TWMS J. App. Eng. Math. V.9, N.3, 2019, pp. 554-562

SYMMETRIC BI-T-DERIVATION OF LATTICES

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ABSTRACT. In this paper, the notion of a new kind of derivation is introduced for a lattice $(L, \vee, \wedge)$, called symmetric bi-T-derivations on L as a generalization of derivation of lattices and characterized some of its related properties. Some equivalent conditions provided for a lattice L with greatest element 1 by the notion of isotone symmetric bi-T-derivation on L. By using the concept of isotone derivation, we characterized the modular and distributive lattices by the notion of isotone symmetric bi-T-derivation on L.

Keyword: Lattice, Derivation of lattice, Symmetric bi-T-derivation of lattice, Modular lattice and Distributive lattice.

AMS Subject Classification: 03G16, 06C05, 17A36

1. INTRODUCTION

The notion of lattice theory introduced by [1]. After the initiation of lattices many researchers studied lattice theory in different point of view such as, Balbes and Dwinger [2] gave the concept on distributive lattices and Hoffmann gave the notion of partially ordered set (Poset). The application of lattice theory plays an important role in different areas such as information theory by [3], information retrieval by Carpineto and Romano [4], information access controls by [5] and cryptanalysis by [6]. Recently, the properties of lattices were studied by some authors [7] in analytic and algebraic point of view.

Derivations is a very interesting research area in the theory of algebraic structure in mathematics. Posner [8] provided the concept of derivation on rings. Based on this concept Bell and Kappea [9] studied that rings in which derivations satisfy certain algebraic conditions. The notion of generalized derivation in ring introduced by Braser [10] and Hvala [11]. This concept of derivation further carried out by many authors Argaç and Albas [12], Jana et al. [20] studied derivation on KUS-algebras, Gölbaşi and Kaya [13] in prime rings and lie ideal in prime rings. Jana et al. [14-19] have been studied lot of works on $BCK/BCI/G$-algebras. The study of derivation in lattice theory is an important topic in application of different mode. Xin et al. [22] introduced the notion of derivation in lattices and discussed its properties. Thereafter, many authors generalized this idea in lattices such as, symmetric

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§ Manuscript received: June 21, 2017; accepted: August 24, 2017.
TWMS Journal of Applied and Engineering Mathematics, Vol.9, No.3

bi-derivation studied by Maksa [23, 24] many researchers introduced this concept to study symmetric bi-derivation on rings and near-rings by Ozturk and Sapancy [25, 26, 27, 28] and we focused to the study of symmetric bi-derivation on lattices and investigated some properties on it by Çven [29].

In this article, we applied a new approach to the study of derivation in lattice theory by the concept of t-derivation of complicated subtraction algebra is defined by Jana et al. [21]. This work is enough to motivated us and best of our knowledge there is no work available on symmetric bi-T-derivation of lattices. In this paper, the notion of symmetric bi-T-derivation on lattices is introduced, which is a generalization of derivation in lattices is introduced and studied some properties of it. We gave some equivalent condition for which a derivation to be an isotone symmetric bi-T-derivation for a lattices with greatest element. We characterized modular lattices and distributive lattices by the concept of isotone symmetric bi-T-derivation.

2. Preliminaries

Definition 2.1. [1] *Let L be a non-empty set endowed with operations $\wedge$ and $\vee$. Then set $(L, \wedge, \vee)$ is called lattices if for all $x, y, z \in L$ satisfies the following conditions:*
$(L1)$ $x \wedge x = x$, $x \vee x = x$
$(L2)$ $x \wedge y = y \wedge x$, $x \vee y = y \vee x$
$(L3)$ $(x \wedge y) \wedge z = x \wedge (y \wedge z)$, $(x \vee y) \vee z = x \vee (y \vee z)$
$(L4)$ $(x \wedge y) \vee x = x$, $(x \vee y) \wedge x = x$.

