Conformable Fractional Order PI Controller Design and Optimization for Permanent Magnet Synchronous Motor Speed Tracking System

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ABSTRACT
The use of permanent magnet synchronous motor (PMSM) is increasing rapidly to meet the need to increase efficiency in variable speed drive systems used in the industry, in recent years. This paper aims to improve the speed control performance of the PMSM based systems. To achieve this, a PMSM speed controller is designed based on the conformable fractional order proportional integral (CFOPI) method. CFOPI controller coefficients $k_p$, $k_i$ and $\gamma$ are optimized using response surface method (RSM). To validate the success of the proposed scheme, the CFOPI controller and the integer order PI (IOPi) controller are tested under the same simulation model and the results are compared. The proposed method grants robust performance with less computational load then the classical fractional order controllers for variable referenced PMSM speed tracking systems. The CFOPI controller can be applied easily for industrial variable speed drive systems which is using PMSM to improve the performance and stability of the systems.

Keywords: Fractional calculus, Response surface methodology, Permanent magnet motors, Variable speed drives

Sürekli Mıknatıslı Senkron Motorun Hız Takip Sistemi için Uygun Kesirli PI Kontrolör Tasarımı ve Optimizasyonu

ÖZET
Son yıllarda, endüstride kullanılan değişken hızlı tahrik sistemlerinde verimliliği artırma ihtiyacı karşılamak için sürekli mıknatıslı senkron motor (PMSM) kullanımı hızla artmaktadır. Bu makalede PMSM tabanlı hız kontrol sistemlerinin performansının artırılması hedeflemektedir. Bu amaçla, uygun kesirli oransal-integral (CFOPI) tabanlı PMSM hız kontrolörü tasarlanmıştır. CFOPI kontrolör katsayları $k_p$, $k_i$ ve $\gamma$ yanıt yüzeyi metodu (RSM) kullanılarak optimize edilmiştir. Önerilen sistem başarılarının kanıtı için CFOPI ve tamsayı PI (IOPi) kontrolörler aynı simülaion modeli üzerinde test edilmiş ve sonuçları karşılaştırılmıştır. Önerilen yöntem klasik kesirli kontrolörlerle göre daha az hesaplama yüküne sahiptir ve değişken referanslı PMSM hız izleme sistemleri için daha dayanıklı performans sağlamaktadır. CFOPI kontrolörü, PMSM kullanılan endüstriyel değişken hızlı tahrik sistemlerinin performansını ve kararlılığını artırmak için kolaylıkla uygulanabilir.

Anahtar Kelimeler: Kesirli analiz, Yanıt yüzeyi metodu, Sabit mıknatıslı motorlar, Değişken hızlı sürücüler
I. INTRODUCTION

In recent years, with the increasing competition in the industry, the use of more efficient systems has gained great importance. Increasing the performance of variable speed control systems, which are widely used in the industry, has also become mandatory. Therefore, the use of permanent magnet synchronous motor (PMSM), which have a robust construction and high efficiency in variable speed control systems, is increasing rapidly. Two types of control methods are applied in PMSMs, scalar control and vector control [1]. Scalar control method has a simple algorithm. In this method, the \( v/f \) (voltage / frequency) ratio should be kept constant [2]. Although this method is sufficient for simple applications, it cannot be used in every application since it cannot work at low-speed values. The vector control method has more complex algorithm than the scalar method. In response to this, it is more preferred because it can work efficiently at low speeds.

Fractional calculus has become popular in control engineering applications in recent years. Thanks to its memory structure, it can control the systems more effectively. It is also more flexible as it has more coefficients compared to the integer order controllers. Besides these advantages, it is a disadvantage that there is excessive computational load. In addition, traditional fractional order definitions cannot satisfy some cases such as Leibniz rule, product rule, chain rule and formula of the derivative of the quotient [3]. In order to find solutions to these problems, a new fractional order definition was proposed by Khalil et al. in 2014. This method is called conformable fractional order (CFO) calculus. A distinct advantage of this method is its simple structure, so its computational load is very low compared to the traditional fractional order methods.

