MANAS Journal of Engineering
ISSN 1694-7398 | e-ISSN 1694-7398

Volume 8 (Issue 1) (2020) Pages 59-67

A new approach on the stability of fractional singular systems with time-varying delay

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ABSTRACT

In this research article, we discussed the asymptotic stability of fractional singular systems with Riemann–Liouville (RL) derivative and constructed some sufficient conditions. The proposed stability criteria are based upon the linear matrix inequalities (LMIs) approach, which can be easily checked using meaningful Lyapunov-Krasovskii functionals. Finally, we presented two simple numerical examples with their simulations to demonstrate the effectiveness and benefits of the proposed method. The theoretical results obtained in this research contribute to existing ones in the literature.

ARTICLE INFO

Research article

Received: 31.03.2020
Accepted: 22.05.2020

Keywords:
Fractional singular system,
RL derivate,
LMI,
Lyapunov-Krasovskii functional,
asymptotic stability

1. Introduction

Singular systems, which are also referred implicit or descriptor systems are dynamic systems. The study of stability problem for dynamic systems with time delay or without has theoretical and practical significance. There are many books and articles made in this sense, for example: [1-27]. In this direction, there are many studies on the qualitative features of solutions of singular and non-singular systems with and without delay [26]. However, there are very few studies on the qualitative behaviors of fractional-order singular systems. During last years, studies on the stability analysis of fractional dynamic systems, including stability, chaos and bifurcation, have become a hot topic of investigation. Although fractional analysis has a long history, interest in common science and engineering practices is growing commonly [13,18]. The singular systems with time delays have already been applied in many areas such as electrical circuits, memorization, moving robots, locomotion, economic systems and many other systems, see References [6, 8–10]. Therefore, the stability problem for differential and RL fractional differential singular systems has attracted researchers.

Investigating the stability of fractional-order singular systems is more complex than the integer-order singular systems. Therefore, it is necessary and interesting to examine the stability of fractional-order singular systems playing important role in both theory and applications. When the related literature is searched in particular, various approaches towards the stability of fractional-order singular systems have attracted attention in recently [5, 15, 17, 20]. In this direction, the Lyapunov-Krasovskii functional method presents a very powerful approach for analyzing the qualitative properties of the fractional-order singular systems. However, the fractional derivative of the Lyapunov functionals is computationally quite difficult. That is the main reason why there are very few works for stability of delayed fractional-order singular systems.

On the basis of the above discusses, this research article deals with the asymptotically stability of fractional singular systems with RL fractional derivate. When compared to integer-order singular systems with delay, it is seen that the works related to the stability of delayed fractional-order singular systems are still in the process of benefiting. The goal of this research, difficulty and its contribution can be summarized as follows:

(i) According to nonsingular ones, it is not easy to meet the existence of solutions in the stability analysis of singular systems since the initial conditions may not be consistent. Also, it is not easy to calculate the fractional-order derivatives of Lyapunov functionals constructed for these systems. To overcome this challenge, we derived a meaningful Lyapunov functional including the terms fractional derivative and integral. The method employed in this article is advantageous in that integer-order derivatives of the Lyapunov functionals can directly calculate.

(ii) The basic goal of this research article is to search the asymptotically stability of fractional singular systems with RL derivate. As proof technique, this article includes the LMIs, Lyapunov functional method and some fundamental inequalities.

(iii) We consider that the theoretical results achieved in this research will contribute to the related literature and studies on the qualitative properties of fractional-order singular systems.

Notations: $\Re^n$ states the $n-$dimensional Euclidean space; $\|.\|$ states the Euclidean norm for vectors; X^T states the transpose of the matrix X; Y is negative-definite (or positive-definite) if $\langle Yx, x\rangle < 0$ or $\langle Yx, x\rangle > 0$) for all $x \neq 0$; $\|Z\| = \sqrt{\lambda_{\max}(Z^T Z)}$ states the spectral norm of matrix Z; $\lambda_{\max}(P)$ states the maximum of eigenvalues of the matrix P

2. Preliminaries

In the current this research motivated by above discussions, we consider the following fractional-order singular system with time-varying delay

$$E\, {}_{t_0}^{RL}D_t^q x(t) = Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau(t)}^{t} x(s)ds, \tag{1}$$

with the given initial condition

$${}_{t_0}^{RL}D_t^{q-1} x(t) = \vartheta(t), \quad t \in [-\tau, 0], \tag{2}$$

where $x(t) \in \Re^n$ is state vector; ${}_{t_0}^{RL}D_t^q x(.)$ states a q order RL derivative of $x(.)$ with $q \in (0,1)$; $A,\ B,\ C \in \Re^{n\times n}$ are known constant matrices with suitable dimensions; the matrix $E \in \Re^{n\times n}$ may be singular, that is rank $E = r \leq n$, differentiable function $\tau(t)$ is a variable delay for all $t \geq 0$,

$$0 \leq \tau(t) \leq \tau \quad \text{and} \quad \dot{\tau}(t) \leq \delta < 1. \tag{3}$$

Before giving our main result, the following property, fundamental definitions and lemmas are presented.