Definition 2.2. [1] *A Lattice $(L, \wedge, \vee)$ is called distributive lattice if for all $x, y, z \in L$ satisfies the following conditions:*
$(L5)$ $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
$(L6)$ $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

It is notified that in a Lattice the conditions $(L5)$ and $(L6)$ are equivalent.

Definition 2.3. [1] *Let $(L, \wedge, \vee)$ be a lattice. A binary relations $(\leq)$ on L defined by $x \leq y$ is holds if and only if $x \wedge y = x$ and $x \vee y = y$.*

Definition 2.4. [2] *A lattice $(L, \wedge, \vee)$ is called a modular lattice if for all $x, y, z \in L$ satisfies the following conditions:*
$(L7)$ *If $x \leq y$ implies $x \vee (y \wedge z) = (x \vee y) \wedge z$.*

Definition 2.5. [22] *Let $(L, \wedge, \vee)$ be a lattice. Then the binary relation $(\leq)$ which is defined in Definition 2.3. Then $(L, \leq)$ is a poset i.e. is a partially ordered set and for any $x, y \in L$, $x \wedge y$ is the g.l.b of $\{x, y\}$, and $x \vee y$ is the l.u.b of $\{x, y\}$.*

Proposition 2.1. [22] *Let L be a lattice and d be a derivation on L. Then for all $x, y \in L$, the following are holds:*

(1) $d(x) \leq x$
(2) $d(x) \wedge d(y) \leq d(x \wedge y) \leq d(x) \vee d(y)$.

Definition 2.6. [22] *Let L be a lattice and d be a derivation on L*

(1) *$x \leq y$ implies $d(x) \leq d(y)$, then d is called an isotone derivation*
(2) *If d is one-to-one, then d is called a monomorphic derivation*
(3) *If d is onto, then d is called an epimorphic derivation.*

Definition 2.7. [29] *Let $(L, \wedge, \vee)$ be a lattice. A function $D(.,.) : L \times L \to L$ is called symmetric if satisfies the condition $D(x, y) = D(y, x)$ for all $x, y \in L$.*

Definition 2.8. [29] *Let L be a lattice. A function $d : L \times L \to L$ defined by $d(x) = D(x, x)$ is called trace of $D(.,.)$, where $D(.,.) : L \times L \to L$ is a symmetric function.*

Definition 2.9. [29] *Let L be a lattice and Let $D : L \times L \to L$ be a symmetric function on L. Then D is called symmetric bi-derivation on L if satisfies the following identity:*

$$D(x \wedge y, z) = (D(x, z) \wedge y) \vee (x \wedge D(y, z))$$

for all $x, y, z \in L$. Also, A symmetric bi-derivation D satisfies the following relation

$$D(x, y \wedge z) = (D(x, y) \wedge z) \vee (y \wedge D(x, z))$$

for all $x, y, z \in L$.

3. Symmetric bi-T-derivations on lattices

In this section, the following definition introduced symmetric bi-T-derivation on a lattice.

Definition 3.1. *Let L be a lattice. Then for any $T \in L$, we define a self-map $D_T : L \times L \to L$ by $D_T(x, y) = (x \wedge y) \wedge T$ for all $x, y \in L$.*

Definition 3.2. *Let L be a lattice. Then for any $T \in L$, a self-map $D_T : L \times L \to L$ is defined as for any $T \in L$, $D_T(x, y) = (x \wedge y) \wedge T$ for all $x \in L$. Then function $D_T : L \times L \to L$ is called symmetric bi-T-derivation of L if satisfies the following condition:*

$$D_T(x \wedge y, z) = (D_T(x, z) \wedge y) \vee (x \wedge D_T(y, z))$$

for all $x, y, z \in L$. Also, A symmetric bi-T-derivation D_T satisfies the following relation

$$D_T(x, y \wedge z) = (D_T(x, y) \wedge z) \vee (y \wedge D_T(x, z))$$

for all $x, y, z \in L$.

