



Increasing the Efficiency of Percentile Parameter Estimation Method for Weibull Distribution

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Abstract

The Weibull distribution is one of the most widely used probability distributions in statistical applications. The percentile parameter estimation method is commonly used in parameter estimation of the two-parameter Weibull distribution in terms of easy computability and efficiency. The effectiveness of the percentile method depends on the selected percentile points and chosen empirical distribution function. In this study, the appropriate empirical distribution function and percentile points were determined to increase the efficiency of the percentile method for two-parameter Weibull distribution by simulation study.

Keywords: Weibull distribution; Percentile estimation; Empirical distribution function.

Weibull Dağılımı için Persentil Parametre Tahmin Yönteminin Etkinliğinin Artırılması

Öz

Weibull dağılımı istatistiksel uygulamalarda en yaygın kullanılan olasılık dağılımlarından biridir. Percentil parametre tahmin yöntemi kolay hesaplanama ve etkinlik açısından iki parametrelili Weibull dağılımının parametre tahmininde yaygın olarak kullanılır. Percentil yönteminin etkinliği seçilen percentil noktalara ve seçilen ampirik dağılım fonksiyonuna bağlıdır. Bu çalışmada, simülasyon çalışması ile iki parametrelili Weibull dağılımı için percentil yönteminin etkinliğini artırmak amacıyla uygun ampirik dağılım fonksiyonu ve percentil noktaları belirlenmiştir.

Anahtar Kelimeler: Weibull dağılımı; Percentil tahmin; Ampirik dağılım fonksiyonu.

1. Introduction

The Weibull distribution is one of the widely used probability distributions which has many different applications and used for solving a variety of problems from many different disciplines. The Weibull distribution has many useful properties because its form is flexible to model different shapes. Weibull distribution has increasing, decreasing or stable failure rates depending on the values of the shape parameter. This feature has provided widespread use of the Weibull distribution. Because of its importance, many different estimation methods have been proposed for estimating parameters of Weibull distribution.

Percentile method is one of the commonly used methods for estimating the parameters of Weibull distribution and has some advantages unlike the other estimation methods in terms of ease compute and efficiency in parameter estimation [1].

The percentile method may be applied via two different approaches. The approach 1 is based on percentiles and is structurally similar to the traditional method of moments. The approach 2 is mainly obtained by minimizing the squared Euclidean distance between the sample percentile and population percentile [2].

The effectiveness of the percentile estimators depends on the selected percentile points and chosen empirical distribution function.

Dubey [3] proposed 17th and 97th percentiles for shape parameter and 40th and 82th percentiles for scale parameter. Seki and Yokoyama [4] suggested 31st and 63rd percentiles to estimate shape and scale parameters. Wang and Keats [5] proposed 15th and 63rd percentiles for parameter estimation of the Weibull distributions. Marks [6] stated that the best results were obtained with 10th and 90th percentiles for the shape and scale parameters.

In this study, we aimed to determine the appropriate empirical distribution and the percentile points to increase the effectiveness of the percentile estimators for the Weibull distribution. We used mean squared error (*MSE*) and mean total error (*MTE*) as performance criteria in the simulation study.

2. Weibull Distribution

The two-parameter Weibull distribution is specified by the probability density function (pdf)

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha}, \tag{1}$$

where $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters, respectively. The corresponding cumulative distribution function (cdf) and survival function (sf) are given as bellow:

