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Effect of Turkish mortality development on the expected lifetime and annuity using entropy measure

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Abstract

Over the past three centuries, there has been a steady and gradual decline in mortality rates due to a considerable process of eliminating hazards to survival, which have increased the life expectancy at birth remarkably. The improvements in mortality rates often been underestimated by annuity and pension providers. To measure the effect of mortality rate developments on life expectancy or annuity, many demographers use the idea of entropy. We use the entropy measure to explain the effect of any change in mortality rates on the annuity along with Gompertz's mathematical model for the force of mortality to obtain a more theoretically satisfactory conclusion. Entropy measures using the Turkish life table are calculated for different age, year, discount factor, and gender parameters.

Keywords: Annuity, Entropy, Expected lifetime, Mortality rate, Pension funds, Rectangularization.

Öz

Türkiye'deki ölümlülük değişimlerinin annüite fiyatları üzerindeki etkisinin entropi ölçüsü

Geçtiğimiz üç yüzyıl boyunca yaşam olasılığını etkileyen risklerin azaltılması sonucu, mortalite hızlarında istikrarlı bir şekilde kademeli düşüler olmuştur. Böylece doğumda beklenen yaşam süresi önemli biçimde artmıştır. Mortalite hızlarındaki bu düşüşler genellikle annüite ve emeklilik ürünleri satan kuruluşlar tarafından olduklarından az tahmin edilmişlerdir. Mortalite hızlarındaki gelişmelerin beklenen yaşam süresi ve annüite değerleri üzerindeki etkisinin ölçülebilmesi için birçok demograf entropi kavramını kullanmaktadır. Bu çalışmada, mortalite hızlarındaki değişimin annüite değerleri üzerindeki etkisinin açıklanması için entropi ölçütü kullanılmış, ayrıca mortalite hızı için Gompertz mortalite modeli benimsenerek teorik açıdan daha güvenilir sonuçlar elde edilmesi amaçlanmıştır. Entropi ölçütü Türkiye hayat tabloları kullanılarak farklı yaş, yıl, iskonto oranı, ve cinsiyet parametreleri için hesaplanmaktadır.

Anahtar Sözcükler: *Annüite, Entropi, Beklenen yaşam süresi, Mortalite hızı, Emeklilik fonu, Dikdörtgenselleşme.*

1. Introduction

Throughout history, there exist significant developments behind the observed reductions in mortality rates. Over the years, disease-related mortality is mostly eradicated, and the residuary vaolatility in the age-at-death generally stems from the genetic factors [1].

Total fertility rate of 2.1, is considered to be the replacement rate of populations. Turkish fertility rate for 2010 was 2.170, for 2015 was 2.105, and for 2020 is 2.046 and it is expected to drop more according to the United Nations's world population prospects [2]. Over the years, decrease in the mortality rates at all ages caused population mean age to increase, and decreasing fertility rates supports this increase. Due to these reasons the population ratio between old people and young people increased over time, putting pressure on the balance of pension systems.

Regression levels for the Turkish Life Tables over the time interval between 1931 and 2015 is obtained from the Construction of Turkish Life, and Annuity Tables Project [3]. Although the Project aims to construct the life tables of male and female insured/non-insured population for 2010, along the study regression levels between the years 1931 and 2015 are constructed. Considering that the Turkish census data is very scarse (for example; according to the law enacted no census was carried out in 1995; the annexation of Hatay to the Turkey in 1938 also effected the census data, etc.), the data used and taken from the Project to provide the yearly regression levels is probably based on the best and only data available for Turkish population.

The project assumes that Turkish mortality follows the Coale-Demeny West Model Life Tables [4] mortality. It uses fifteen Turkish population census registers between 1927 and 2000 provided from Turkish Statistical Institute (TUİK). Preston Bennett method is used to obtain suitable life table levels for Turkish mortality. Life table levels between 1931 and 2000 are modeled by using the exponential regression method, and finally life table levels consisting of the force of mortality rates for both genders and ages between 0 to 100 are computed using exponential extrapolation method for the years until 2015.

Figure 1 demonstrates the considerable improvements in force of mortality of Turkey for females over the years 1931 to 2015, constructed using the regression levels and the corresponding Coale-Demeny West Model Life Tables. The decline in mortality rates are often underestimated by annuity and pension providers. Developments in the mortality rates for post-retirement ages does have a substantial financial impact on the sustainability of pension plans. Due to the tax-paid structure, the deficiencies of public finance aggravated by mortality improvements.

Many researchers use entropy to measure the effect of decreasing mortality rates on life expectancy or annuity costs. Pollard [5] focused on the function of the mortality rate and its effects on life expectancy. Keyfitz and Caswell [6] introduced the concept of entropy as a measure of the interaction of the expected lifetime and the developments in mortality rates.

Higher values of entropy measure point out that the expected lifetime has a higher sensitivity to a deviation in the mortality rates. Lower values of entropy measure mean that life expectancy is relatively insensitive to future deviations in mortality rates [7]. The most critical attraction of entropy measure is that it allows to define the influence of the mortality rates on the expected lifetime by a single index number.

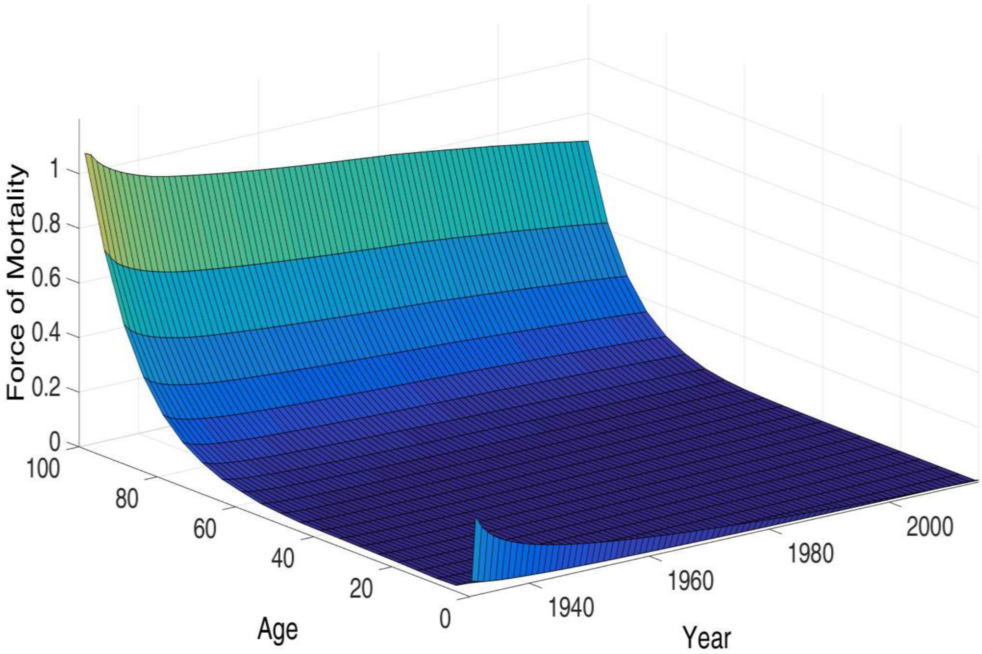

Figure 1: Mortality rates of Turkish female population over the years 1931 to 2015.