Example 3.1. *Let $L = \{0, a, b, 1\}$ be a lattice shown by the Hasse diagram of Figure 1 Define the mapping $\mathcal{D}_T$ as follows:*
for $T = 0$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in L \times L$
for $T = a$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in \{(0, 0), (0, a), (a, 0), (b, 0), (0, b), (1, 0), (0, 1)\}$
$\mathcal{D}_T(x, y) = a$ for all $(x, y) \in \{(a, a), (a, b), (b, a), (a, 1), (1, a), (b, b), (b, 1), (1, b), (1, 1)\}$
for $T = b$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in \{(0, 0), (a, 0), (0, a), (0, b), (b, 0), (1, 0), (0, 1)\}$, $\mathcal{D}_T(x, y) = a$ for all $(x, y) \in \{(a, a), (a, b), (b, a), (a, 1), (1, a)\}$ and $\mathcal{D}_T(x, y) = b$ for all $(x, y) \in \{(b, b), (b, 1), (1, b), (1, 1)\}$
For $T = 1$, $\mathcal{D}_T(x, y) = 0$ for all $(x, y) \in \{(0, 0), (0, a), (a, 0), (b, 0), (0, b), (1, 0), (0, 1)\}$, $\mathcal{D}_T(x, y) = a$ for all $(x, y) \in \{(a, a), (a, b), (b, a), (a, 1), (1, a)$, $\mathcal{D}_T(x, y) = b$ for all $(x, y) \in \{(b, b), (b, 1), (1, b)\}$ and $\mathcal{D}_T(x, y) = 1$ for $(x, y) = (1, 1)$. then it is verified that for each $T \in L$, $\mathcal{D}_T$ is a symmetric bi-T-derivation on L.

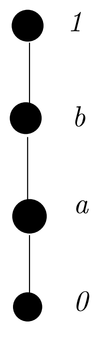

Figure 1. The lattice in example 3.3

Proposition 3.1. *Let L be a lattice with least element 0. Then For $T = 0 \in L$, we have $D_0(x, y) = 0$ for all $x, y \in L$.*

Proof: For $T = 0 \in L$, we have $D_0(x, y) = (D_0(x \wedge x, y) = (D_0(x, y) \wedge x) \vee (x \wedge D_0(x, y))$
$= (((x \wedge y) \wedge 0) \wedge x) \vee ((x \wedge ((x \wedge y) \wedge 0)))$
$= (0 \wedge x) \vee (x \wedge 0) = 0 \vee 0 = 0.$ □

Theorem 3.1. *Let L be a lattice and d_T be a trace of symmetric bi-T-derivation D_T. Then following conditions are hold for all $x, y \in L$.*

(1) $D_T(x, y) \leq x$ and $D_T(x, y) \leq y$
(2) $D_T(x, y) \wedge D_T(w, y) \leq D_T(x \wedge w, y) \leq D_T(x, y) \vee D_T(w, y)$
(3) $D_T(x \wedge w, y) \leq x \vee y$
(4) $D_T(x, y) \leq x \wedge y$
(5) $d_T(x) \leq x$
(6) $d_T^2(x) = d_T(x)$.

Proof:

