Empirical Parametrization of COVID-19 Virus Pandemic

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Abstract — *Published global data for the infections and mortalities caused by the corona virus for the period between January-22 and June-1-2020 are used to build and test an empirical model equation that uses past trends in the data to predict near and medium future behavior of the pandemic. Extrapolation of the empirical model equation suggested for the time development of the numbers of infections and deaths is used to make approximate predictions concerning the disease leveling of the ultimate numbers of infections and deaths. Such analysis can be useful in the assessment of the particular measures adapted by each country in its efforts to hinder the spread of the virus infections.*

Keywords: COVID-19, Corona, Modeling, Pandemic modeling.
Mathematics Subject Classification: 93A30, 97M10.

1 Introduction

It is now a trivial fact that the whole world has been taken off guard as far as the COVID-19 pandemic is concerned. Furthermore, this pandemic exposed the insufficiency of the degree of readiness of most twenty first century healthcare systems round the world to handle such pandemic. The main reason for such uncoordinated handling of the situation is related to the unique combination of two intrinsic characteristics of this virus. The first is the virus ability to cause infection through the respiration inlets [1, 2, 3]. The second is the virus relatively long incubation period of about two weeks [4, 5, 6]. Such incubation period allows an infected asymptomatic individual to travel round the world several times, unknowingly transmitting the disease to whoever he comes in nearby with. These two properties played a crucial role in assisting the wide spread of infections through the utilization of the world air traffic. All this has caused the fast spread of the pandemic worldwide.

Under such desperate situation of an exponential worldwide pandemic, world countries found themselves almost helpless in taking any measures to combat the pandemic apart from imposing one degree or another social distancing measure. This was coupled with some *ad hoc* efforts to find a vaccine or treatment for the disease by the most advanced countries. In spite of all such latter efforts, the nearest projected date for an effective medical treatment or vaccine seems months away in the least [7, 8]. With world modern civilization at an almost complete halt, the only current hope and course of action so far is to wait and see if nature will play some role in bringing the pandemic to a halt or a slow-down at least. The only helping hands man can give are the commitment to social distancing [9, 10, 11], and the full obedience to the rules of general hygiene.

After the initial shock wave, the major part of the world efforts in combating this pandemic has concentrated on finding medical treatments and vaccines that can help in halting the virus spread. Such effort have been going side by side with many other efforts devoted to establishing and refining statistical, mathematical and empirical modeling related to the nature and properties of this pandemic [12-20]. Such modeling plays an important role in the decision making process regarding the adaptation of the optimum strategies aiming to hinder the virus spread on one hand and minimize the resulting economic damages on the other hand.

It is the purpose of this work to suggest and test a collective empirical model that can be used in assessing the development and spread of this pandemic in the world as a whole and in any particular country. Such model can help decision makers in their evaluation of the different actions that need to be taken.

2 The Data

The data used in this work came from the world meter web site [21]. Starting on January-22-2020, the data used are update up to June-1-2020. This site is updated daily. It provides data for the numbers of infections, deaths and recoveries for all world countries and their corresponding global totals. It must be said however that the site compiles data issued by government of all countries listed. The accuracies of these data depend on each particular government openness about the matter.

Figure 1 shows a summary of the percentage daily increases in infections and deaths registered globally between Jan-22 and June-1-2020. The figure shows that at the early stage when the majority of infection took place in China, the percentage daily increases were so high reaching the value of about 35% at some stage. However, this ratio dropped to only about 2% some time by the end of February when China announced that it had the pandemic under control. At that stage, the number of infections elsewhere were small compared to those in China. This situation did not last long and soon enough Europe and the United States began to feel the burden, registering increasing numbers of infections and mortalities by the first week of March. The situation worsened with daily increases reaching 12% by the second half of March.

Social distancing which may be coupled with other still not well understood environmental conditions helped in bringing the pandemic under some kind of contagious control. This resulted in a drop in daily increased infections ratio to about 2% again after about four months from the original onset of the pandemic.