Definition 1. ([5]) The RL fractional integral and RL fractional derivative are defined as, respectively

$${}_{t_0}D_t^{-q} x(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}x(s)ds, \qquad (q>0),$$

$${}_{t_0}D_t^{q} x(t) = \frac{1}{\Gamma(n-q)}\frac{d^n}{dt^n}\int_{t_0}^{t}\frac{x(s)}{(t-s)^{q+1-n}}ds, \qquad (n-1 \leq q < n),$$

where Γ is a Gamma function.

Property 1. ([5]) For $x(t) \in \Re^n$, if $p > q > 0$, then the following relation holds

$$ {}^{RL}_{t_0}D_t^{q}\left({}_{t_0}D_t^{-p}x(t)\right) = {}^{RL}_{t_0}D_t^{q-p}x(t). $$

Definition 2. ([18])

(i) System (1) or the pair (E, A) is said to be regular if $\det(s^q E - A)$ is not identically zero.

(ii) The pair (E, A) is said to be impulse free if $\deg(\det(s^q E - A)) = rank(E)$.

Lemma 1. ([18]) For a vector of differentiable function $x(t) \in \Re^n$ and constant matrix $W = W^T (\geq 0) \in \Re^{n\times n}$, then

$$ \frac{1}{2}\,{}^{RL}_{t_0}D_t^{q}\left\{x^T(t)Wx(t)\right\} \leq x^T(t)W\,{}^{RL}_{t_0}D_t^{q}\left\{x(t)\right\}, \qquad q \in (0,1), $$

for all $t \geq 0$.

Lemma 2. ([1]) For any symmetric positive definite matrix $\Sigma \in D^{n\times n}$, scalar $\lambda > 0$ and vector function $\tilde{g} : [0, \lambda] \to \Re^n$, such that the integrations given in the following are well defined, then

$$ \lambda \int_0^{\lambda} \tilde{g}^T(s)\Sigma\tilde{g}(s)ds \geq \left[\int_0^{\lambda}\tilde{g}(s)ds\right]^T \Sigma \left[\int_0^{\lambda}\tilde{g}(s)ds\right]. $$

3. Stability

In this section, we provide a few novel sufficient conditions for the stability of delayed fractional singular systems by constructing meaningful Lyapunov functional including fractional integral and derivative terms.

Theorem 1. If there exist a matrix P and matrices $Q = Q^T > 0$, $R = R^T > 0$, $S = S^T > 0$ and $U = U^T > 0$ such that $E^T P = P^T E \geq 0$ and

$$ \Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{12}^T & \Xi_{22} & \Xi_{23} \\ \Xi_{13}^T & \Xi_{23}^T & \Xi_{33} \end{bmatrix} < 0, \tag{4} $$

where

$$ \Xi_{11} = P^T A + A^T P + Q + \tau^2 S + A^T\left(E^T RE + \tau^2 E^T UE\right)A, $$
$$ \Xi_{12} = P^T B + A^T\left(E^T RE + \tau^2 E^T UE\right)B, $$
$$ \Xi_{13} = P^T C + A^T\left(E^T RE + \tau^2 E^T UE\right)C, $$
$$ \Xi_{22} = B^T\left(E^T RE + \tau^2 E^T UE\right)B - (1-\delta)Q, $$
$$ \Xi_{23} = B^T\left(E^T RE + \tau^2 E^T UE\right)C, $$
$$ \Xi_{33} = -S + C^T\left(E^T RE + \tau^2 E^T UE\right)C. $$

Then the trivial solution of fractional singular system (1) with (2) is asymptotically stable.