(1) Since $D_T(x, y) = D_T(x \wedge x, y) = (D_T(x, y) \wedge x) \vee (x \wedge D_T(x, y)) = x \wedge D_T(x, y)$ from which we get $D_T(x, y) \leq x$. In similar manner we shown $D_T(x, y) \leq y$ for all $x, y \in L$.
(2) Since $D_T(x, y) \leq x$ and $D_T(w, y) \leq w$. Then, we have $D_T(x, y) \wedge D_T(w, y) \leq x \wedge D_T(w, y)$, and from (1) $D_T(x, y) \wedge D_T(w, y) \leq w \wedge D_T(x, y)$ for all $x, y, w \in L$. Hence, $D_T(x, y) \wedge D_T(w, y) \leq (x \wedge D_T(w, y)) \vee (w \wedge D_T(x, y)) = D_T(x \wedge w, y)$. Also, since $x \wedge D_T(w, y) \leq D_T(w, y)$ and $w \wedge D_T(x, y) \leq D_T(x, y)$, and obtained $(x \wedge D_T(w, y)) \vee (w \wedge D_T(x, y) \leq D_T(x, y) \vee D_T(w, y)$. Thus, $D_T(x \wedge w, y) \leq D_T(x, y) \vee D_T(w, y)$.
(3) Since $D_T(x, y) \wedge w \leq w$ and $x \wedge D_T(w, y) \leq x$. Therefore, $(D_T(x, y) \wedge w) \vee (x \wedge D_T(w, y)) \leq x \vee w$. Hence, $D_T(x \wedge w, y) \leq x \vee w$.
(4) From (1) it is clear that $D_T(x, y) \leq x \wedge y$ for all $x, y \in L$.
(5) Since $d_T(x) = D_T(x \wedge x, x) = (D_T(x, x) \wedge x) \vee (x \wedge D_T(x, x)) = x \wedge D_T(x, y)$ from which we obtained $d_T(x) \leq x$ for all $x \in L$.
(6) From (5) it is seen that $d_T^2(x) = d_T(d_T(x)) \leq d_T(x) \leq x$ and also from (1) gives $D_T(x, d_T(x)) \leq d_T(x)$. Then, we have $d_T^2(x) = d_T(d_T(x)) = d_T(x \wedge d_T(x))$
$= D_T(x, d_T(x)) \vee (x \wedge d_T^2(x)) \vee (d_T(x) \wedge x)$
$= D_T(x, d_T(x)) \vee d_T^2(x) \vee d_T(x) = D_T(x, d_T(x)) \vee d_T(x).$ □

Corollary 3.1. *Let L be a lattice and D_T be a symmetric bi-T-derivation on L with least element 0 and greatest element 1. Then $D_T(0, x) = 0$ and $D_T(1, x) \leq x$ for all $x \in L$.*

Proof: The proof of the corollary is trivial by Proposition 3.1(1). □

Theorem 3.2. *Let L be a lattice and D_T be symmetric bi-T-derivation of L and d_T be the trace of symmetric bi-T-derivation D_T. Then, $d_T(x \wedge y) = D_T(x, y) \vee (x \wedge d_T(y)) \vee (y \wedge d_T(x))$ for all $x, y \in L$.*

Proof: Using the Proposition 3.1 (1) and (5), we have

$$\begin{aligned}
d_T(x \wedge y) &= D_T(x \wedge y, x \wedge y) \\
&= (D_T(x \wedge y, x) \wedge y) \vee (D_T(x \wedge y, y) \wedge x) \\
&= D_T(x \wedge y, x) \vee D_T(x \wedge y, y) \\
&= ((d_T(x) \wedge y) \vee (x \wedge D_T(x, y))) \vee ((D_T(x, y) \wedge y) \vee (x \wedge d_T(y))) \\
&= ((d_T(x) \wedge y) \vee D_T(x, y)) \vee (D_T(x, y) \vee (x \wedge d_T(y))) \\
&= D_T(x, y) \vee (x \wedge d_T(y)) \vee (y \wedge d_T(x)).
\end{aligned}$$

Corollary 3.2. *Let L be a lattice and D_T be symmetric bi-T-derivation of L and d_T be the trace of symmetric bi-T-derivation d_T. Then followings are hold: for all $x, y \in L$*

(1) $D_T(x, y) \leq d_T(x \wedge y)$
(2) $x \wedge d_T(y) \leq d_T(x \wedge y)$
(3) $y \wedge d_T(x) \leq d_T(x \wedge y)$
(4) $d_T(x) \wedge d_T(y) \leq d_T(x \wedge y)$.