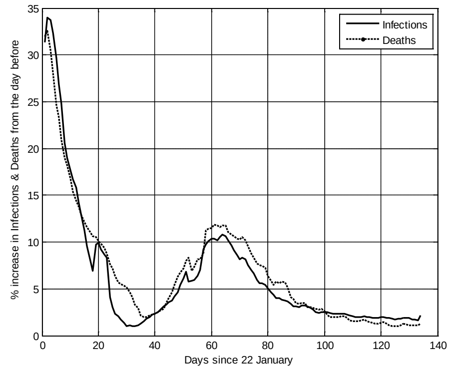

Figure 1. Percentage daily global increases of COVID-19 infections and death

3 Empirical Modelling of the Pandemic

Severe enough as it is, the current coronavirus pandemic should not deviate in its behavior from that of any other typical pandemic. Such behavior is characterized by three stages. The first is the initial onset which is usually relatively slow. The second is the fast and sharp rise in the number of infections and casualties. The third is the leveling of the number of daily infections and deaths. This chain of events are expected to take place even with the worst case scenario of no vaccine and no treatment. In such a case, and as far as COVID-19 is concerned, it has become a well-established fact that the mortality rate within those infected is few percent. This means that infections leveling must be reached even if the whole world population is to be infected at the super heavy price of several hundred million casualties. Consequently, one can safely assume the universality of the above three stages in all pandemics. The main differences between one pandemic and another are the speed by which infections increase during the second stage, and the time needed to reach the third stage. Such typical behavior can be mathematically described by a tangent hyperbolic function (***tanh***). This function is characterized by all above three properties of initial slow buildup, a following sharp rise and an ultimate saturation. Consequently, one can describe the accumulated number of infections or deaths (*N*) by an empirical equation of the form

$$N = e^{a_1 tan\,h\left[\frac{(t-a_2)}{a_3}\right]} + a_4 \tag{1}$$

The exponentiation of the tangent hyperbolic function is introduced for enabling the equation to describe the asymmetry between the start of the rising part and the approach to leveling saturation of the data.

With t being the time in days, a_1, a_2, a_3, and a_4 are free fitting parameters to be determined by the data. These four fitting parameters carry physical significance. The parameter a_1 and a_4 form a direct measure of the total number of infections registered throughout the pandemic when leveling off takes place this number is equal to $(e^{a_1} - a_4)$. The parameter a_2 determines the amount of horizontal shift on the time axis. The third parameter a_3 governs the speed by which the pandemic is spreading. Such behavior applies to all pandemics including the COVID-19 virus.

Equation (1) is used to fit COVID-19 infection data for nineteen world countries and those for the whole world. In all cases, good convergent fits with over 95% confidence level are obtained using MATLAB nonlinear fitting facility.

Although finding a suitable empirical parametrization fitting equation for any data is regarded as one of the standard accepted scientific research tools, such parametrization usually acquires added importance if the fitted empirical equation can be extrapolated to give predictions outside the domain of the measured data. Acting within this criteria, equation (1) must not only be able to describe known registered infection data, but it should also be able to produce reasonable predictions concerning the future development of the pandemic within any country and for the world. Furthermore, it will be an added advantage if the equation is able to produce reasonable predictions on the approximate date when the pandemic will level off in any particular country. Such estimations need to be made at the earliest possible date from the pandemic onset. There is only little advantage in making predictions when the pandemic is approaching near the leveling off stage.

In order to achieve the above goals, world countries are categorized into four groups based on data available by June-1-2020. Group (I), includes countries where the pandemic has already reached or closely approached the leveling off stage. These countries include China, South Korea, Greece, and Switzerland. Group (II) category includes countries which are close to reaching the leveling off saturation of infections. Examples of such countries are Spain, Italy, France, and Turkey. Group (III) includes countries where the pandemic is still on the rise with indication of slowing down. This group includes both Russia, UK and USA. The whole world fall into this category. The forth category represents countries where infections are in the fast increasing stage. Examples are India, Chile, and Brazil.

Data for the period January-22 and June-1-2020 obtained from reference [21] for nineteen countries and the whole world within the four categories defined above are fitted to equation (1). Good fits with over 95% confidence levels are obtained using the MATLAB nonlinear fitting facility [22]. Typical examples for the four groups are presented in Figure 1. All fits are extrapolated to 300 days after the last data point on June-1-2020. Each plot in Figure 1 also shows as a black square the last data point belonging to June-1-2020 used in the fit indicating its position relative to the fit ultimate horizontal saturation which marks infections leveling off. Furthermore, the plots show the position of the curve inflection point as a black circle. This point will have an important role in the following analysis.

From mathematical point of view, the inflection point is defined as the point at which the curvature of a curve shifts from being concave upwards to concave down wards or vice versa. Mathematically, the second derivative of the curve at this point is equal to zero. Applying this condition on equation (1) and solving gives the position (X_I) of the inflection point for the general curve in equation (1) as:

$$X_I = a_3 tanh^{-1}\left[\frac{(1+a_1^2)^{1/2}-1}{a_1}\right] - a_2 \tag{2}$$

It is well known fact that the quality and robustness of any empirical fitting procedure are governed by two factors. The first is the suitability of the fitting equation in describing the fitted data. The second is the number of data points. The larger this number the better the fit. A general rule of thumb in this respect is that the number of data points should be much larger than the number of free fitting parameters within the fitted equation. It is clear from Figure 1 that equation (1) satisfies both above conditions. Fits with good qualities similar to those shown in Figure 1 are obtained for all nineteen countries plus those of world data studied.