$$F(x; \alpha, \beta) = 1 - e^{-\left(\frac{x}{\beta}\right)^\alpha}, \tag{2}$$

$$S(x; \alpha, \beta) = e^{-\left(\frac{x}{\beta}\right)^\alpha}. \tag{3}$$

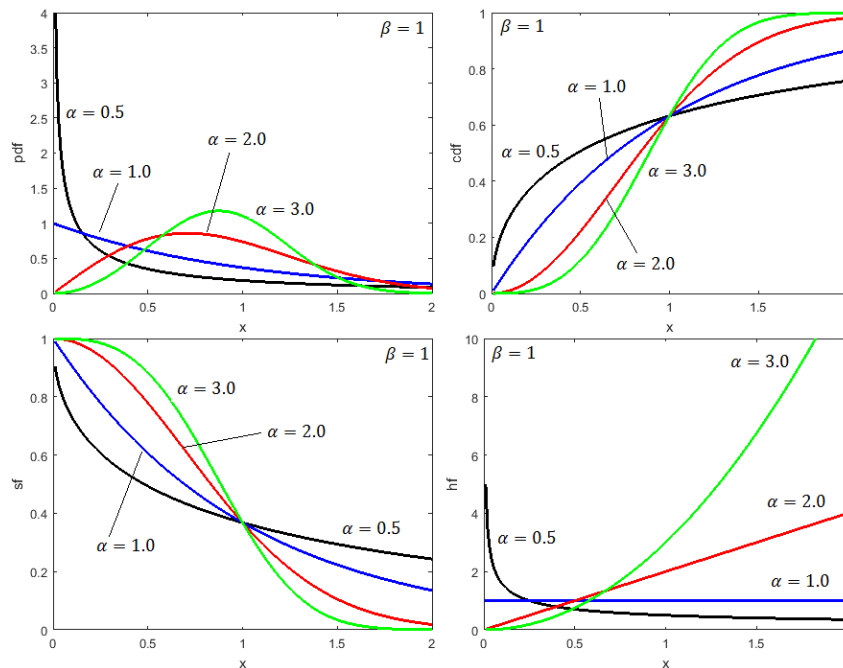


Figure 1: The curves of the pdf, cdf, sf and hf of Weibull distribution for selected parameter values

The hazard function (hf) is given

$$h(x; \alpha, \beta) = \alpha \beta^{-\alpha} x^{\alpha-1}, \tag{4}$$

which can be increase, decrease or constant depending on , $\alpha > 1$, $\alpha < 1$ or $\alpha = 1$, respectively.

The curves of the pdf, cdf, sf and hf of Weibull distribution are shown in Fig. 1 for selected parameter values.

3. Percentile Method (PM)

The quantile function corresponding to Eqn. (1) is

$$x_p = \beta[-\ln(1 - p)]^{1/\alpha}. \quad (5)$$

Percentile estimations of the Weibull distribution are calculated based on the obtained equations from selected two percentile points in the quantile function. The percentile estimators of the shape and scale parameters for Weibull distribution are respectively given by

$$\hat{\alpha} = \frac{\ln[-\ln(1-p_1)] - \ln[-\ln(1-p_2)]}{\ln(x_{p_1}) - \ln(x_{p_2})}, \quad (6)$$

$$\hat{\beta} = \frac{x_{p_1}}{(-\ln(1-p_1))^{1/\hat{\alpha}}}, \quad (7)$$

where p_1 and p_2 are selected two percentile points and x_{p_1} and x_{p_2} are observation values corresponding to p_1 and p_2 , respectively. There are many approaches for selection of the p_1 and p_2 . For example, we can obtain an estimate for scale parameter by using one of these approaches like setting $p_1 = 1 - \exp(-1) \cong 0.632$ (63.2th percentile point) into Eqn. (7). In this approach, the percentile estimators for Weibull distribution are given

$$\hat{\beta} = x_{1-\exp(-1)}, \quad (8)$$

$$\hat{\alpha} = \left(\frac{\ln[-\ln(1-p_2)]}{\ln(x_{p_2}) - \ln(\hat{\beta})} \right), \quad (9)$$

where $0 < x_{p_2} < x_{0.632}$ [7]. There are some suggestions for the optimum value of p_2 . Wang and Keats [5] showed by simulation that 0.15 would be the correct choice for p_2 . Seki and Yokoyama [8] suggested $p_2 = 0.31$.

The percentile based estimators for α and β of Weibull distribution according to approach 2 can be obtained by minimizing

$$\sum_{i=1}^n \left\{ \ln x_{(i)} - \ln \beta - \frac{1}{\alpha} \ln(-\ln(1 - \hat{F}_{(i)})) \right\}^2, \quad (10)$$

with respect to α and β . The percentile estimators for α and β of the Weibull distribution are given by

$$\hat{\alpha} = \frac{A^2 - nB}{AC - nD}, \quad (11)$$

$$\hat{\beta} = \exp\left(\frac{1}{n}\left(C - \frac{A}{\hat{\alpha}}\right)\right), \quad (12)$$

where $A = \sum_{i=1}^n \ln(-\ln(1 - \hat{F}_{(i)}))$, $B = \sum_{i=1}^n \{\ln(-\ln(1 - \hat{F}_{(i)}))\}^2$, $C = \sum_{i=1}^n \ln x_i$ and $D = \sum_{i=1}^n \ln x_i \ln(-\ln(1 - \hat{F}_{(i)}))$.