Proof. For this theorem, we define the following Lyapunov-Krasovskii functional including the fractional derivative and integral terms

$$V(t) = \sum_{j=1}^{3} V_j(t), \tag{5}$$

$$V_1(t) = {}_{t_0}^{RL}D_t^{q-1}\left(x^T(t)P^T Ex(t)\right),$$

$$V_2(t) = \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds + \int_{-\tau(t)}^{0} ({}_{t_0}^{RL}D_t^q x(t+s))^T E^T RE({}_{t_0}^{RL}D_t^q x(t+s))ds,$$

$$V_3(t) = \tau\int_{-\tau}^{0}\int_{t+\beta}^{t} x^T(s)Sx(s)dsd\beta + \tau\int_{-\tau}^{0}\int_{t+\beta}^{t} ({}_{t_0}^{RL}D_s^q x(s))^T E^T UE({}_{t_0}^{RL}D_s^q x(s))dsd\beta.$$

From Definition 1, we know that $V_1(t), V_2(t)$ and $V_3(t)$ are positive definite functionals. By Property 1 and Lemma 1, computing the differential of $V(t)$ along the solutions of fractional singular system in (1)

$$\dot{V}(t) = \sum_{j=1}^{3} \dot{V}_j(t)$$

for

$$\begin{aligned} \dot{V}_1(t) &= {}_{t_0}^{RL}D_t^q(x^T(t)P^T Ex(t)) \le 2x^T(t)P^T E\, {}_{t_0}D_t^q(x(t)) \\ &= 2x^T(t)P^T[Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau(t)}^{t} x(s)ds] \\ &= x^T(t)(P^T A + A^T P)x(t) + 2x^T(t)P^T Bx(t-\tau(t)) \\ &\quad + 2x^T(t)P^T C\int_{t-\tau(t)}^{t} x(s)ds. \end{aligned} \tag{6}$$

By (3), computing the differential of $V_2(t)$, we obtained

$$\begin{aligned} \dot{V}_2(t) &= x^T(t)Qx(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))Qx(t-\tau(t)) \\ &\quad + ({}_{t_0}^{RL}D_t^q x(t))^T E^T RE({}_{t_0}^{RL}D_t^q x(t)) \\ &\quad - (1-\dot{\tau}(t))({}_{t_0}^{RL}D_t^q x(t-\tau(t)))^T E^T RE({}_{t_0}^{RL}D_t^q x(t-\tau(t))) \\ &\le x^T(t)Qx(t) - (1-\delta)x^T(t-\tau(t))Qx(t-\tau(t)) \\ &\quad + ({}_{t_0}^{RL}D_t^q x(t))^T E^T RE({}_{t_0}^{RL}D_t^q x(t)) \\ &\quad - (1-\delta)({}_{t_0}^{RL}D_t^q x(t-\tau(t)))^T E^T RE({}_{t_0}^{RL}D_t^q x(t-\tau(t))) \\ &\le x^T(t)Qx(t) - (1-\delta)x^T(t-\tau(t))Qx(t-\tau(t)) \\ &\quad + ({}_{t_0}^{RL}D_t^q x(t))^T E^T RE({}_{t_0}^{RL}D_t^q x(t)). \end{aligned} \tag{7}$$

By Lemma 2, computing the differential of $V_3(t)$, we have

$$\begin{aligned}\dot{V}_3(t) &= \tau^2 x^T(t)Sx(t) - \tau \int_{t-\tau}^{t} x^T(s)Sx(s)ds \\ &+ \tau^2 ({}^{RL}_{t_0}D^q_t x(t))^T E^T UE({}^{RL}_{t_0}D^q_t x(t)) \\ &- \tau \int_{t-\tau}^{t} ({}^{RL}_{t_0}D^q_s x(s))^T E^T UE({}^{RL}_{t_0}D^q_s x(s))ds \\ &\le \tau^2 x^T(t)Sx(t) - \tau \int_{t-\tau}^{t} x^T(s)Sx(s)ds \\ &+ \tau^2 ({}^{RL}_{t_0}D^q_t x(t))^T E^T UE({}^{RL}_{t_0}D^q_t x(t)).\end{aligned} \tag{8}$$

For any $s \in [t-\tau, t]$,

$$-\tau \int_{t-\tau}^{t} x^T(s)Sx(s)ds \le -\tau \int_{t-\tau(t)}^{t} x^T(s)Sx(s)ds \le -\left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T S\left(\int_{t-\tau(t)}^{t} x(s)ds\right). \tag{9}$$