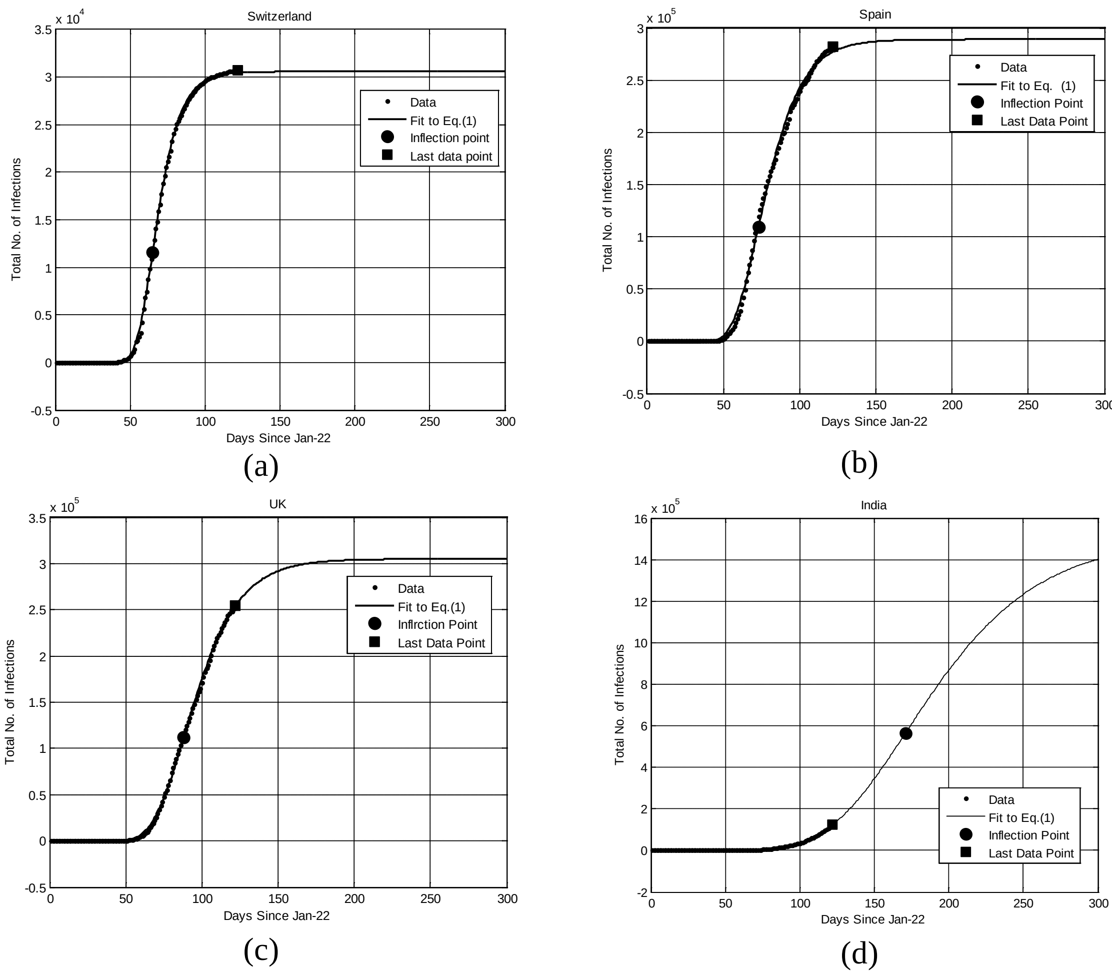

Figure 2. Examples of groups I, II, III, and IV COVID-19 infections data fitted to equation (1). (a) Switzerland. (b) Spain, (c) UK, and (d) India respectively

As it is the case with any empirical fitting procedure, its usefulness is limited to the academic purpose of demonstrating the ability of the suggested empirical equation to describe the data unless the fit is useable in describing situations outside the data domain used in the fit. Equation (1) is shown to provide a reasonable description of all available COVID-19 data. However, the questions on whether fits to this equation can be safely extrapolated to produce estimations on magnitude and date of infections saturation in any particular country still remained unanswered. The following discussion is based on the assumption that there will be no sudden or drastic change in factors affecting the virus spread on the ground. Examples of such factors can be the introduction of a working vaccine or treatment drug, or the sudden abandonment of current social distancing measures. Under such assumption, it can be reasonably argued that the situation with group (I) countries is relatively easy. Once the pandemic in a particular country reaches the saturation part of the curve, flat horizontal extrapolation of the leveling off is trivial. Few fluctuations from the flat leveling off are to be expected of course, but the overall trend will remain the same. For such countries, extracting new information from the fit is limited to the monitoring of any increases in infection above the already achieved leveling off.