4. Simulation Study

In this study, we designed a simulation according to different sample size and different parameter values with the following purposes:

- To determine the best empirical distribution function respect to *MSE* for approach 1 in the percentile method.
- To choose the best selected second percentile point p_2 when setting $p_1 = 1 - \exp(-1) \cong 0.632$ for approach 1.
- To determine the best percentile pair among from several selected percentile pairs for approach 1 in the percentile estimation.
- To determine the best empirical distribution function for the best successful estimation according to *MTE* in the percentile estimation by approach 2.

Table 1: The selected empirical distribution functions

Kaplan-Meir	$\hat{F}_1(x_{(i)}) = \frac{i}{n}$
Herd-Johnson (Mean rank)	$\hat{F}_2(x_{(i)}) = \frac{i}{n+1}$
Median (Symmetrical)	$\hat{F}_3(x_{(i)}) = \frac{i-0.5}{n}$
Median rank	$\hat{F}_4(x_{(i)}) = \frac{i-0.3}{n+0.4}$
Approximate Normal	$\hat{F}_5(x_{(i)}) = \frac{i-\frac{3}{8}}{n+\frac{1}{4}}$

Firstly, we examined fitness between empirical distribution functions and actual cumulative distribution function values of the Weibull distribution for approach 1 in the percentile estimation method. The selected EDFs are given in Table 1.

We used mean squared error (*MSE*) in the comparison of the fitness between empirical distribution functions and actual cumulative distribution function values of Weibull distribution for the different parameter values and sample sizes.

$$MSE_m = \frac{1}{100000} \sum_{r=1}^{100000} \sum_{i=1}^n \frac{[F(x_{(i)}; \alpha, \beta) - \hat{F}_m(x_{(i)})]^2}{n} \tag{13}$$

In the simulation study, 100000 random samples of sizes 10, 30, 50 and 100 were generated from Weibull distributions having $\alpha = 0.5, 1, 1.5, 2, 3.4, 5$ and $\beta = 1, 10$. The comparisons of the empirical distribution functions respect to *MSE* are given in Table 2.

In accordance with Table 2, the empirical distribution function that best matches the actual cumulative distribution function values of the Weibull distribution is Hard-Johnson's empirical distribution function for all the cases. Hard-Johnson's function will be used as empirical distribution function in the determination of the best percentile pairs for the percentile estimations according to approach 1.

At this stage of the study, we will try to determine the best percentile point for the $p_1 = 1 - \exp(-1) \cong 0.632$ in the approach 1. For this, the $MSE(\hat{\alpha})$ values for all candidate points were determined with 0.01 increases from 0 to 0.63.

$$MSE(\hat{\alpha}) = \frac{1}{10000} \sum_{r=1}^{10000} (\alpha - \hat{\alpha}_r)^2 \tag{14}$$

In the simulated study for the different parameter values and sample sizes, the percentile point with the smallest $MSE(\hat{\alpha})$ was determined, and the results are given in Table 3.

Table 2: The comparison of the EDFs according to MSE for different sample sizes and parameter values