One of the biggest problems in industrial control systems is the performance improvement. To do this, many optimization methods are used. These are artificial neural network [4-6], fuzzy logic [7-9], particle swarm algorithm [10-12], genetic algorithm [13, 14], response surface method (RSM) [15-20] etc. Applications using RSM have been increasing rapidly in recent years. The ability of modelling the systems easily on an experimental basis is effective in this increase. RSM can give successful results using only a few experimental data.

There are many studies about fractional order control and parameter tuning methods used in PMSM systems: Haghighatnia and Shandiz presented a study about CFO sliding mode controller [21]. Proposed controller is tested on three different nonlinear system such as dynamic model of a gyro system, a second-order nonlinear spring damper system and a fractional order Arneodo system. The simulation results show that the proposed controller have faster convergence speed and lower chattering effect. Luo et al. proposed a fractional-order robust controller for position and velocity control of a PMSM servo system [22]. The study aims the cogging effect compensation. The proposed controller is compared with a traditional integer order controller in simulation and experimental tests. The results indicate the success of the proposed method against the integer order method. Zong et al. presented the fractional order proportional integral (PI) controller application for PMSM speed adjusting system [23]. They used Riemann-Liouville definition in integral controller part of the PI controller. The simulation results demonstrate the fractional order PI controller has great performance on disturbance rejection of the PMSM system. Zhang et al. presented a study about fractional order sliding mode velocity control of a PMSM [24]. In this study, the proposed controller is designed according to Lyapunov stability method. Simulation and experimental results show that the proposed method has smaller chattering effect than the integer order sliding mode controller. Also, it has robust structure to external load disturbance and parameter variations. Zheng and Pi studied on optimization method for tuning the fractional order PI controller of a PMSM system [25]. They used differential evolution algorithm as the tuning method. Integral of time absolute error and the phase margin values is taken for the optimization criterion. To validate the robustness and the dynamic response performance of the proposed method, speed tracking experiments are performed. Experimental results illustrate the proposed tuning method has a robust structure and optimal dynamic response performance under gain variations. Qiao et al. presented an adaptive fractional order two degree of freedom PI speed control of PMSM [26]. Fractional order
The generalized predictive control method is used as an adaptation mechanism. Simulation and experimental results demonstrate the effectiveness of the proposed method. Rajasekhar et al. presented an optimization study about tuning the fractional order controller for PMSM drive system [27]. In this study, hybrid differential artificial bee colony algorithm is used in tuning the fractional order controller. They compared the proposed method with conventional methods in simulations. The results show the success of the proposed algorithm. Tabatabaei proposed an adaptive fractional order velocity controller for PMSM system [28]. Lyapunov method is used as an adaptation mechanism. The simulation results demonstrate the proposed controller has robust performance under the external load torque and the mechanical parameter uncertainties. Karthikeyan et al. presented a study about speed and current regulation of permanent magnet synchronous generator (PMSG) wind turbine [29]. In this study, fractional order nonlinear adaptive control method is used as a speed and current controller. PMSG model is simulated in LabVIEW environment and the results show the success of the proposed method. Saraji and Ghanbari proposed a fractional order PID speed controller for PMSM in aerospace applications [30]. They stated the tuning advantage of the fractional order controller due to it has more parameters than the integer order controller. The simulations are implemented in MATLAB environment for comparison of the fractional order controller and the integer order controller. The results demonstrate the proposed controller has better performance than the other method.

CFO operator advantages are given in many studies [3, 31-34]. On the other hand, using CFO as a controller is a new study area. This paper aims to design a CFO based PI controller to improve the performance of PMSM speed control system.

The paper is organized as the following order: Dynamic model of PMSM is explained in Section 2; Conformable Fractional Calculus expressions are given in Section 3; Conformable Fractional Order PI (CFOPI) Controller design is given in Section 4; the simulation studies and results are presented in Section 5; and the conclusion is given in Section 6.