The situation with countries where the pandemic is short from reaching leveling off stage is somehow different. As mentioned earlier, the robustness of any fit is highly dependent on the number of data points used. The question is can one safely use the fit extrapolated leveling off horizontal line to predict the ultimate saturation level? The answer is a simple NO. Extrapolations are good enough only in cases of straight lines fittings. Even then, the addition or removal of new points to or from the fitted data can significantly change the slope and intercept of the fitted straight line. The situation with more complicated nonlinear equation like equation (1) fitting is much more difficult. In short, and as far as equation (1) is concerned, the saturation leveling off value is significantly affected by the number of data points used in the fit. To demonstrate this fact, The COVID-19 infections data from Switzerland as an example of a country where leveling off can be considered to have been reached are selected. These data have been showing a good degree of leveling off in numbers of infections for at least a week before May-27-2020. The increase in the number of infections during this week is less than 250, and it represents only 0.8% of the saturation level of 30700 cases. These data can thus be considered a reasonable and perhaps the only available example of being nearest to being leveled off. Multiple fits using equation (1) are performed using Switzerland's data with different number of data points. The smallest set of data points are those which are truncated at the inflection point. One data point is added to the set and the fit is repeated each time until the final available point in the data set is reached. The results of such fits are shown in Figure 2. The main conclusion drawn from such analysis is that the value of the extrapolated curve saturation leveling off is significantly affected by the number of points included in the fit in the region before actual leveling off is achieved. The difference between the extrapolated saturation values for the data truncated at the inflection point and the complete data set is about 20%. Similar analysis for other countries showed similar pattern but with no systematic relation between the curve saturation value and the number of data points used in the fit. Furthermore three important properties regarding the curve inflection point are worth mentioning. The first is that all fits carried out using different truncation levels are identical for the region prior to the inflection point. The second is that for most fits carried out for different countries, the data post inflection point represent about two thirds of the complete data set for that country. The third is that most attempts to carry out fits to equation (1) using data truncated below the inflection point have failed to produce acceptable convergence. This leads to the conclusion that data portion post inflection point have the major important role to play in the fit quality and results.

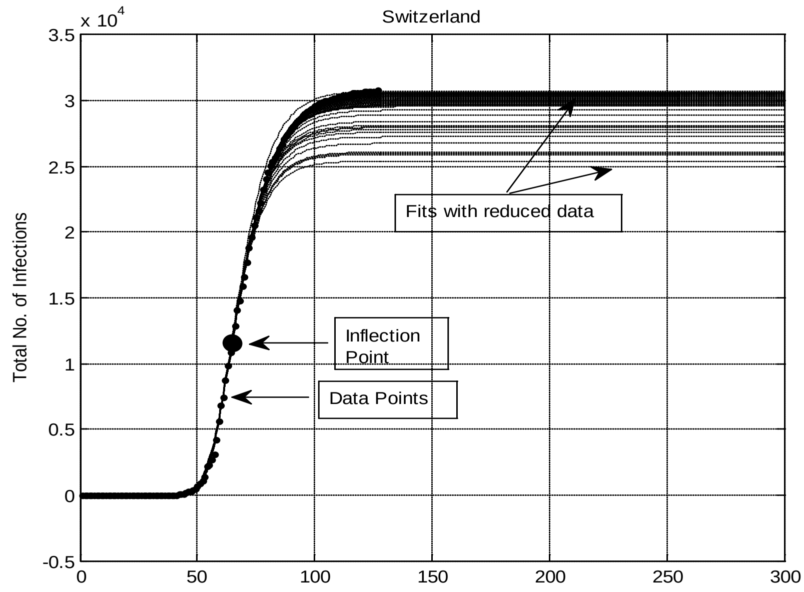

Figure 3. Results of fits with different sizes of truncated data

Figure 3. clearly suggests that the extrapolated behaviors is dependent upon the number of data points used in the fit. It becomes thus necessary to apply some correction to the extrapolated leveling off predictions of equation (1). The correction is based on the assumption that future data daily increases have the same statistical distributions of past data. For such purpose, percentage daily increases in infections for the whole World, Germany, Italy and France are plotted and shown in Figure 3. Results for other countries are much similar. It is clear from this figure that daily percentage increases in the number of infections is always in the range of few percent.

Adopting the above hypothesis that the extrapolated statistical distribution of the percentage daily increases of infections in future data are to have the same statistical distribution as those of the prior data, the mean and the standard deviation for the percentage daily changes for each country is calculated. Ten new projected points belonging to the immediate extrapolated fit are added to the real data set. These projected points are modulated by being lifted up to two different levels. The higher level which represents the worst case scenario. This involves lifting these points up by a percentage equals to the mean plus three standard deviations of the daily increases in infections. The lower level represents the best case scenario where the lifting up is only by a percentage equal to the mean. The reason behind selecting the number ten is to be discussed later. Both sets of modified are fitted to equation (1) and two new extrapolations of leveling are obtained. These represent the upper and lower limit estimation of the projected pandemic development. Sample results of such procedure are shown in Figure (6-c) and (7-c).