α	n	$\beta = 1$					$\beta = 10$				
		\hat{F}_1	\hat{F}_2	\hat{F}_3	\hat{F}_4	\hat{F}_5	\hat{F}_1	\hat{F}_2	\hat{F}_3	\hat{F}_4	\hat{F}_5
0.5	10	0.01825	0.01503	0.01567	0.01523	0.01537	0.01830	0.01522	0.01588	0.01544	0.01557
	30	0.00583	0.00548	0.00556	0.00551	0.00552	0.00573	0.00537	0.00546	0.00540	0.00542
	50	0.00342	0.00328	0.00332	0.00330	0.00330	0.00342	0.00328	0.00331	0.00329	0.00330
	100	0.00167	0.00163	0.00164	0.00164	0.00164	0.00168	0.00165	0.00166	0.00166	0.00166
1	10	0.01851	0.01534	0.01604	0.01558	0.01572	0.01842	0.01530	0.01595	0.01551	0.01564
	30	0.00578	0.00539	0.00549	0.00543	0.00545	0.00574	0.00537	0.00545	0.00540	0.00542
	50	0.00340	0.00326	0.00329	0.00327	0.00328	0.00342	0.00329	0.00332	0.00330	0.00330
	100	0.00169	0.00166	0.00167	0.00167	0.00167	0.00168	0.00165	0.00165	0.00165	0.00165
1.5	10	0.01798	0.01479	0.01545	0.01500	0.01514	0.01837	0.01522	0.01595	0.01547	0.01562
	30	0.00574	0.00534	0.00544	0.00538	0.00540	0.00571	0.00532	0.00542	0.00536	0.00538
	50	0.00339	0.00325	0.00328	0.00326	0.00326	0.00346	0.00332	0.00335	0.00333	0.00334
	100	0.00166	0.00162	0.00163	0.00163	0.00163	0.00168	0.00165	0.00166	0.00165	0.00166

2	10	0.01822	0.01512	0.01577	0.01533	0.01546	0.01829	0.01508	0.01573	0.01529	0.01542
	30	0.00577	0.00543	0.00551	0.00546	0.00547	0.00564	0.00530	0.00539	0.00533	0.00535
	50	0.00337	0.00324	0.00327	0.00325	0.00326	0.00342	0.00331	0.00334	0.00332	0.00333
	100	0.00167	0.00164	0.00165	0.00165	0.00165	0.00169	0.00166	0.00166	0.00166	0.00166
3.4	10	0.01833	0.01530	0.01598	0.01553	0.01567	0.01829	0.01502	0.01573	0.01526	0.01541
	30	0.00578	0.00540	0.00549	0.00543	0.00545	0.00578	0.00541	0.00549	0.00543	0.00545
	50	0.00344	0.00331	0.00335	0.00333	0.00333	0.00340	0.00327	0.00330	0.00328	0.00329
	100	0.00167	0.00164	0.00164	0.00164	0.00164	0.00167	0.00163	0.00164	0.00163	0.00163
5	10	0.01833	0.01520	0.01589	0.01543	0.01557	0.01859	0.01529	0.01599	0.01553	0.01567
	30	0.00570	0.00536	0.00545	0.00539	0.00541	0.00573	0.00535	0.00544	0.00538	0.00540
	50	0.00346	0.00332	0.00335	0.00333	0.00334	0.00340	0.00327	0.00330	0.00328	0.00329
	100	0.00168	0.00165	0.00166	0.00165	0.00165	0.00170	0.00167	0.00168	0.00167	0.00167

Table 3: The percentile points values which have the smallest $MSE(\hat{\alpha})$ for different sample sizes and parameter values

Sample Size	$\beta = 1$						$\beta = 10$						
	α												
	n	0.5	1	1.5	2	3.4	5	0.5	1	1.5	2	3.4	5
10	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
30	0.11	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
50	0.12	0.12	0.12	0.14	0.10	0.12	0.12	0.10	0.12	0.14	0.10	0.14	0.14
100	0.11	0.12	0.11	0.12	0.12	0.13	0.14	0.13	0.12	0.13	0.13	0.13	0.12

According to Table 3, the most suitable percentile points is found ranges 0.10 to 0.14 for the $p_1 \cong 0.632$ in the approach 1. The best percentile point was 0.13 with respect to $MSE(\hat{\alpha})$ in the shape parameter estimation of the Weibull distribution by this approach when the small sample size is $n = 10$.

Here, the best percentile pair will be determined from the selected percentile pairs in the percentile estimation by approach 1 of the Weibull distribution parameters. The comparison is based on MTE criterion defined by

$$MTE = \frac{1}{10000} \sum_{r=1}^{10000} \left(\left(\frac{\hat{\alpha}_r - \alpha}{\alpha} \right)^2 + \left(\frac{\hat{\beta}_r - \beta}{\beta} \right)^2 \right). \tag{15}$$

In this study, the selected percentile pairs are denoted in Table 4.