Entropy is also used as an indicator of the rectangularization of the survival function. Rectangularization showed up over the last 100 years in mostly the developed countries of the world, which the transformation from high mortality to low mortality is nearly complete [8]. It occurs as a result of increased survival rate and expectation of life. Improvements in the mortality rates have caused people to die at older ages, and it results in a reduction in the volatility of the age-at-death, and deaths to compact in the older years of life. An entirely rectangular survival function means that entropy is zero, and people live to a certain age and then die at that age.

We use entropy to measure the effect of changing Turkish mortality rates on the cost of an annuity. Then we work on the mortality parameters to see their effects on the entropy measure. First, we use period life tables to calculate entropy measure from annuity values and life expectations. Then we consider a mortality model to obtain a more theoretically satisfactory conclusion.

2. Entropy in actuarial science

2.1. Expected lifetime defined by entropy

In actuarial science, entropy is a single figure index that quantifies the effect of a same level change in the mortality rates at all ages on expected lifetime. Entropy determines the expected uncertainty on the result of an experiment and presents information about the foreseeability of the result of a random variable. The larger the entropy, the less compressed the distribution (less rectangularization in poupulation), and observation about random variable provide fewer information [9].

As an example; if the force of mortality of a person aged x is μ_x, and the force of mortality changes by 100φ percent for all ages and could be a positive or a negative value depending on whether the change is beneficial to human life or not. Then, the new force of mortality of a person aged x is defined as; $\mu_x^* = \mu_x(1+\varphi)$. This new force of mortality leads to a new survivorship curve, and the new probability to survive from age 0 to x becomes

$$\begin{aligned} {}_x p_0^* &= \exp\left(-\int_0^x \mu_a^* da\right) \\ &= ({}_x p_0)^{1+\varphi}. \end{aligned} \tag{1}$$

The new complete expected lifetime at birth is

$$e_0^{0*} = \int_0^w ({}_a p_0)^{1+\varphi} da, \tag{2}$$

Where, w is the limiting age of the mortality table. The derivative of Equation 2 is considered with respect to φ:

$$\frac{\partial e_0^{0*}}{\partial \varphi} = \int_0^w \ln({}_a p_0)({}_a p_0)^{1+\varphi} da\,. \tag{3}$$

In the neighborhood of φ based on a Taylor expansion, we have the following approximation

$$\frac{\Delta e_0^0}{e_0^0} = \left(\frac{\int_0^w (\ln {}_a p_0)\, {}_a p_0\, da}{\int_0^w {}_a p_0\, da}\right)\varphi\,. \tag{4}$$

To make H a positive quantity we define it as minus the expression in brackets.

$$\frac{\Delta e_0^0}{e_0^0} = -H\varphi\,. \tag{5}$$

Equation 5 interprets as; the proportional change in expected lifetime can be approximated as H times φ. A disadvantage of this method is that; it cannot be used when mortality change, φ, is not proportional across all ages [10].

Therefore, the measure H can be displayed as

$$H = \frac{\int_0^w (\ln {}_a p_0)\, {}_a p_0\, da}{\int_0^w {}_a p_0\, da}. \tag{6}$$

As ${}_t p_x$ declines from one to zero over the age span, $-\ln {}_t p_x$ increases from zero to infinitely large values. There is no apparent mathematical explanation of why $H_x(\delta)$ should be bounded from above [11], and life tables may have entropy values above one.

Entropy measure depends directly on the concavity of the survival function (same as ${}_t p_x$, or μ_x) [12]. As mortality improves over time, or the curve of ${}_t p_x$ turns down more sharply, which is the same as saying that μ_x turns up more sharply, more people die in a narrow range of old ages before w. This leads to H

being closer to zero. Hence, a change in μ_x has almost no effect on life expectancy. Lower the H, greater the tendency for all of us to die at about the same age, a tendency that accompanies mortality decline. In the case of every person dying at the same age (defined as complete rectangularization), H would be 0, independent of the age at every person dies.

Analyses point out that there is an inverse relationship between life expectation and the entropy measure. The decline in H, on a large scale, is an outcome of the process of reducing deaths in infancy. Another outcome is an alteration in the ages, where additional improvements in mortality would be most effective in adding to the expected lifetime. About a century ago, a higher potential to increase life expectancy was situated in the first couple of years of childhood (age 0 to 5); today, in most of the developed countries is in old-ages. In the course of mortality developments, and life expectancy increases, there is a shift to the right in the ages where additional declines in mortality would be most effective in increasing expected lifetime. This gives rise to expected lifetime being less sensitive to additional declines in the mortality rates, and cause entropy to decline over time [13].

Rectangularization shows an increased building up near a certain old age, as the survival function takes more and more rectangular shape. It is mostly seen as a sign of expected lifetime approaching to its biological limit. Achieving a biological limit means that increases in the total expected lifetime must decelerate. Figure 2 presents an example of rectangularization of the Turkish female survival curve for selected calendar years of 1931, 1950, 1970, 1990, and 2015. Rectangularization is gradually evolving over the last 8 decades, which is a sign of decreasing entropy over the years. Figure shows that in the earlier years survival rate for the infants (ages 0 to 5) increased substantially, and in the latter years the survival curve improvements are concentrated at later years of life (ages 60 to 100). Women made more significant gains as in mortality compared to males, and remarkable improvements are registered in the probabilities of survival to an older age.