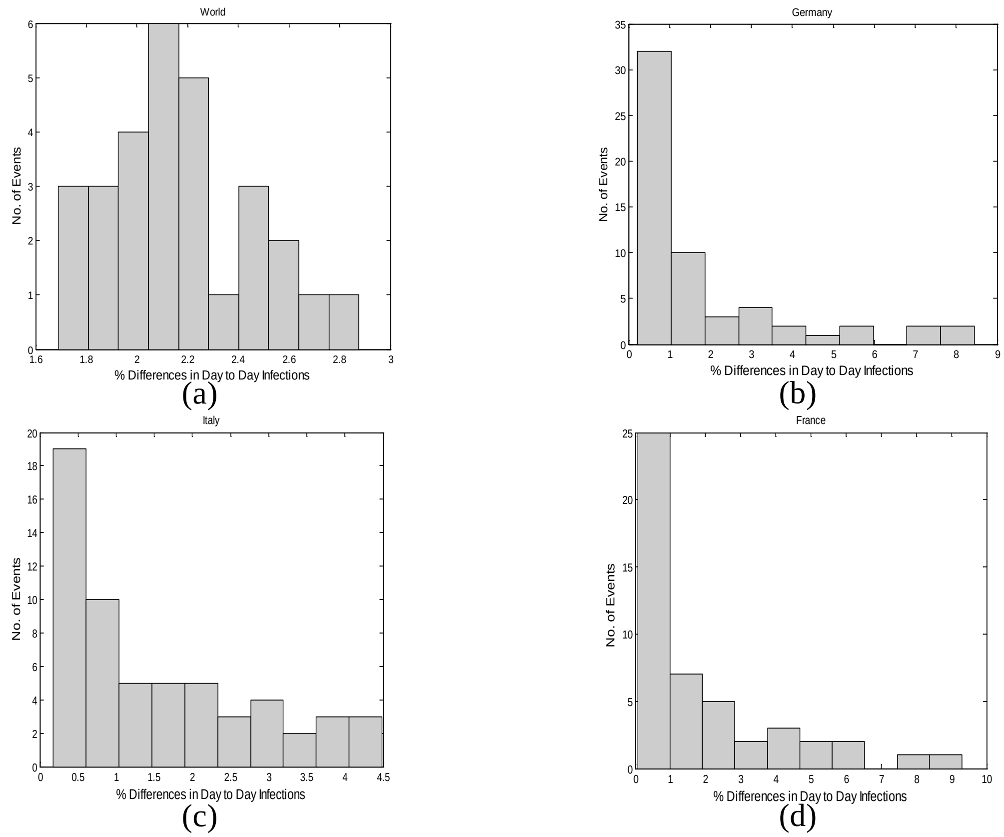

Figure 4. Four examples of the distributions of the percentage daily increases in infections (a) World, (b) Germany, (c) Italy, (d) France.

In order to estimate the approximate date of the leveling off, two points on the upper and lower extrapolated curves are selected such that their values are larger by not more than 0.01% than their immediate predecessor points.

It is important to state that all above analysis is equally applicable for the total number of death cases caused by COVID-19 in any country because death numbers follow almost exactly the same pattern as that of infections.

4 The Software

All above calculations are contained within a free downloadable MATAB code which performs the fitting, calculate the corrections and make the estimations. The software is available on the author's MATLAB file exchange website [23]. The software input data are those for the number of infections or deaths in any particular country. These should be pre-saved as a MATAB data file under a given name. The software is activated by entering the statement

>> COVID_19(name of data file)

The software package include infections data files for twenty countries and deaths data files for ten countries. These data files are up to date until June-1-2020. The software will output a plot of the actual data, its extrapolated fit to equation (1), and the upper and lower modified ten points, and the two extrapolated fits. Numerical output of the program includes the upper and lower estimated saturation leveling off values together with their estimated dates. The figure title assumes start of data file to be January-22-2020. Other starting dates can be used with changing the plot x-label to the correct date. The software will fail to produce results if the size of the data is too small such that the inflection point obtained from the first fit lays outside the data range. This can happen in countries where the pandemic is still in the early or the fast rise stage and the inflection point has not been reached. Examples of such countries are Chile and India.

5 Model Cross Test

Two types of cross tests are applied to assess the robustness of the modeling and the software. The first involves running the software several times with the addition of different numbers of projected data points above the actual ones. Sets between one and 100 projected data points are added and the analysis is repeated each time. The percentage differences between upper and lower leveling off values are plotted against the number of data points. Typical results for three countries are shown in Figure 5. These and similar plots for other countries indicate the existence of an optimum number of projected points that can be added. Adding this optimum number of points to the data produces maximum separation between best case and worst case leveling off scenarios. In the majority of cases, this optimum number of points fall between 5 and 15. Consequently, the software is set to add ten projected points to the actual data set. This number can be modified easily within the software if necessary.