Table 4: The selected pairs of percentage point

$A: P\left(\frac{1}{n+1}, \frac{n}{n+1}\right)$	$B: P(0.25, 0.75)$	$C: P\left(0.25, \frac{n}{n+1}\right)$	$D: P\left(\frac{1}{n+1}, 0.75\right)$
$E: P\left(0.5, \frac{n}{n+1}\right)$	$F: P\left(\frac{1}{n+1}, 0.5\right)$	$G: P(0.632, 0.30)$	$H: P(0.05, 0.95)$
$I: P(0.10, 0.90)$	$J: P(0.17, 0.97)$	$K: P(0.24, 0.93)$	$L: P(0.632, 0.12)$

In the simulation study, the obtained *MTE* values for different parameter values and sample sizes are given in Table 5.

According to Table 5, in case of the scale parameter is equal 1, *J* and *K* percentile pairs point are more successful than other percentile pairs for ≥ 1 . The *B* percentile pair is more successful than other pairs for $\alpha < 1$ and $\beta = 1$. In case of the scale parameter is set as $\beta = 10$, *B* percentile pair is more successful than other percentile pairs for $\alpha \leq 1$. The *K* percentile pair is more successful than other pairs for $\alpha = 2$ and $\beta = 10$. In addition, *J* percentile pair showed more performance than other pairs for $\alpha = 5$ and $\beta = 10$.

Finally, we performed a simulation study to investigate the performance of empirical distribution functions in the percentile estimates for parameters of the Weibull distribution by approach 2. The comparison results of the empirical distribution functions according to *MTE* for different sample sizes and parameter values in the approach 2 are given in Table 6.

Table 5: The comparison results of the selected percentage points according to *MTE*

β	α	<i>n</i>	A	B	C	D	E	F	G	H	I	J	K	L
1	0.5	10	1.725	1.201	1.376	1.126	1.405	3.677	1.617	1.725	1.725	1.538	1.376	1.679
		30	0.899	0.265	0.418	0.288	0.320	1.478	0.394	0.457	0.348	0.544	0.328	0.408
		50	0.689	0.150	0.262	0.178	0.182	0.530	0.176	0.260	0.186	0.226	0.165	0.177
		100	0.553	0.070	0.148	0.104	0.091	0.308	0.090	0.131	0.086	0.108	0.078	0.094
	1	10	0.341	0.462	0.315	0.336	0.540	0.640	0.428	0.341	0.341	0.312	0.315	0.552
		30	0.201	0.109	0.104	0.132	0.106	0.280	0.132	0.124	0.103	0.119	0.092	0.160
		50	0.167	0.064	0.068	0.093	0.066	0.155	0.075	0.077	0.061	0.062	0.054	0.075
		100	0.143	0.031	0.040	0.065	0.036	0.107	0.038	0.042	0.030	0.032	0.027	0.046
	1.5	10	0.264	0.523	0.271	0.332	0.620	0.586	0.424	0.264	0.264	0.250	0.271	0.610
		30	0.143	0.111	0.077	0.139	0.098	0.237	0.126	0.096	0.084	0.081	0.073	0.168
		50	0.117	0.065	0.050	0.101	0.062	0.146	0.075	0.059	0.051	0.046	0.045	0.076
		100	0.098	0.030	0.029	0.076	0.034	0.107	0.037	0.033	0.025	0.024	0.023	0.049
	2	10	0.279	0.647	0.304	0.391	0.773	0.685	0.503	0.279	0.279	0.270	0.304	0.752
		30	0.142	0.131	0.079	0.167	0.112	0.266	0.147	0.099	0.090	0.079	0.078	0.203
		50	0.113	0.076	0.050	0.123	0.071	0.170	0.088	0.061	0.055	0.047	0.049	0.089
		100	0.093	0.035	0.029	0.094	0.039	0.127	0.043	0.034	0.028	0.024	0.024	0.059
3.4	10	0.405	1.052	0.464	0.615	1.262	1.079	0.798	0.405	0.405	0.401	0.464	1.220	
	30	0.191	0.207	0.112	0.264	0.172	0.408	0.230	0.141	0.131	0.107	0.113	0.324	
	50	0.149	0.120	0.070	0.196	0.110	0.267	0.138	0.087	0.080	0.065	0.072	0.140	
	100	0.118	0.055	0.040	0.152	0.061	0.201	0.067	0.047	0.041	0.033	0.036	0.094	
5	10	0.576	1.534	0.668	0.891	1.843	1.566	1.159	0.576	0.576	0.575	0.668	1.780	
	30	0.268	0.300	0.158	0.384	0.249	0.590	0.333	0.200	0.188	0.150	0.161	0.472	
	50	0.208	0.174	0.099	0.285	0.159	0.387	0.201	0.123	0.114	0.092	0.102	0.203	
	100	0.163	0.080	0.056	0.222	0.088	0.292	0.097	0.067	0.058	0.046	0.051	0.137	
10	0.5	10	17.08	10.58	13.31	10.63	11.52	34.29	14.61	17.08	17.08	14.93	13.31	14.45
		30	8.534	2.384	4.152	2.538	2.950	13.81	3.678	4.403	3.298	5.195	3.177	3.661
		50	6.913	1.280	2.456	1.468	1.617	4.867	1.534	2.410	1.679	2.083	1.498	1.525
		100	5.311	0.602	1.454	0.796	0.804	2.826	0.779	1.204	0.781	1.018	0.707	0.775