II. DYNAMIC MODEL OF PERMANENT MAGNET SYNCHRONOUS MOTOR

d-q axis rotating reference frame model of the PMSM can be described as following equation [35]

\[
\frac{di_d}{dt} = -\frac{R_s}{L_d} i_d + \omega_e \frac{L_q}{L_d} i_q + \frac{1}{L_d} v_d \\
\frac{di_q}{dt} = -\omega_e \frac{L_d}{L_q} i_d - \frac{R_s}{L_q} i_q - \frac{\lambda_f}{L_q} + \frac{1}{L_q} v_q
\]  

(1)

where \(i_d\) and \(i_q\) are the d-q axis currents; \(v_d\) and \(v_q\) are the d-q axis voltages; \(R_s\) is the stator winding resistance; \(L_d\) and \(L_q\) are the d-q axis inductances; \(\lambda_f\) and \(\omega_e\) are the permanent magnet flux linkage and the electrical angular speed, respectively. The electromagnetic torque equation can be written as

\[
T_e = \frac{3p}{4} \lambda_f i_q, \quad \left( K_i = \frac{3p}{4} \lambda_f \right)
\]

(2)

this equation can be simplified as follows

\[
T_e = K_i i_q
\]

(3)

In Equation (2) \(T_e\) is the electromagnetic torque and \(p\) is the pole pairs. Dynamic equation of PMSM for mechanical load can be described as
\[ J_m \frac{d}{dt} \omega_r + B_m \omega_r = T_r - T_L \quad (4) \]

where \( J_m \) is the rotor inertia, \( \omega_r \) is the rotor speed, \( B_m \) is the friction factor and \( T_L \) is the load torque.

## II. CONFORMABLE FRACTIONAL ORDER PI CONTROLLER

Let consider \( f : [0, \infty) \to \mathbb{R} \) as a function. The conformable derivative of \( f \) of order \( \gamma \) can be defined as [3]

\[
T^\gamma_r (f)(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\gamma}) - f(t)}{\varepsilon} 
\]

\[
(5)
\]

where \( t > 0, \gamma \in (0, 1) \). \( T^\gamma_r \) is the CFO derivative operator. Let \( h = \varepsilon t^{1-\gamma} \) in Equation (5) and \( \varepsilon = ht^{\gamma-1} \).

If \( f(t) \) is a \( \gamma \) differentiable function, CFO derivative of \( f(t) \) can be defined as follows

\[
T^\gamma_r (f)(t) = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{ht^{\gamma-1}} 
= t^{1-\gamma} \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} 
= t^{1-\gamma} \frac{df}{dt}(t) 
\]

\[
(6)
\]

The CFO integral of \( f(t) \) can be defined as Equation (7) or (8)

\[
I^\gamma_r (f)(t) = \int_0^t f \left( \frac{t}{t^{1-\gamma}} \right) dt 
\]

\[
I^\gamma_r (f)(t) = \sum_{i=0}^{\infty} \frac{f(t_i)}{t_i^{\gamma}} dt_i, \quad (8)
\]

where \( i \geq 0, \gamma \in (0, 1) \) and \( I^\gamma_r \) is the CFO integral operator.

Integer order IOPI controller input expression is follows:

\[
u(t) = k_pe(t) + k_i \int e(t) dt \quad (9)
\]

where \( e(t) \) is the error function, \( k_p \) is the proportional control coefficient and \( k_i \) is the integral control coefficient. CFOPI controller can defined as changing of the integral operator part of the integer order PI (IOPI) controller. CFOPI controller input equation can be described as below:

\[
u(t) = k_pe(t) + k_i I^\gamma_r [e(t)] dt \quad (10)
\]

where \( I^\gamma_r \) is the CFO integral operator from the Equation (7), \( e(t) \) is the error function, \( k_p \) is the proportional control coefficient, \( k_i \) is the integral control coefficient and \( \gamma \) is the order of the conformable fractional integral.
III. OPTIMIZATION AND SIMULATIONS

In this study, Three-phase 1.1 kW, 50 Hz PMSM is used and its parameters are listed in Table 1. The proposed simulation model of the system is designed on MATLAB/Simulink program. CFOPI based PMSM speed control system block diagram is shown in Figure 1. The proposed CFOPI controller algorithm is written in a function block and it is used as speed controller in Figure 1.