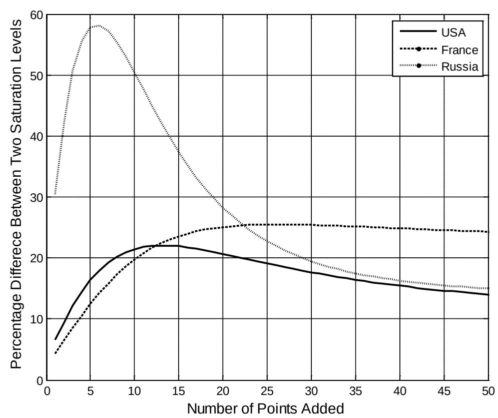

Figure 5: Effect of changing the number of points added to the actual data on the percentage change on the span between upper and lower saturation levels

The second cross check performed is related to the validity of the basic idea of adding projected data points. It is true that projected data points are points inferred from the fitted actual data rather than being real data themselves. To assess the correctness of such procedure, a backward analysis is performed. Analysis related to upper and lower leveling off are carried out using deliberately truncated data. Figures (6 & 7) show two examples of such analysis performed of USA using data truncated on May – 15 and June -1, and for the UK using data which terminate on May-2, and May-17. In both cases, the analysis show that and in spite of the fact that the extrapolated fit leveling off obtained using the truncated data is lower than the corresponding one derived from the full set of available data, the extrapolation is always within the margin between the upper and lower limits projected.

5.1 Model Predictions

After carrying out all above tests and cross checks on the model, it becomes useful to use the model in making some predictions. For this purpose, the software is run on data from nineteen countries which registered the highest infections on June-1-2020 and the world. The software and the data used are available on the MATLAB file exchange website. The data can be updated and used to obtain more accurate predictions. It is worth mentioning that the analysis and the software are equally valid for both infection and mortality data. The summary of result for infections in the countries studied are presented in table (1). Results related to deaths in nine countries and the world are presented in table (2).