According to (7) and (8), we have

$$\begin{aligned}&({}^{RL}_{t_0}D^q_t x(t))^T E^T RE({}^{RL}_{t_0}D^q_t x(t)) + \tau^2 ({}^{RL}_{t_0}D^q_t x(t))^T E^T UE({}^{RL}_{t_0}D^q_t x(t)) \\ &= [Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau(t)}^{t} x(s)ds]^T \left(E^T RE + \tau^2 E^T UE\right) \\ &\times [Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau(t)}^{t} x(s)ds] \\ &= x^T(t)A^T\left(E^T RE + \tau^2 E^T UE\right)Ax(t) \\ &+ x^T(t)A^T\left(E^T RE + \tau^2 E^T UE\right)Bx(t-\tau(t)) \\ &+ x^T(t)A^T\left(E^T RE + \tau^2 E^T UE\right)C\int_{t-\tau(t)}^{t} x(s)ds \\ &+ x^T(t-\tau(t))B^T\left(E^T RE + \tau^2 E^T UE\right)Ax(t) \\ &+ x^T(t-\tau(t))B^T\left(E^T RE + \tau^2 E^T UE\right)Bx(t-\tau(t)) \\ &+ x^T(t-\tau(t))B^T\left(E^T RE + \tau^2 E^T UE\right)C\int_{t-\tau(t)}^{t} x(s)ds \\ &+ (\int_{t-\tau(t)}^{t} x(s)ds)^T C^T\left(E^T RE + \tau^2 E^T UE\right)Ax(t) \\ &+ (\int_{t-\tau(t)}^{t} x(s)ds)^T C^T\left(E^T RE + \tau^2 E^T UE\right)Bx(t-\tau(t)) \\ &+ (\int_{t-\tau(t)}^{t} x(s)ds)^T C^T\left(E^T RE + \tau^2 E^T UE\right)C\int_{t-\tau(t)}^{t} x(s)ds.\end{aligned} \tag{10}$$

From (6)-(10), we can conclude that

$$\dot{V}(t) \leq \mu^T(t)\Xi\mu(t),$$

where $\mu^T = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & (\int_{t-\tau(t)}^{t} x(s)ds)^T \end{bmatrix}$ and Ξ is defined (4). If $\Xi < 0, \dot{V}(t)$ is negative definite for $\mu(t) \neq 0.$ Therefore, the fractional singular system defined in (1) is asymptotically stable.

4. Illustrative examples with numeric simulations

To show the usefulness of the employed method, we present the following examples with simulation results.

Example 1 For $n = 2,$ as a special state of (1), we take into account the following singular system with a constant delay and RL derivate

$$E\,{}_{t_0}^{RL}D_t^q x(t) = Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau}^{t} x(s)ds, \qquad t \geq 0, \tag{11}$$

where $0 < q \leq 1,\ \tau = 0.4$,

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -6.2 & 1.6 \\ 1.6 & -7.6 \end{bmatrix},\ B = \begin{bmatrix} 0.6 & -1.2 \\ -1.1 & 1.3 \end{bmatrix},\ C = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}.$$

Let us choose

$$P = \begin{bmatrix} 3.6 & 0 \\ 0 & 2.8 \end{bmatrix},\ Q = \begin{bmatrix} 1.8 & 0.12 \\ 0.12 & 1.5 \end{bmatrix}, R = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.16 \end{bmatrix},$$

$$S = \begin{bmatrix} 12 & 0.1 \\ 0.1 & 8 \end{bmatrix} \text{ and } U = \begin{bmatrix} 0.4 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}.$$

By MATLAB-Simulink, under the above assumptions, we can easily obtain that all the eigenvalues in LMI defined in (4) are $\lambda_{\max}(\Xi) \leq -0.4621$. Thus, according to Theorem 1, this shows that the origin of system (11) is asymptotically stable.

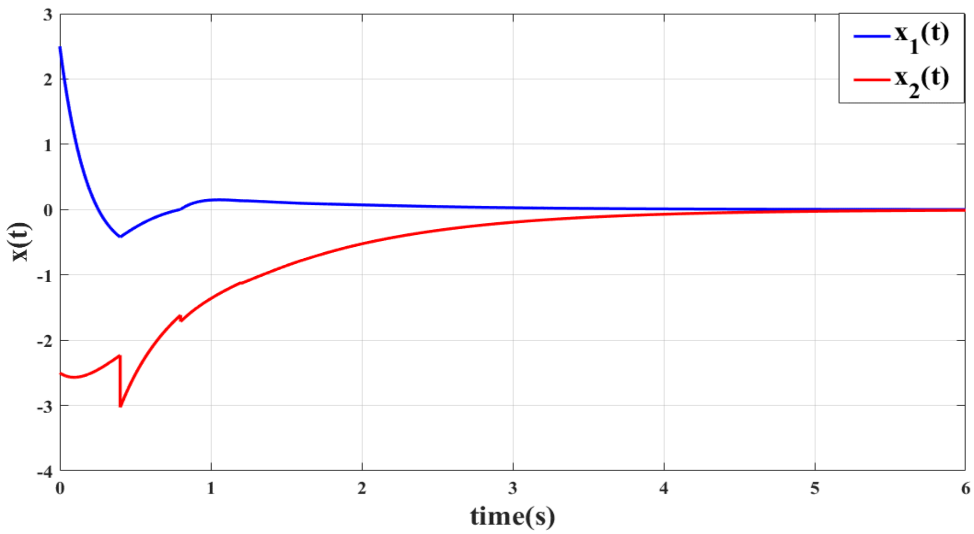

Figure 1. *The simulation of* $x(t)$ *in Example 1 for* $\tau = 0.4$.