1	10	2.365	1.891	1.938	1.779	2.033	3.527	2.508	2.365	2.365	2.076	1.938	2.173
	30	1.519	0.549	0.764	0.621	0.617	1.753	0.765	0.878	0.692	0.906	0.641	0.732
	50	1.321	0.315	0.491	0.401	0.368	0.847	0.393	0.540	0.390	0.442	0.343	0.376
	100	1.115	0.156	0.305	0.243	0.194	0.537	0.202	0.288	0.193	0.233	0.173	0.194
1.5	10	1.091	1.139	0.911	0.934	1.245	1.679	1.630	1.091	1.091	0.949	0.911	1.128
	30	0.717	0.305	0.351	0.354	0.315	0.785	0.426	0.422	0.339	0.404	0.307	0.377
	50	0.629	0.174	0.227	0.240	0.192	0.423	0.234	0.263	0.195	0.208	0.171	0.208
	100	0.538	0.086	0.140	0.156	0.102	0.279	0.117	0.140	0.097	0.111	0.087	0.105
2	10	0.738	1.021	0.648	0.731	1.138	1.297	1.567	0.738	0.738	0.649	0.648	0.898
	30	0.467	0.244	0.230	0.286	0.233	0.549	0.353	0.283	0.234	0.257	0.208	0.287
	50	0.407	0.137	0.148	0.202	0.145	0.319	0.198	0.176	0.136	0.137	0.120	0.164
	100	0.347	0.067	0.090	0.138	0.077	0.217	0.097	0.094	0.067	0.073	0.060	0.081
3.4	10	0.562	1.264	0.573	0.748	1.443	1.368	2.099	0.562	0.562	0.526	0.573	0.961
	30	0.305	0.257	0.165	0.301	0.217	0.487	0.395	0.207	0.183	0.170	0.160	0.283
	50	0.255	0.141	0.105	0.224	0.138	0.315	0.225	0.128	0.109	0.096	0.098	0.167
	100	0.209	0.068	0.060	0.164	0.074	0.226	0.107	0.067	0.054	0.050	0.048	0.080
5	10	0.645	1.740	0.712	0.973	1.999	1.810	2.945	0.645	0.645	0.628	0.712	1.271
	30	0.318	0.338	0.187	0.395	0.274	0.610	0.529	0.233	0.215	0.184	0.186	0.364
	50	0.256	0.183	0.117	0.299	0.176	0.409	0.302	0.143	0.128	0.107	0.117	0.217
	100	0.204	0.088	0.065	0.223	0.093	0.142	0.074	0.063	0.063	0.055	0.057	0.103

Table 6: The comparison results of the empirical distribution functions according to *MTE* for different sample sizes and parameter values in the approach 2