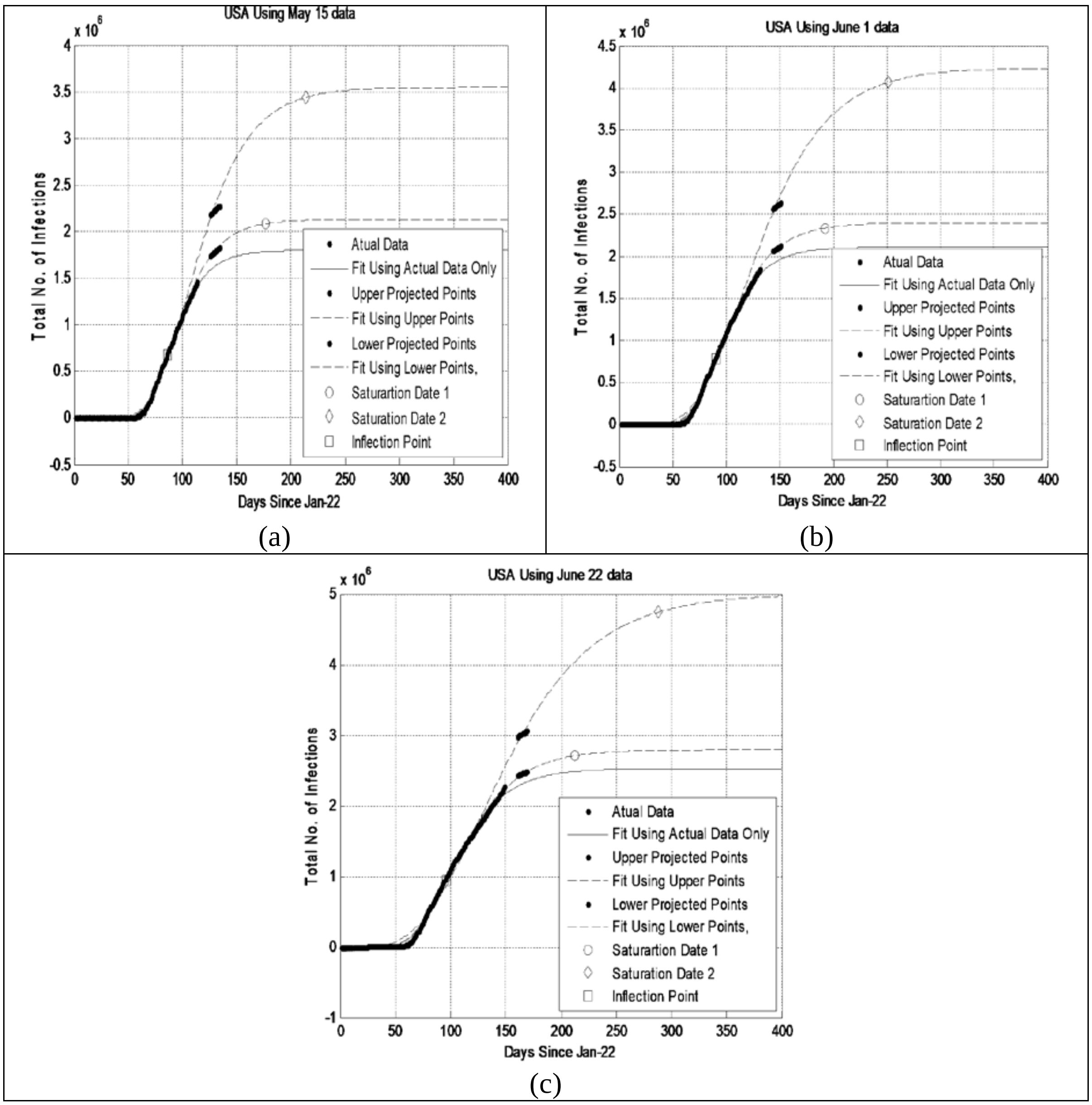

Figure 6: Effect of reduction of number of USA data points on the final model projections

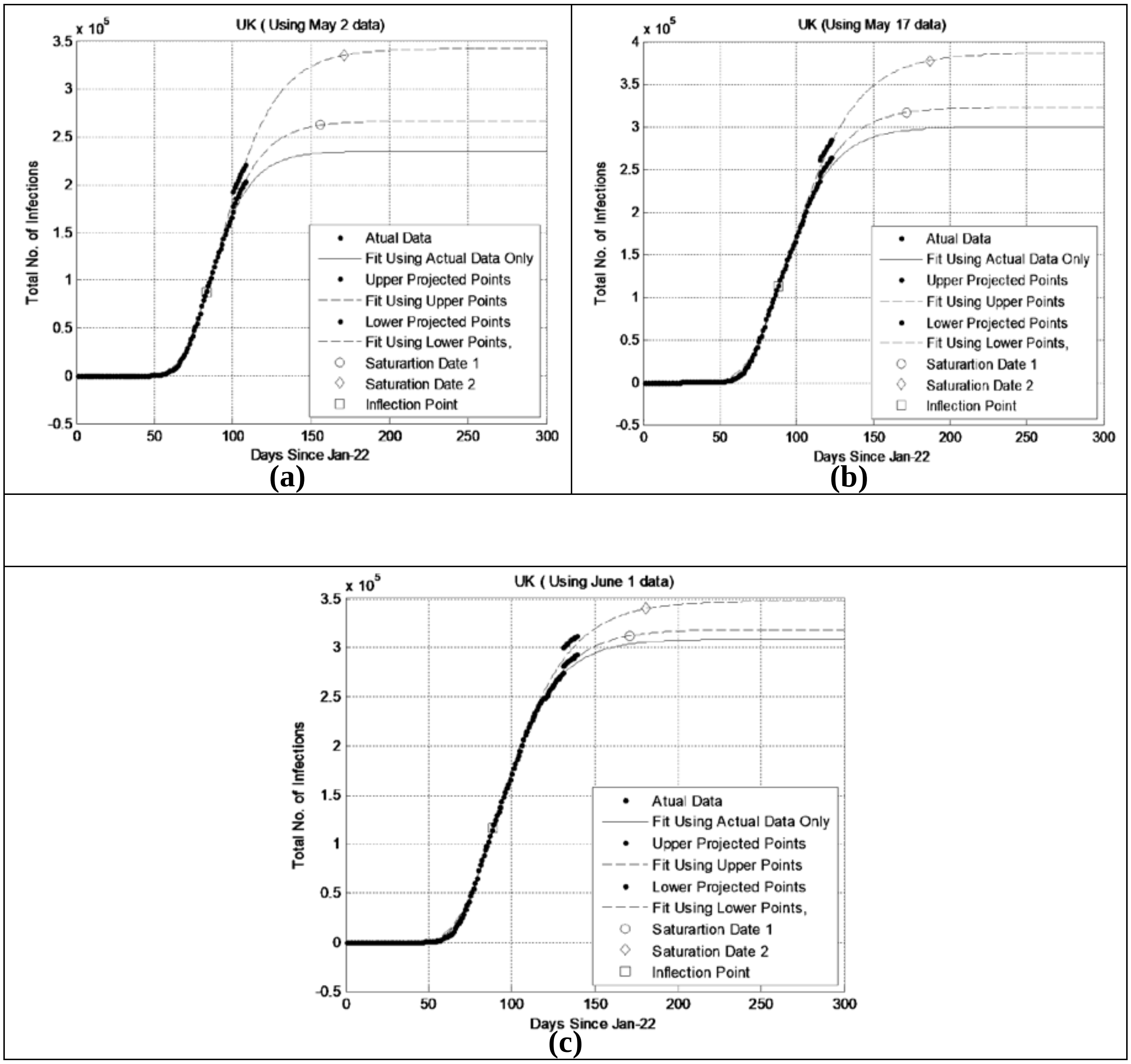

Figure 7. Effect of reduction of number of UK data points on the final model projection

Table 1. Summary of projected total infections leveling off magnitudes and dates in twenty counteries