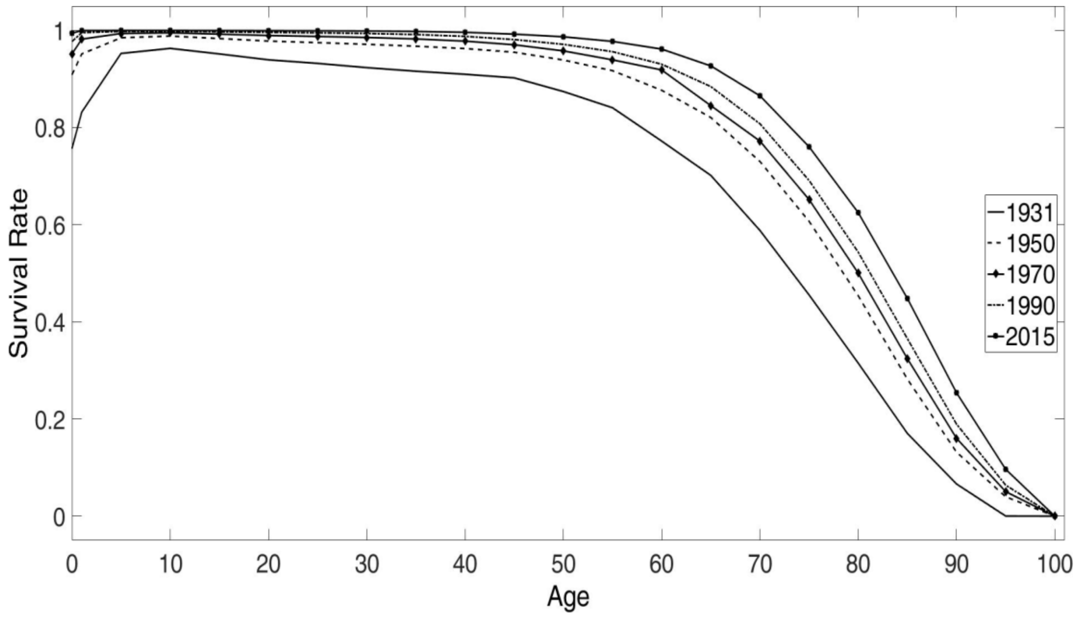

Figure 2: Turkish female population survival curve for selected calendar years.

2.2. Annuity cost defined by entropy

If the effect of changing mortality rates on the annuity is not recognized, it can cause companies to fail to meet their liabilities. In this section we define the annuity cost using the entropy measure based on the method described in the paper of M. Khalaf-Allah [7]. The mortality-decrease-affected value of an annuity product at age x can be defined as:

$$\bar{a}_x^* = \int_0^\infty ({}_t p_x)^{1+\varphi} \exp(-\delta t)dt. \tag{7}$$

The derivative of Equation 7 with respect to φ is,

$$\frac{d\bar{a}_x^*}{d\varphi} = \int_0^\infty (\ln {}_t p_x)({}_t p_x)^{1+\varphi} \exp(-\delta t)dt. \tag{8}$$

Using a Taylor expansion for the neighborhood of $\delta = 0$, it concludes that,

$$\frac{\Delta\bar{a}_x}{\bar{a}_x} = \left(\frac{\int_0^\infty (\ln {}_t p_x)({}_t p_x)\exp(-\delta t)dt}{\int_0^\infty ({}_t p_x)\exp(-\delta t)dt} \right)\varphi. \tag{9}$$

To make $H_x(\delta)$ a positive quantity again, we define it as minus the expression in brackets, as the integrals are always negative.

$$\frac{\Delta\bar{a}_x}{\bar{a}_x} = -H_x(\delta)\varphi. \tag{10}$$

The measure $H_x(\delta)$ dependent on the interest rate, δ, can be displayed as:

$$H_x(\delta) = \frac{-\int_0^\infty (\ln {}_t p_x)({}_t p_x)\exp(-\delta t)dt}{\int_0^\infty ({}_t p_x)\exp(-\delta t)dt}. \tag{11}$$

Over the course of mortality rate decreases, we expect a higher percentile of deaths to concentrate at older ages because of the decrease in the volatility of age-at-death distribution. Also, the value of $H_x(\delta)$ decreases and becomes closer to 0. The rate at which this happens depends on the concavity of the survival function. As interest rates increase, $H_x(\delta)$ decreases as the discount factor increase, because any change of the mortality rate would have a lower effect on the cost of an annuity.

Equation 11 can be written as a weighted average of $\bar{a}_{x+s}$ as:

$$H_x(\delta) = \frac{-\int_0^\infty \mu_{x+s}\, {}_s p_x \exp(-\delta s)\bar{a}_{x+s}\, ds}{\bar{a}_x}. \tag{12}$$

$H_x(\delta)$ can be easily seen as a measure of heterogeneity, as ${}_s p_x \mu_{x+s}$ is the probability density function of the time-of-death of a person at age x. To compute the values of entropy measures, we first need to arrange the integrals in Equation 11, with a numerical approach designed by Pollard [5]. Let

$$_{t}Q_{x}=\int_{0}^{t}\mu_{x+u}\,du=-\ln\left(\frac{l_{x+t}}{l_{x}}\right)=-\ln{}_{t}\,p_{x}, \tag{13}$$

and

$$_{t}E_{x}={}_{t}\,p_{x}\exp(-\delta t). \tag{14}$$

Using Equation 13 and 14 and the mean value theorem; Equation 11 can be expressed as the sums of the one-year integrals. Entropy measure is calculated as:

$$H_{x}(\delta)\approx\frac{\sum_{t=0}^{w}{}_{t+\frac{1}{2}}Q_{x}\,{}_{t+\frac{1}{2}}E_{x}}{\sum_{t=0}^{w}{}_{t+\frac{1}{2}}E_{x}}. \tag{15}$$

3. Turkish mortality tables and entropy measures

Here, we calculated the entropy measure $H_{x}(\delta)$ using the Turkish Life Tables over the time interval between 1931 and 2015. For the applications in this paper we are using MATLAB R2014a software. We consider the plots of mortality rates to determine the properties of population data. From Figure 3, decreasing mortality rates can be seen. To facilitate understanding these rates have been plotted on a logarithmic scale. These life tables will allow us to investigate the entropy measure changes over an almost 85 year period.