Table 1. PMSM parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Voltage (line-line)</td>
<td>220 V</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>750 rpm</td>
</tr>
<tr>
<td>Stator Resistance ($R_s$)</td>
<td>2.875 Ω</td>
</tr>
<tr>
<td>Armature Inductance ($L_s$)</td>
<td>0.00153 H</td>
</tr>
<tr>
<td>Permanent magnet flux linkage ($\lambda_d$)</td>
<td>0.175 Wb</td>
</tr>
<tr>
<td>Pole Pairs ($p$)</td>
<td>4</td>
</tr>
<tr>
<td>Rotor Inertia ($J$)</td>
<td>0.002 kg.m$^2$</td>
</tr>
<tr>
<td>Friction Factor ($F$)</td>
<td>0.0008 N.m.s</td>
</tr>
</tbody>
</table>

Figure 1. CFOPI based PMSM speed control system block diagram.

In Figure 1, flux and torque controllers are used in simple PI controller type and pulse width modulation (PWM) based full bridge inverter with gate driver is used in 3-phase inverter block. Flux controller parameters are $k_p=0.1$, $k_i=10$ and torque controller parameters are $k_p=0.1$, $k_i=10$. In vector control method, the rated flux and the maximum speed is limited by the stator voltages, the rated current and the back emf of the PMSM. This limited speed is called the rated speed. In order for the motor to run above this speed, the back emf must be reduced. If the $I_d$ current is reduced to zero, the back emf decreases and thus the motor speed rises above the rated speed. This method is called as field weakening control of the motor.

In Figure 1, CFOPI speed controller parameters $k_p$, $k_i$, $\gamma$ are optimized by using RSM for minimizing the steady-state error in ramps ($e_{ss,r}$) and constant sections ($e_{ss,c}$), chattering effect in error ($e_{ch}$) and the settling time ($T_s$).

RSM is a mathematical and statistical technique used in the development of a functional relationship between a response $y$ and a number of associated input variables $x_1$, $x_2$, ..., $x_k$. In general, low-degree polynomial model of RSM can be defined as [36]
where $x = (x_1, x_2, \ldots, x_k)^T$, $f(x)$ is a vector function of $p$ elements that consists of powers and cross-products of powers of $x_1$, $x_2$, ... , $x_k$ up to a certain degree denoted by $d \geq 1$. $\beta$ is a vector of unknown constant coefficients of $p$. $e$ is a random experimental error (approximately zero). This is conditioned on the belief that the model provides an appropriate representation of the response. The quantity $f(x)\beta$ represents the mean response, the expected value of $y$ and is defined by $\mu(x)$. Two models are generally used in RSM. The first-order model is

$$Y_u = \beta_0 + \sum_{i=1}^{n} \beta_i X_{iu} + e_u$$

and the second-order model can be defined as follows

$$Y_u = \beta_0 + \sum_{i=1}^{n} \beta_i X_{iu} + \sum_{i<j} \beta_{ij} X_{iu} X_{ju} + \sum_{i=1}^{n} \beta_{ii} X_{iu}^2 + e_u$$

In Equation (12) and (13), $Y_u$ is the system response; $X_{iu}$ and $X_{ju}$ are coded values of $i^{th}$ and $j^{th}$ input parameters, respectively; $\beta_0$, $\beta_i$, and $\beta_{ij}$ are the regression coefficients; $i$ and $j$ are the linear and quadratic coefficients; $e_u$ is the residual experimental error of $u^{th}$ observation.

