac{\mathrm{d}\Phi_0(x)}{\mathrm{d}x} \tag{7}$$

By following the same procedure as in Fick's law, diffusion equation can be obtained for the conservation of the neutron flux in a medium without source when Eq. (7) is inserted into Eq. (5),

$$\frac{\mathrm{d}^2\Phi_0(x)}{\mathrm{d}x^2} - 2\sigma_T^2(1-c)\Phi_0 = 0 \tag{8}$$

Since Eq. (8) is obtained with the same procedure as in P_1 approximation, the diffusion length (L) in T_1 approximation can be given according to Eq. (8),

$$L = \frac{1}{\sigma_T\sqrt{2(1-c)}} \tag{9}$$

In addition, an analytic expression for the neutron scalar flux can be obtained for T_1 approximation and for $c < 1$ by solving the differential equation given in Eq. (8),

$$\Phi_0(x) = Ae^{\sigma_T\sqrt{2(1-c)}\,x} + Be^{-\sigma_T\sqrt{2(1-c)}\,x}, \tag{10}$$

where the constants A and B can be found from boundary conditions.

Other expressions for diffusion lengths obtained from the method of separation of variables (often referred to as asymptotic relaxation length), traditional P_1 (diffusion) approximation, and U_1 approximation can be given as, respectively;

$$1 = cL\tanh^{-1}\frac{1}{L}, \tag{11}$$

$$L = \frac{1}{\sigma_T \sqrt{3(1-c)}}\,, \tag{12}$$

$$L = \frac{1}{2\sigma_T \sqrt{(1-c)}}\,, \tag{13}$$

These methods are not needed to be discussed here since they can be found elsewhere [9,12].

3. Results and Discussion

The diffusion approximation for one-speed neutrons in a homogeneous slab with isotropic scattering is applied and an analytic expression is derived. For this analysis, neutron angular flux is expanded in terms of the Chebyshev polynomials of first kind and first two terms of the series expansion of the angular flux are taken (n = 0 and n = 1) to obtain the equations of moments, i.e. T_1 approximation. Then, numerical results for the diffusion lengths are calculated from Eqs. (9), (11), (12) and (13) for various values of the collision parameter c and they are tabulated in the tables for comparison. In all cases, the total macroscopic cross section is assumed to be its normalized value, $\sigma_T = 1\ \text{cm}^{-1}$.

The diffusion lengths given in Eqs. (11) and (12) are quoted from Bell and Glasstone [12] and represent the asymptotic relaxation length from the transport theory (exact) and the diffusion length from simple diffusion theory (traditional P_1 approximation), respectively. Eq.(13) gives the result for the diffusion length from U_1 approximation [9]. The diffusion lengths are computed for the values of c ranging from 0 (weakly absorbing medium) to 1 (highly scattering medium).

As seen from Table 1, diffusion lengths obtained from the present method is quite similar to the exact results. Meanwhile, the same manner is also valid for the results obtained from the well-known P_1 and U_1 approximations. The numerical results obtained for the diffusion length from the present T_1 approximation may not seemed to be very close to the results obtained from transport theory (exact), completely. However, this does not show the inefficiency of the T_N method since it has been applied to other types of transport problems successfully in previous studies [7,8]. Therefore, it is aimed in this study that this method may be applied to other problems in science and engineering and may give consistent results.

Table 1. Diffusion lengths L obtained from T_1 approximation and literature values, (cm)

c	T_1 (present work, Eq. (9))	U_1 (Eq. (13))	P_1 (Eq. (12))	Exact (Eq.(11))
0.99	7.071	5.000	5.774	5.797
0.98	5.000	3.536	4.082	4.116
0.95	3.162	2.236	2.582	2.635
0.90	2.236	1.581	1.826	1.903
0.80	1.581	1.118	1.291	1.408
0.50	1.000	0.707	0.816	1.044
0	0.707	0.500	0.577	1.000

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