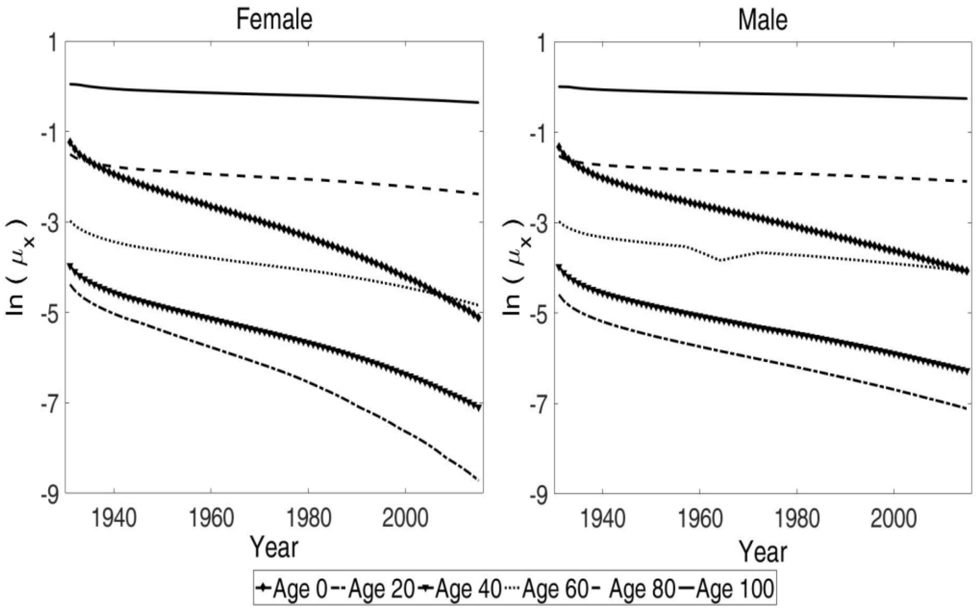

Figure 3: Mortality rates for females and males in log scale over the period 1931-2015.

The current retirement ages in Turkey, until the year 2036, are 60 for males and 58 for females, as stated in law number 4447 [14]. The retirement age of the Social Security Institution will start to increase in 2036, one year in every two years, and reach to 65 for males in 2044, and for females in 2048 [15]. Hence the joint retirement age of two genders for this paper is selected as 65. The center of attraction of the entropy for annuities is for people who retired, mainly between the age ranges [60, 100], and [70, 100] are considered with different rates of interest (e.g., 0%, 2%, 4%, 6%, 8%, 10%). The values for $H_x(\delta)$ at ages 60 and 70 are computed for both genders. Figure 4 displays the entropy measure for females and males, aged 60. For all years, higher the interest rate, lower the value of $H_{60}(\delta)$, and the effect of changes in the mortality on the annuity. As expected, at any interest rate, $H_{60}(\delta)$ is decreasing while levels of mortality get lower from 1931 to 2015. Hence, for the 2015 life table and $\delta = 0.1$ the values of $H_{60}(\delta)$ are lowest. Similar characteristics are valid in the entropy values for males. Generally, male entropy values are higher than the values for females and this interprets as male mortality has more room for development than female mortality rates. Also, higher the interest rates, lower the effect of changes in the force of mortality on the annuity. We expect $H_{60}(\delta)$ to decrease when levels of mortality are lower so that, through 1931 to 2015, the values of $H_{60}(\delta)$ are on the decrease at all rates of interest.

Figure 5 shows the values of $H_{70}(\delta)$ for females and males. As expected, $H_{70}(\delta)$ values are higher than $H_{60}(\delta)$ values at all interest rates. It shows the greater effect of changes in the mortality on the expected lifetime and the cost of a life annuity is in the olger age group (where mortality rates are much higher compared to younger ages) supporting the idea of rectangularization in Turkish mortality rates. Also, similarly to age 60, for all interest rates and for all years, entropy measure is lower for females than males at age 70. This result proves the mortality development potential of male population compared to female population, which already has a higher life expectancy than males, in various different ages like 60 and 70.

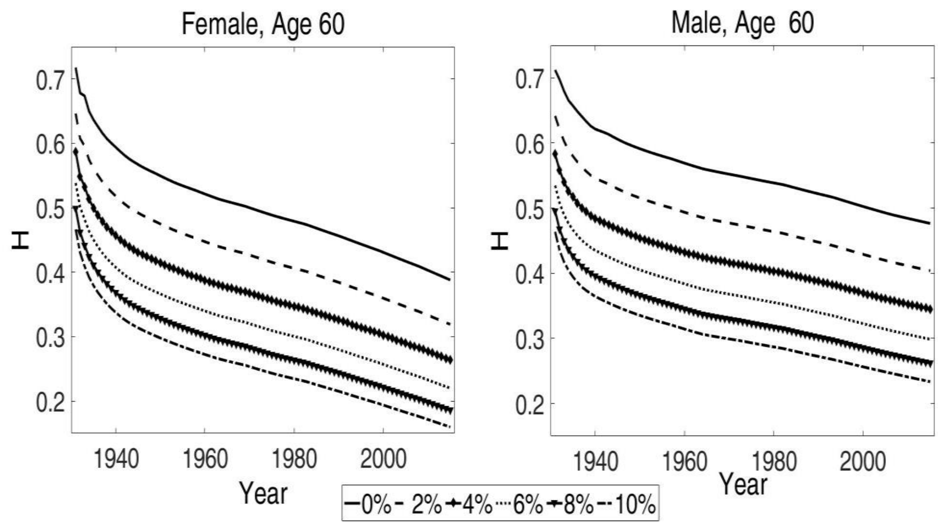

Figure 4: Sensitivity of entropy values to different interest rates, for Turkish life tables, females and males at age 60.

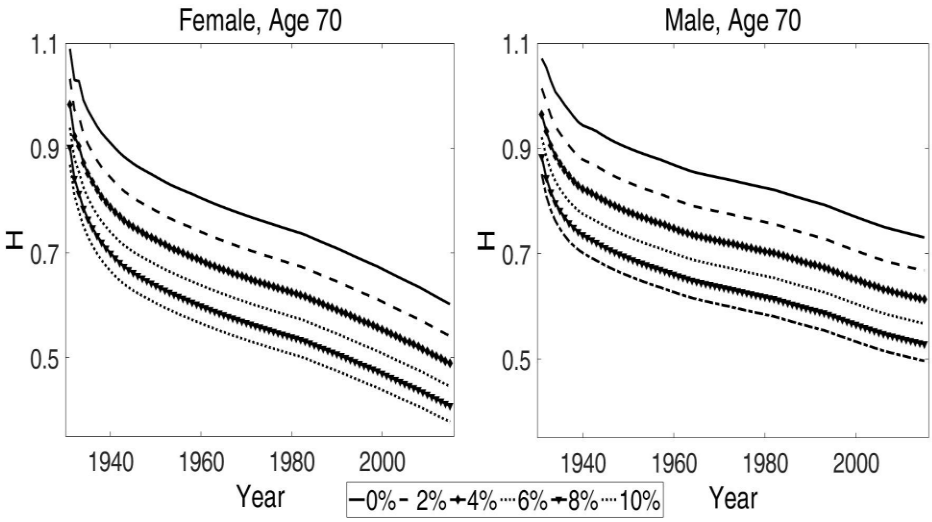

Figure 5: Sensitivity of entropy values to different interest rates, for Turkish life tables, females and males at age 70.