In this study, RSM tool of Minitab program is used. In this program, a simulation table is created according to selected design mode (central composite full design) and the limit values of inputs ($k_p$, $k_i$, $\gamma$ for CFOPI and $k_p$, $k_i$ for IOPI). 20 simulations were performed for optimization of CFOPI controller coefficients. The simulation results for CFOPI controller are given in Table 2.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$\gamma$</th>
<th>$e_{ss,r}$</th>
<th>$e_{ss,c}$</th>
<th>$e_{cht}$</th>
<th>$T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.10</td>
<td>0.1</td>
<td>0.010</td>
<td>0.006</td>
<td>0.076</td>
<td>0.135</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>1.55</td>
<td>0.5</td>
<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
<tr>
<td>3</td>
<td>55</td>
<td>1.55</td>
<td>0.1</td>
<td>0.005</td>
<td>0.004</td>
<td>0.046</td>
<td>0.103</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>3.00</td>
<td>0.1</td>
<td>0.001</td>
<td>0.001</td>
<td>0.085</td>
<td>0.102</td>
</tr>
<tr>
<td>5</td>
<td>55</td>
<td>0.10</td>
<td>0.5</td>
<td>0.119</td>
<td>0.005</td>
<td>0.045</td>
<td>2.200</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>3.00</td>
<td>0.9</td>
<td>0.615</td>
<td>0.014</td>
<td>0.040</td>
<td>2.950</td>
</tr>
<tr>
<td>7</td>
<td>55</td>
<td>1.55</td>
<td>0.5</td>
<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>3.00</td>
<td>0.1</td>
<td>0.050</td>
<td>0.004</td>
<td>0.046</td>
<td>0.102</td>
</tr>
<tr>
<td>9</td>
<td>55</td>
<td>1.55</td>
<td>0.5</td>
<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
<tr>
<td>10</td>
<td>55</td>
<td>1.55</td>
<td>0.9</td>
<td>0.120</td>
<td>0.021</td>
<td>0.041</td>
<td>12.540</td>
</tr>
<tr>
<td>11</td>
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<td>0.9</td>
<td>0.070</td>
<td>0.055</td>
<td>0.070</td>
<td>0.101</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>1.55</td>
<td>0.5</td>
<td>0.011</td>
<td>0.015</td>
<td>0.041</td>
<td>0.142</td>
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<tr>
<td>13</td>
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<td>0.020</td>
<td>0.001</td>
<td>0.041</td>
<td>0.190</td>
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<tr>
<td>14</td>
<td>55</td>
<td>1.55</td>
<td>0.5</td>
<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
<tr>
<td>15</td>
<td>100</td>
<td>1.55</td>
<td>0.5</td>
<td>0.045</td>
<td>0.008</td>
<td>0.068</td>
<td>0.101</td>
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<tr>
<td>16</td>
<td>10</td>
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<td>0.1</td>
<td>0.005</td>
<td>0.015</td>
<td>0.039</td>
<td>0.105</td>
</tr>
<tr>
<td>17</td>
<td>100</td>
<td>3.00</td>
<td>0.9</td>
<td>0.075</td>
<td>0.021</td>
<td>0.065</td>
<td>4.550</td>
</tr>
<tr>
<td>18</td>
<td>10</td>
<td>0.10</td>
<td>0.9</td>
<td>0.640</td>
<td>0.153</td>
<td>0.045</td>
<td>40.500</td>
</tr>
<tr>
<td>19</td>
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<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
<tr>
<td>20</td>
<td>55</td>
<td>1.55</td>
<td>0.5</td>
<td>0.050</td>
<td>0.003</td>
<td>0.043</td>
<td>0.280</td>
</tr>
</tbody>
</table>
According to Table 2, $e_{ss\_r}$, $e_{ss\_c}$, $e_{cht}$ and $T_s$ based mathematical model of the system is obtained from Minitab RSM tool. These are given in Equation (14), (15), (16) and (17), respectively.