Example 2 For $n = 2$, as a special state of (1), we take into account the following singular system with time-varying delay and RL derivate

$$E\,{}^{RL}_{t_0}D^q_t x(t) = Ax(t) + Bx(t-\tau(t)) + C\int_{t-\tau(t)}^{t} x(s)ds, \qquad t \geq 0, \tag{12}$$

where $0 < q \leq 1,\ \tau(t) = 0.1 + 0.2\sin^2 t \leq 0.3 = \tau$,

$$E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -8.2 & 2.8 \\ 2.8 & -7.6 \end{bmatrix}, B = \begin{bmatrix} 0.9 & -1.2 \\ -1.3 & 1.2 \end{bmatrix}, C = \begin{bmatrix} 0.7 & 0.1 \\ 0.2 & 0.6 \end{bmatrix}.$$

Since $\dot{\tau}(t) = 0.2\sin 2t \leq \delta < 1$, it can be selected $\delta = 0.2$. Let us choose

$$P = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 1.8 & 0.14 \\ 0.14 & 1.6 \end{bmatrix}, R = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.08 \end{bmatrix},$$

$$S = \begin{bmatrix} 16 & 0.1 \\ 0.1 & 10 \end{bmatrix} \text{ and } U = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$

By MATLAB-Simulink, under the above assumptions, we can easily obtain that all the eigenvalues in LMI defined in (4) are $\lambda_{\max}(\Xi) \leq -0.0197$. Thus, according to Theorem 1, this shows that the origin of system (12) is asymptotically stable.

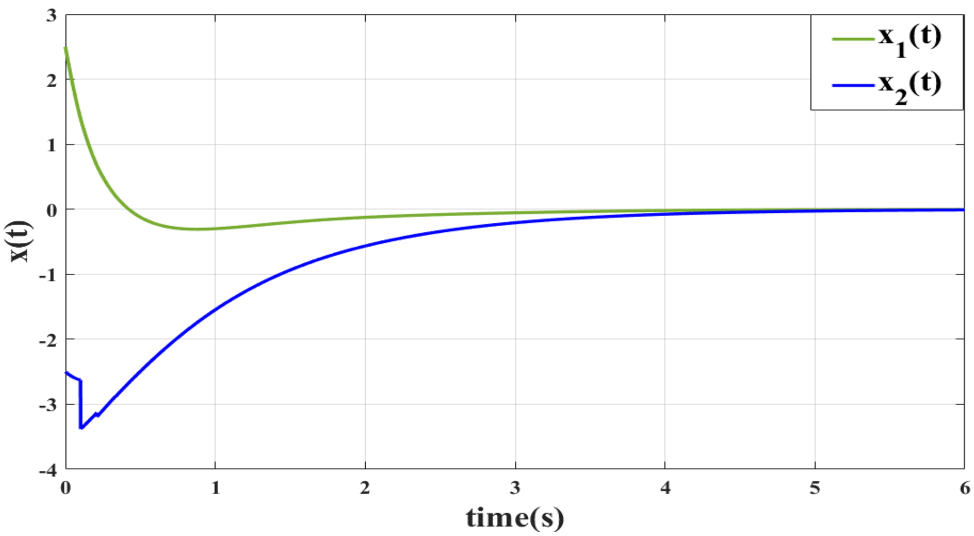

Figure 2. *The simulation of* $x(t)$ *in Example 2 for* $\tau(t) = 0.1 + 0.2\sin^2 t$.

When the theoretical results of the above *Example 1* and *Example 2* are examined, it is seen that the origin of addressed system in the examples is asymptotically stable under different initial conditions. Moreover, the theoretical results of this research article are supported by the simulations in Figure 1 and Figure2.

5. Discussion and conclusion

In this study, we have derived some new sufficient conditions concerning the asymptotically stability of singular systems with time-varying delay and RL derivate. Stability criteria have been derived by constructing meaningful a Lyapunov-Krasovskii functional and expressing in terms of LMI. Using MATLAB-Simulink, two examples with numerical simulation are given to illustrate the usefulness of the stability criteria of the addressed singular system. Consequently, the results obtained in this article provide contribution to the improvement and generalization of the classical integer-order delayed singular systems.

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