4. Gompertz mortality model and entropy measures

Here, the entropy measure is calculated with an assumption that mortality is following a selected mathematical model. The data from the Construction of the Turkish Life and Annuity Tables Project is used as base mortality tables for the years 2000 and 2015. In this manner, mortality does not depend entirely on observed data, and with this, a hypothetical cohort outlook is adopted rather than a period life table outlook. By using the mathematical model assumption, it allows for a better examination of the different characteristics of entropy and how the value changes with different factors of a mathematical model [7].

Gompertz [16] suggested that a geometric progression pervades in mortality after a certain age. The mortality rates, particularly for ages over 30, suitably fit the Gompertz model. The model is often used to explain the distribution of the adult lifespan. There are other and newer models for modeling the specific features of mortality rates, but the Gompertz mortality model has advantages over them for users, such as implementation simplicities and familiarity with formula. In the recent study of Aburto et al. (2019), the threshold age of the lifetable entropy within the Gompertz mortality model is investigated [17].

The Gompertz mortality model defines the force of mortality as:

$$\mu_x = \exp(b + cx), \tag{16}$$

where, b denotes the force of mortality when the age $x = 0$ (location parameter), and c denotes the rate of aging (shape parameter).

Considering the decreasing rates of mortality and an exponential mortality reduction factor (RF) depending only on time past after the base year, such as $RF(t) = \exp(-\alpha t)$, the new force of mortality is defined as

$$\mu_{x+u}^{t} = \mu_{x+u}^{0} RF(t), \tag{17}$$

where, μ_{x+u}^{0} is the force of mortality in the chosen base year at age $x+u$, and μ_{x+u}^{t} is the force of mortality in the t^{th} year after the base year at age $x+u$.

The behavior of the expected lifetime, $e_x^0(t)$, and annuity value as a function of time is studied. As mentioned in Vaupel [13], we can show show that

$$\frac{\partial e_x^0(t)}{\partial t} \approx \frac{\alpha}{c}\left[1-\mu_x^t e_x^0(t)\right]. \tag{18}$$

When considered in the long term, the following equation is achieved:

$$\frac{\partial e_x^0(t)}{\partial t} \approx \frac{\alpha}{c}. \tag{19}$$

The rate of change in the expectated lifetime, over time, reaches a balanced level (the same can be applied to annuity value).

The force of mortality of the base table (time 0), at age $x+u$, defined as:

$$\mu_{x+u}^{0} = \exp\left[b + c(x+u)\right]. \tag{20}$$

Also RF is used which is depending only on time t for simplicity, in the form $\exp(-\alpha t)$, such that $\mu_{x+u}^{t} = \mu_{x+u}^{0}\exp(-\alpha t)$. Hence, the probability of a person at age x lives to be $x+t$ is:

$$\begin{aligned} {}_t p_x^* &= \exp\left(-\int_0^t \mu_{x+u}^{u}\,du\right) \\ &= \exp\left(-\int_0^t \mu_{x+u}^{0} RF(u)\,du\right) \\ &= \exp\left(-\mu_x^0\left(\frac{\exp\left((c-\alpha)t\right)-1}{c-\alpha}\right)\right). \end{aligned} \tag{21}$$

Here, α is a bounded such that $\alpha < c$, so the probability of survival in Equation 21 will be less than 1.

Entropy is then calculated on a hypothetical cohort basis with the use of a reduction function. Hypothetical cohort life tables are preferable to period life tables at projecting a population into the future when the mortality rate is expected to change over time, and for analyzing the general trends in mortality.

Hereafter, for the notation of the entropy, computed as in [7], $H(b,c,\alpha,\delta)$ can be used and defined as:

$$H(b,c,\alpha,\delta) = \frac{\ln({}_t p_x^*)\, {}_t p_x^* \exp(-\delta t)dt}{\int_0^\infty {}_t p_x^* \exp(-\delta t)dt} = \frac{\mu_x^0}{c-\alpha}\left\{ \frac{\int_0^t \exp\left[-\frac{\mu_x^0}{c-\alpha}\left(\exp((c-\alpha)t)-1\right)\right]\exp\left((c-\alpha-\delta)t\right)dt}{\int_0^t \exp\left[-\frac{\mu_x^0}{c-\alpha}\left(\exp((c-\alpha)t)-1\right)\right]\exp\left(-\delta t\right)dt} - 1 \right\}. \tag{22}$$

One beneficial side of using the Gompertz mortality model is that it enables us to see the effects of its parameters on $H(b,c,\alpha,\delta)$. It helps to grasp the behavior of the entropy. Integrals in Equation 22 can be written as the sums of one-year integrals using the same numerical approach in Equation 15.

2000 and 2015 mortality tables are selected as base mortality tables used in calculations. The parameters μ_x^0, c, α, and δ are needed to calculate $H(b,c,\alpha,\delta)$ values from the Equation 22. The Gompertz mortality model is applied to data in order to estimate c. Lower c value indicates that the mortality level of the base table is at a lower level. Table 1 shows the mortality rates and estimated c values for ages 0, 60 and 70, for the base table 2000. Table 2 shows the same features for the base table 2015.

Mortality rates of the base year 2000 and 2015 with the Gompertz models mortality rates are shown in Figure 6 and 7, which gives similar results. Gompertz mortality model is not a suitable fit for ages younger than 30. For ages 60 and 70, mortality rate estimations are consistent with the data. For this reason, age 0 is not taken into consideration in the rest of the application.

Table 1. 2000 base table mortality rates and estimated c values.