\[
e_{ss\_r} = -0.023 + 0.00004k_p - 0.0920k_i + 0.430\gamma
+ 0.000011k_p^2 + 0.0306k_i^2 + 0.358\gamma^2
- 0.000046k_p^2 * k_i - 0.00740k_p * \gamma - 0.0121k_i * \gamma
\]  
\[
e_{ss\_c} = 0.0296 - 0.001011k_p - 0.0149k_i + 0.0531\gamma
+ 0.000006k_p^2 + 0.00218k_i^2 + 0.0881\gamma^2
+ 0.000213k_p^2 * k_i - 0.000549k_p * \gamma - 0.0338k_i * \gamma
\]  
\[
e_{cht} = 0.04143 - 0.000265k_p + 0.00099k_i + 0.00035\gamma
+ 0.000006k_p^2 + 0.000562k_i^2 + 0.01051\gamma^2
+ 0.000004k_p * k_i - 0.000181k_p * \gamma - 0.005603k_i * \gamma
\]  
\[
T_s = 4.04 - 0.047k_p - 4.09k_i + 5.4\gamma
- 0.00026k_p^2 + 0.26k_i^2 + 35.5\gamma^2
+ 0.0804k_p * k_i - 0.270k_p * \gamma - 7.13k_i * \gamma
\]

Optimum values for $k_p$, $k_i$, $\gamma$ are determined by using Minitab RSM tool to minimize the $e_{ss\_r}$, $e_{ss\_c}$, $e_{cht}$ and $T_s$. RSM optimization plot for CFOPI controller is given in Figure 2. The optimum CFOPI controller parameters are shown in Table 3.

![RSM optimization plot for CFOPI controller](image)

**Figure 2. RSM optimization plot for CFOPI controller**

**Table 3. The optimum CFOPI controller parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>$k_p$</td>
<td>27,2727</td>
</tr>
<tr>
<td>$k_i$</td>
<td>1,2717</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0,1889</td>
</tr>
</tbody>
</table>
To show the success of the proposed controller, it is compared with integer order PI (IOPI) controller which is known to all. Similar studies can be seen in reference [37] and [38] for speed tracking of PMSM. IOPI controller parameters $k_p$ and $k_i$ are optimized under the same conditions as the proposed controller in Minitab RSM tool. The simulation results for IOPI controller are given in Table 4.

**Table 4. Simulation table for IOPI controller**

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$k_p$</th>
<th>$k_i$</th>
<th>$e_{ss_r}$</th>
<th>$e_{ss_c}$</th>
<th>$e_{cht}$</th>
<th>$T_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
<td>1.55</td>
<td>0.130</td>
<td>0.049</td>
<td>0.030</td>
<td>47.70</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
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<td>0.130</td>
<td>0.049</td>
<td>0.030</td>
<td>47.70</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>3.00</td>
<td>0.070</td>
<td>0.037</td>
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<td>4</td>
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<td>0.049</td>
<td>0.030</td>
<td>47.70</td>
</tr>
<tr>
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<td>55</td>
<td>3.00</td>
<td>0.120</td>
<td>0.017</td>
<td>0.044</td>
<td>24.35</td>
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<tr>
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<td>55</td>
<td>1.55</td>
<td>0.130</td>
<td>0.049</td>
<td>0.030</td>
<td>47.70</td>
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<td>0.129</td>
<td>0.042</td>
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<tr>
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<td>0.006</td>
<td>0.037</td>
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<tr>
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<td>0.630</td>
<td>0.007</td>
<td>0.041</td>
<td>27.20</td>
</tr>
</tbody>
</table>

According to Table 4, $e_{ss_r}$, $e_{ss_c}$, $e_{cht}$ and $T_s$ based mathematical model of the system is obtained from Minitab RSM tool. These are given in Equation (18), (19), (20) and (21).

$$
\begin{align*}
    e_{ss_r} &= 0.81514 - 0.018447k_p - 0.00531k_i \\
             &\quad - 0.000110k_p^2 + 0.000303k_i^2 + 0.000023k_p^* k_i \\
    e_{ss_c} &= 0.1192 - 0.000330k_p - 0.0625k_i \\
             &\quad - 0.000003k_p^2 + 0.00586k_i^2 + 0.000414k_p^* k_i \\
    e_{cht} &= 0.05411 - 0.000809k_p - 0.01121k_i \\
             &\quad + 0.000009k_p^2 + 0.00239k_i^2 + 0.000088k_p^* k_i \\
    T_s &= 91.14 + 0.202k_p - 42.15k_i \\
         &\quad - 0.00528k_p^2 + 3.95k_i^2 + 0.1764k_p^* k_i
\end{align*}
$$