α	n	$\beta = 1$					$\beta = 10$				
		\hat{F}_1	\hat{F}_2	\hat{F}_3	\hat{F}_4	\hat{F}_5	\hat{F}_1	\hat{F}_2	\hat{F}_3	\hat{F}_4	\hat{F}_5
0.5	10	0.6209	1.2054	0.8843	0.8976	0.9490	5.6093	11.584	8.2614	7.0162	8.9809
	30	0.1872	0.2579	0.2137	0.2210	0.2236	1.6950	2.4080	1.9811	1.8365	2.0847
	50	0.1157	0.1439	0.1238	0.1278	0.1284	1.0498	1.3319	1.1438	1.0858	1.1906
	100	0.0563	0.0647	0.0579	0.0595	0.0594	0.5066	0.5902	0.5293	0.5127	0.5445
1	10	0.2533	0.2632	0.2600	0.2384	0.2500	1.3328	1.6937	1.4358	1.1833	1.4813
	30	0.0791	0.0840	0.0755	0.0751	0.0757	0.4369	0.4981	0.4433	0.4030	0.4545
	50	0.0485	0.0511	0.0457	0.0461	0.0461	0.2704	0.2972	0.2696	0.2523	0.2753
	100	0.0247	0.0258	0.0232	0.0236	0.0235	0.1341	0.1440	0.1332	0.1277	0.1354
1.5	10	0.2562	0.2186	0.2479	0.2147	0.2254	0.7622	0.7778	0.7332	0.6168	0.7262
	30	0.0770	0.0762	0.0696	0.0683	0.0684	0.2385	0.2494	0.2277	0.2120	0.2300
	50	0.0468	0.0473	0.0421	0.0422	0.0420	0.1458	0.1518	0.1391	0.1330	0.1407
	100	0.0242	0.0247	0.0218	0.0222	0.0220	0.0728	0.0758	0.0700	0.0683	0.0709
2	10	0.2998	0.2418	0.2884	0.2444	0.2567	0.5974	0.5413	0.5556	0.4810	0.5305
	30	0.0889	0.0864	0.0791	0.0773	0.0772	0.1810	0.1815	0.1671	0.1610	0.1668
	50	0.0540	0.0540	0.0477	0.0479	0.0474	0.1099	0.1117	0.1018	0.1004	0.1023
	100	0.0281	0.0285	0.0249	0.0254	0.0251	0.0555	0.0569	0.0519	0.0519	0.0524
3.4	10	0.4657	0.3656	0.4500	0.3761	0.3950	0.5763	0.4645	0.5415	0.4838	0.4879
	30	0.1374	0.1326	0.1212	0.1182	0.1177	0.1700	0.1646	0.1514	0.1538	0.1483
	50	0.0834	0.0831	0.0730	0.0732	0.0724	0.1030	0.1026	0.0916	0.0949	0.0911

	100	0.0436	0.0442	0.0383	0.0391	0.0385	0.0531	0.0539	0.0476	0.0497	0.0479
	10	0.6727	0.5264	0.6515	0.5433	0.5705	0.7258	0.5715	0.6938	0.6165	0.6133
	30	0.1984	0.1913	0.1749	0.1705	0.1697	0.2137	0.2060	0.1888	0.1932	0.1838
5	50	0.1205	0.1200	0.1053	0.1056	0.1043	0.1296	0.1289	0.1138	0.1189	0.1129
	100	0.0631	0.0639	0.0552	0.0565	0.0556	0.0675	0.0683	0.0595	0.0627	0.0599

In the large sample sizes ($n = 50$ and 100), the difference between *MTE* values of the empirical distribution functions was not significant according to Table 6. In case of the shape parameter is set as $\alpha = 0.5$, Kaplan-Meir's empirical distribution function has lower *MTE* value than other empirical distribution functions for $\beta = 1$ and 10 .

5. Conclusion

In this paper, we aimed at improving the effectiveness of the percentile method for two parameter Weibull distribution. Based on simulation study, the empirical distribution function that best fits the actual Weibull cumulative distribution function is the empirical distribution function of Hard Johnson according to *MSE*. In conclusion, the most suitable percentage points ranged from 0.10 to 0.14 when p_1 value was nearly set as 0.632 in the percentile method by approach 1. In general, (0.17, 0.97) and (0.24, 0.93) percentile pairs were the best successful percentile pairs for $\alpha \geq 1$. In case of the $\alpha < 1$, (0.25, 0.75) percentile pair was more successful than other percentile pairs. Also, median (symmetrical) empirical distribution function has been more successful in case of large sample size ($n = 100$) for percentile method by approach 2.

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