	Female		Male	
Age	μ_x^0	c	μ_x^0	c
0	0.014789	0.0741	0.026659	0.0664
60	0.011753	0.1037	0.020131	0.0924
70	0.037185	0.0996	0.052360	0.0910

The sensitivity of $H_x(b,c,\alpha,\delta)$ to α and δ parameters are tested. Various levels of improvement in mortality rates are considered by allowing α to vary between -0.1 and 0.1. Positive values of α stand for decreasing mortality rates, and negative values of α stands for increasing mortality rates over time. Also, different rates of interest are considered from 0% to 10%. As can be seen from Figures 8 and 9 the value of entropy, $H_x(b,c,\alpha,\delta)$, decreases in all four cases as the interest rate increases. When the interest rate is lower, the value of entropy is at its highest, and it is directly proportional to the mortality rate improvement level. $H_x(b,c,\alpha,\delta)$ increases with the α in direct proportion up until a level (breaking point of H) which thereafter any improvement in mortality rate decreases H. Hence mortality rate improvements start to lose their effect on the annuity value. As the interest rate increase, entropy decreases almost continuously and there is no peak point as mortality improves.

Table 2. 2015 base table mortality rates and estimated c values.

	Female		Male	
Age	μ_x^0	c	μ_x^0	c
0	0.006029	0.0869	0.017054	0.0722
60	0.007890	0.1112	0.017071	0.0953
70	0.028844	0.1050	0.046953	0.0932

H values and the breaking points are consistently smaller for women than men; this reflects the relative difference in favor of women in the survival probabilities, annuity costs, and life expectations of various ages. Increasing age from 60 to 70 causes H values and the breaking points for both genders to increase.

As a summary with $\alpha = 0.05$ and $\delta = 0.04$ changes in the annuity values are found with the Equation 10. For the base year 2000, the change in the annuity value for males; at age 60 is 1.68%, and at age 70 is 2.59%. For the base year 2015, these values change to 1.54% at age 60 and 2.46% at age 70. For the base year 2000, the change in the annuity value for females; at age 60 is 1.26% and at age 70 is 2.19%. For the base year 2015, these values change to 1.01% at age 60 and 1.93% at age 70.

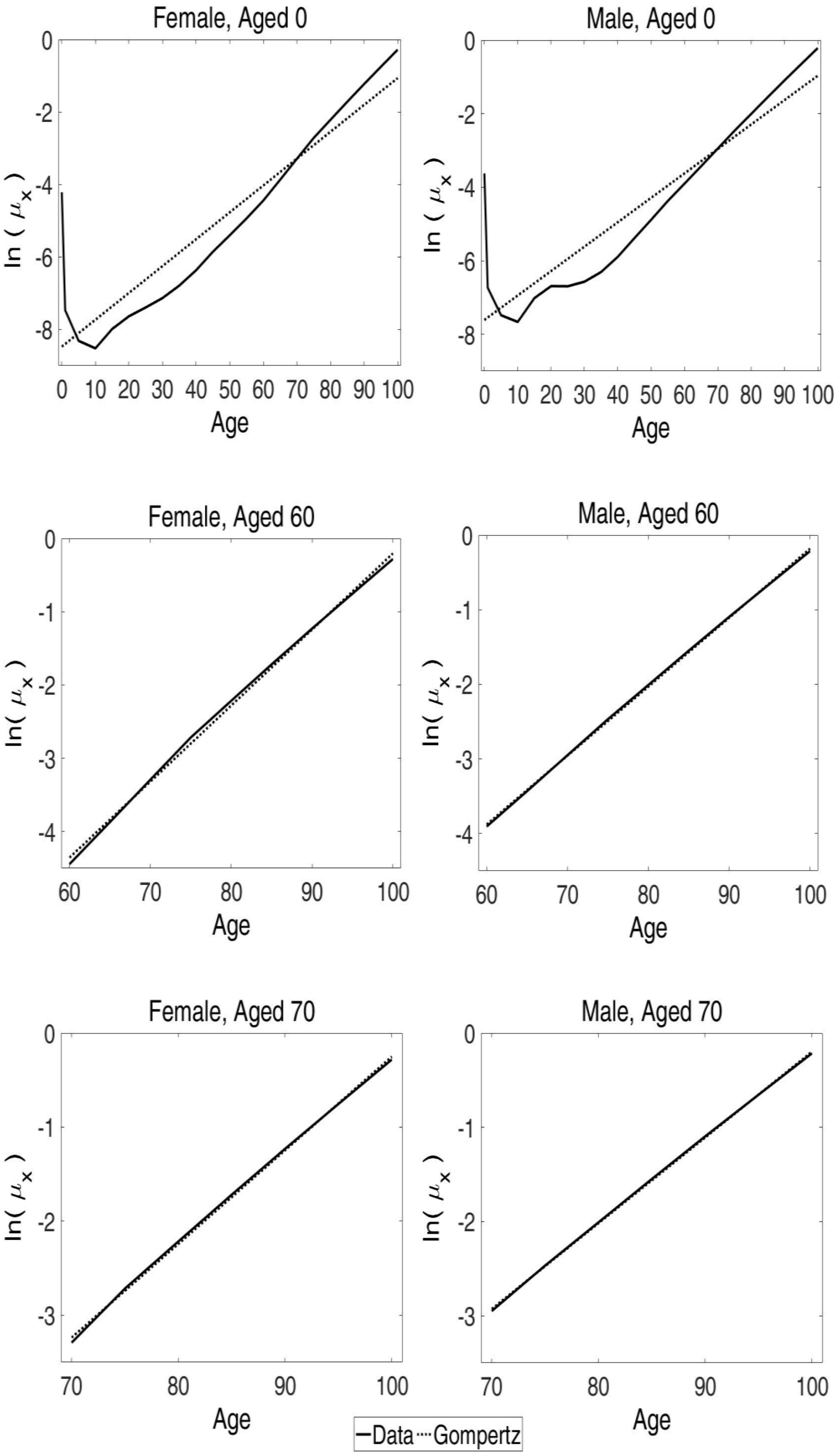

Figure 6: Data and Gompertz model comparisons of mortality rates of 2000, in logarithmic scale at ages 0, 60 and 70.