Optimum values for $k_p$ and $k_i$ are determined by using Minitab RSM tool to minimize the $e_{ss_r}$, $e_{ss_c}$, $e_{cht}$ and $T_s$. RSM optimization plot for IOPI controller is given in Figure 3. The optimum IOPI controller parameters are shown in Table 5.
The speed controller models CFOPI and IOPI are tested with the optimum parameters which are given in Table 3 and Table 5.

In first test, PMSM is started with nominal load. It reaches 700 rpm reference speed at 0.1\textsuperscript{th} second. Reference speed and actual speed trends are compared in Figure 4 for CFOPI controller and Figure 7 for IOPI controller. The zoomed graphs are also given in Figure 5 and Figure 8 so that the difference is clearly visible. Finally, error graphs for each controller are shown in Figure 6 and Figure 9.
Figure 5. Comparison of the reference and the actual speeds (zoomed graph) for CFOPI controller (Test 1).

Figure 6. CFOPI controller error (Test 1).

Figure 7. Comparison of the reference and the actual speeds for IOPI controller (Test 1).

Figure 8. Comparison of the reference and the actual speeds (zoomed graph) for IOPI controller (Test 1).
When Figure 6 is examined for CFOPI controller, the $e_{ss,r}$, $e_{ss,c}$, $e_{sl}$ and $T_s$ values are 0.004, 0.005, 0.039 and 0.104, respectively. The $e_{ss,r}$, $e_{ss,c}$, $e_{sl}$ and $T_s$ values for IOPI controller are also examined on Figure 9 and the values are obtained as 0.18, 0.016, 0.036 and 31.95, respectively.

In second test, reference speed is setting in different values while operating. This test shows the CFOPI and IOPI controllers’ speed tracking performance. PMSM is started with nominal load and tracks the reference speed value. Reference speed and actual speed trends are compared in Figure 10 for CFOPI controller and Figure 12 for IOPI controller. Error graphs for each controller are shown in Figure 11 and Figure 13.
When Figure 11 and Figure 13 are examined for both controllers, it is seen that the IOPI controller error value changes depending on the reference speed value. In response to this, the CFOPI controller error is almost the same value for each sector of the speed tracking test. This shows that the proposed controller has a robust structure. The results show that the CFOPI controller has less steady-state error on ramp and constant sections. Also, it has better settling time than IOPI controller. It is observed that the IOPI controller is better only at error chattering amplitude value. When the results of references [37] and [38] and the results of the CFOPI controller are examined, it is seen that the proposed method is successful.

IV. CONCLUSION

In this study, CFOPI controller is designed for PMSM speed control system. The proposed controller is established on conformable fractional integral definition proposed by Khalil et al. in 2014. The controller coefficients $k_p$, $k_i$ and $\gamma$ are optimized using RSM for minimizing the $e_{ss,r}$, $e_{ss,c}$, $e_{cht}$ and $T_s$ values. CFOPI controller coefficients are obtained as $k_p=27.2727$, $k_i=1.2717$ and $\gamma=0.1889$. The optimum $e_{ss,r}$, $e_{ss,c}$, $e_{cht}$ and $T_s$ values are obtained as 0.004, 0.005, 0.039 and 0.104, respectively. The proposed controller compared with IOPI controller to validate the success. The simulations results show that the CFOPI controller has a little much bigger error chattering amplitude than IOPI controller. In response to this it has less steady-state error on ramp and constant sections, and shorter settling time. It is also shown that the proposed controller has a robust structure for variable reference speed tracking systems. As a result, the CFOPI controller can be used effectively in industrial variable speed systems because of its robust structure and simple algorithm.
V. REFERENCES