	Country	Total Infection Cases	Inflection point Days after 22 January	Estimated Upper leveling off	Estimated Lower Leveling off	Estimated Date for Upper Leveling off Days after Jan 22	Estimated Date for Lower Leveling off Days after Jan-22	Comments All data used apart from World and US are up to June 1 2020
	World	**8,908,556**	**128**	**3.6092e+007**	**1.8880e+007**	**307**	**360**	**Data June 22**
1	**USA**	**2,263,651**	**91**	**4.9670e+006**	**2.7916e+006**	**212**	**288**	**Data June 22**
2	**Brazil**	**514,849**	**217**	**1.0388e+008**	**3.1382e+007**	**517**	**517**	**??? Inflection not Reached**
3	**Russia**	**405,843**	**112**	**1.5585e+006**	**7.7941e+005**	**262**	**213**	
4	**Spain**	**286,509**	**73**	**3.9272e+005**	**3.0364e+005**	**180**	**145**	
5	**UK**	**274,762**	**89**	**5.3422e+005**	**3.4004e+005**	**227**	**179**	
6	**Italy**	**232,997**	**68**	**2.9085e+005**	**2.4353e+005**	**166**	**140**	
7	**India**	**190,609**	**197**	**6.228e+008**	**1.1392e+007**	**497**	**497**	**??? Inflection not Reached**
8	**France**	**188,882**	**74**	**2.214e+005**	**1.8963e+005**	**151**	**128**	
9	**Germany**	**183,494**	**70**	**2.1671e+005**	**1.8477e+005**	**152**	**128**	
10	**Peru**	**164,476**	**144**	**1.1735e+007**	**1.1315e+006**	**444**	**313**	**? Close to inflection**
11	**Turkey**	**163,942**	**84**	**2.7261e+005**	**1.7508e+005**	**204**	**152**	
12	**Iran**	**151,466**	**82**	**6.9483e+005**	**1.9311e+005**	**388**	**206**	
13	**Chile**	**99,688**	**???**	**???**	**???**	**???**	**???**	**??? Data not Enough**
14	**Canada**	**90,947**	**93**	**2.163e+005**	**1.2054e+005**	**252**	**191**	
15	**Mexico**	**87,512**	**159 9**	**1.0265e+007**	**1.2164e+006**	**459**	**351**	**??? Inflection not Reached**
16	**Saudi Arabia**	**85,261**	**110**	**1.3912e+006**	**2.6835e+005**	**350**	**238**	
17	**China**	**83,001**	**16**	**82266**	**82266**	**49**	**49**	
18	**Pakistan**	**69,496**	**157**	**1.7755e+008**	**7.594e+005**	**457**	**354**	**??? Inflection not Reached**
19	**Belgium**	**58,381**	**77**	**78025**	**61337**	**175**	**143**	
20	**Qatar**	**56,910**	**146**	**1.5125e+006**	**1.9126e+005**	**435**	**268**	**??? Inflection not Reached**

Table 2. Summary of projected total deaths leveling off magnitudes and dates in twenty countries

	Country	Number of Deaths June 1	Inflection Point	Upper Estimated Deaths Leveling off	Lower Estimated Death Leveling off	Upper Leveling off Data	Lower Leveling off Date
1	**World**	**380,265**	**90**	**6.3256e+005**	**5.2998e+005**	**228**	**203**
2	**USA**	**107,620**	**92**	**2.2587e+006**	**1.2974e+005**	**394**	**183**
3	**Russia**	**5,037**	**135**	**2.6867e+005**	**24,407**	**442**	**292**
4	**Spain**	**27,127**	**74**	**34142**	**29,112**	**161**	**136**
5	**UK**	**39,369**	**87**	**55711**	**43,999**	**198**	**165**
6	**Italy**	**33,530**	**73**	**40014**	**35,137**	**169**	**147**
7	**France**	**28,940**	**79**	**33459**	**29659**	**155**	**136**
8	**Germany**	**8,643**	**84**	**10670**	**9153**	**172**	**149**
9	**Canada**	**7,395**	**101**	**13909**	**9665**	**230**	**188**

6 Discussion and Conclusions

Beside demonstrating the ability of equation (1) in describing the behavior of COVID-19 accumulated infections and deaths data, the devised extrapolation procedure applied provides reasonable future estimations of upper and lower limits of infections and deaths together with approximate dates for the establishment of leveling offs. Furthermore, these estimations are supported by the consistencies of results between infections and deaths. These consistencies can be observed through the comparisons of expected upper and lower saturation dates. These differences are in the range of few days only. One further proved consistency in the predictions is related to group (I) countries which have almost reached saturated leveling off. The model succeeded in predicting the dates when such leveling off have been reached. Examples of such countries are China and Switzerland. These consistencies give added weight to other projected predictions made.

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Conflict of Interest Declaration

The author declares that there is no conflict of interest statement.

Ethics Committee Approval and Informed Consent

The author declares that declare that that there is no ethics committee approval and/or informed consent statement.

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