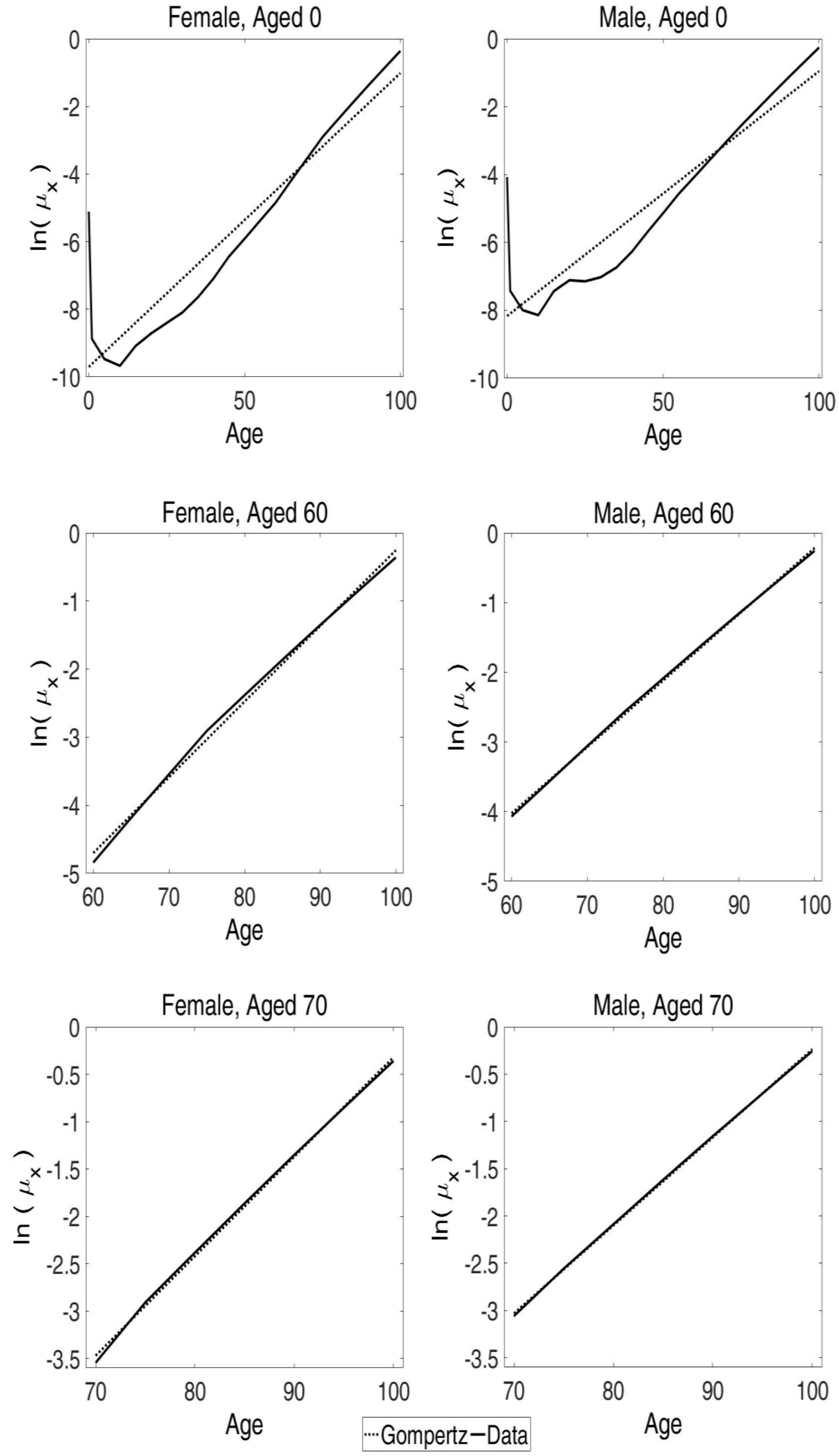

Figure 7: Data and Gompertz model comparisons of mortality rates of 2015, in logarithmic scale at ages 0, 60 and 70.

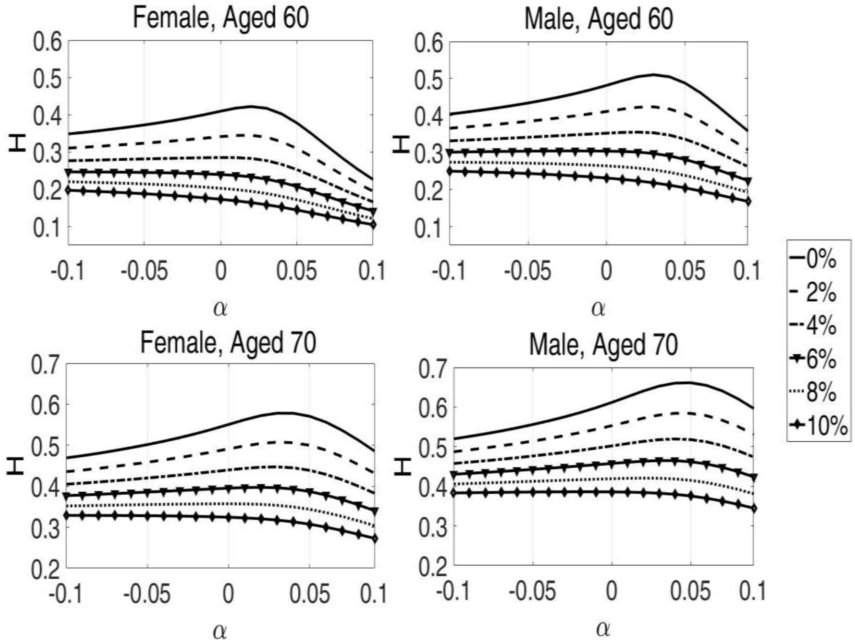

Figure 8: Sensitivity of $H_x(b,c,\alpha,\delta)$ in year 2000 to α and δ for ages 60 and 70.

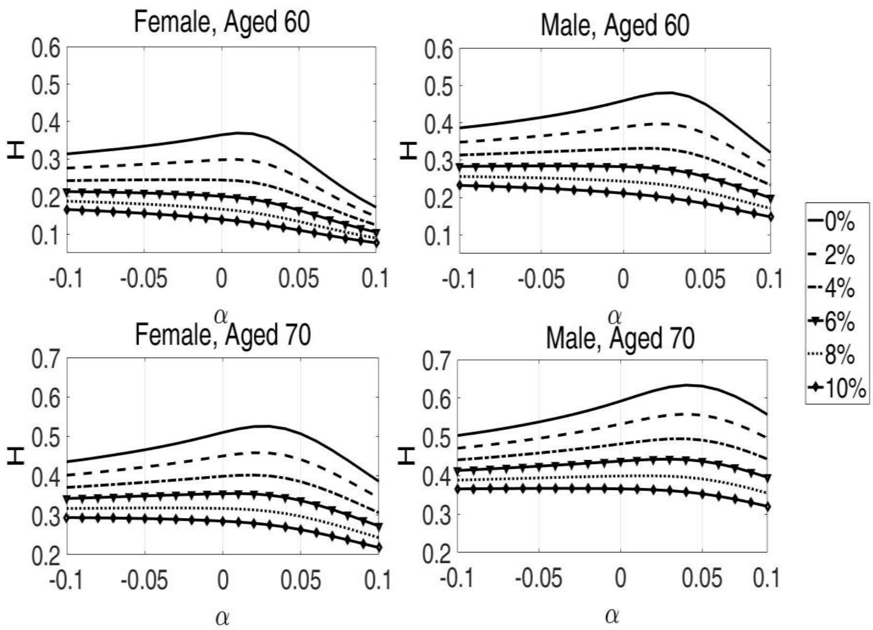

Figure 9: Sensitivity of $H_x(b,c,\alpha,\delta)$ in year 2015 to α and δ for ages 60 and 70.

Effect of the base mortality table on entropy can be tested by different values of c. To exemplify, the values of c between 0.09 and 0.115 is taken into consideration. Figures 10 and 11 shows results for $\delta = 0\%$. $H_x(b,c,\alpha,\delta)$, as a function of α, is peaked at one point (breaking point of H). When the RF is greater than one (when α is negative), entropy values are higher in lower mortality level ($c = 0.09$) than the ones in higher mortality levels ($c = 0.0115$). On the contrary, when the RF is less than one (when α is positive), this relationship is reversed with improving mortality.

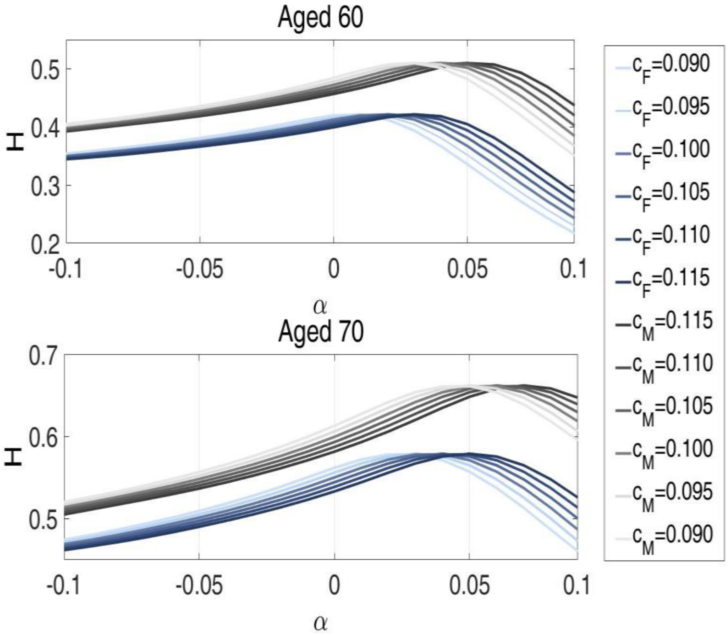

Figure 10: Entropy values for the base year 2000 with a range of c values, and $\delta = 0\%$.

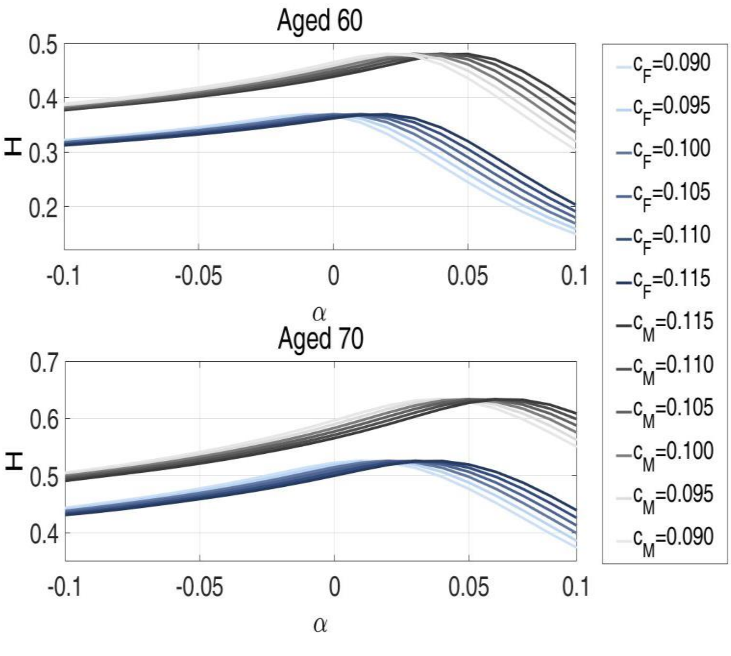

Figure 11: Entropy values for the base year 2015 with a range of c values, and $\delta = 0\%$.

Initially, verifying the conclusion that annuity cost with a high c value (high mortality group) is less effected from the changes in the mortality rate than with a low c value when the mortality rate is high. At decreased levels of mortality, this is reversed and the annuity cost with a high c value is more effected from the changes in the mortality rate than a low c value.

5. Conclusion

The decrease in mortality rates was expected globaly, mostly due to elimination of disease-related factors and with better living conditions. The unexpected part of the decrease was the continuity. The persistent decrease in mortality rates is causing several new challenges and questions at multiple levels, such that is there a limit of human life expectancy? Challenges include Turkish annuity and pension providers, as they have to update the policy tariffications continuously. Mortality reduction and rising longevity for post-retirement ages have a remarkable financial effect on the annuity and pension plans. Because of the tax-paid structure of the pension system, the deficiencies of public social security system are aggravated by mortality improvements.

We examined the entropy measure under various scenarios to measure the effect of any change in the mortality on the annuity value. The entropy measure is calculated with different mortality, age, gender, and interest rate parameters. Results show that entropy values are higher for the males at different ages, suggesting that male population has more room for mortality developments rather than female population. Entropy is found to be highly sensitive to age, selected base year, interest rate, and mortality changes. As the age increase, entropy value follows this increase to the older ages in life. In most developed countries infant mortality is mostly eliminated and further increases in life expectancy and developments in mortality are concentrated in later years of life, as suggested by Fernandez and Beltrán-Sánchez [18]. We see that annuity and entropy are inverse proportion to interest rate and directly proportional to age, supporting the idea of rectangularization in the Turkish mortality rates. In all scenarios, lower the interest rate, higher the value of entropy. This highlights the importance that there is a higher effect of mortality risk on the net single premium of annuity payments; in particular with a low-interest economical environment.

At very high and low levels of mortality developments, the numerical results suggest that the effect of mortality changes on the annuity value is minimal and less likely to have a significant effect on the the annuity values. In this case, even if mortality continues to decline in the future, it will approach to a country specific level and any more future decline would not affect the annuity premiums.

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