Proof: The proof of (1),(2) and (3) are trivial by Theorem 3.2. (4) can be proved by using (2), (3) and Proposition 3.1(5). □

Corollary 3.3. *Let L be a lattice with least element 0 and greatest element 1, and D_T be symmetric bi-T-derivation of L and d_T be the trace of symmetric bi-T-derivation d_T, then followings are hold:*

(1) *If $x \geq d_T(1)$, then $d_T(x) \geq d_T(1)$*
(2) *If $x \leq d_T(1)$, then $d_T(x) = x$*
(3) *If $x \leq y$ and $d_T(y) = y$, then $d_T(x) = x$.*

Proof: Straight forward. □

Theorem 3.3. *Let L be a lattice with greatest element 1 and let d_T be a trace of a symmetric bi-T-derivation D_T. Then following conditions are equivalent:*

(1) *d_T is an isotone mapping*
(2) $d_T(x) = x \wedge d_T(1)$
(3) $d_T(x \wedge y) = d_T(x) \wedge d_T(y)$
(4) $d_T(x) \wedge d_T(y) \leq d_T(x \vee y)$.

Proof: Proof of theorem is straight forward. □

Theorem 3.4. *Let L be a lattice with greatest element 1 and d_T be a trace of symmetric bi-T-derivation D_T. Then followings are equivalent for all $x, y, z \in L$*

(1) *d_T is isotone*
(2) $d_T(x) = x \wedge d_T(1)$
(3) $d_T(x \wedge y) = d_T(x) \wedge d_T(y)$
(4) $d_T(x) \wedge d_T(y) \leq d_T(x \vee y)$.

Proof:

(1) (1) $\Rightarrow$ (2). Since d_T is isotone and $x \leq 1$, we have $x \leq d_T(1)$ and by Theorem 3.1 (5), $d_T(x) \leq x$, and so obtained $d_T(x) \leq x \wedge d_T(1)$. Also, by Corollary 3.2, we get $x \wedge d_T(1) \leq d_T(x)$. Therefore, $d_T(x) = x \wedge d_T(1)$ for all $x \in L$.
(2) (2) $\Rightarrow$ (3). Let $d_T(x) = x \wedge d_T(1)$.
Then $d_T(x \wedge y) = (x \wedge y) \wedge d_T(1)$
$= (x \wedge y) \wedge (d_T(1) \wedge d_T(1)) = (x \wedge d_T(1)) \wedge (y \wedge d_T(1)) = d_T(x) \wedge d_T(y)$ for all $x, y \in L$.
(3) (3) $\Rightarrow$ (1). Let $d_T(x \wedge y) = d_T(x) \wedge d_T(y)$ and $x \leq y$ and so, $d_T(x) = d_T(x \wedge y) = d_T(x) \wedge d_T(y)$. Hence, $d_T(x) \leq d_T(y)$.
(4) (1) $\Rightarrow$ (4). Let d_T be isotone. Since $x \leq x \vee y$ and $y \leq x \vee y$. Then $d_T(x) \leq d_T(x \vee y)$ and $d_T(y) \leq d_T(x \vee y)$. Thus, $d_T(x) \wedge d_T(y) \leq d_T(x \vee y)$.
(5) (4) $\Rightarrow$ (1). Let $x \leq y$. Then $d_T(x) = d_T(x \vee y) \leq d_T(y)$. Hence, $d_T(x) \leq d_T(y)$. □

Proposition 3.2. *Let L be a lattice with greatest element 1 and D_T be a symmetric bi-T-derivation. Then followings are holds.*

(1) *If $x \leq D_T(1, y)$, then $D_T(x, y) = x$*
(2) *If $x \geq D_T(1, y)$, then $D_T(x, y) \geq D_T(1, y)$.*

Proof:

(1) Let $x \leq D_T(1,y)$, then $D_T(x,y) = D_T(x \wedge 1, y) = (D_T(x,y) \wedge 1) \vee (x \wedge D_T(1,y)) = D_T(x,y) \vee x$. Hence, $x \leq D_T(x,y)$ and $D_T(x,y) = x$ by
(2) Let $x \geq D_T(1,y)$. Then, $D_T(x,y) = D_T(x \wedge 1, y) = (D_T(x,y) \wedge 1) \vee (x \wedge D_T(1,y)) = D_T(x,y) \vee D_T(1,y)$. Thus, $D_T(1,y) \leq D_T(x,y)$ for all $x, y \in L$. □

Proposition 3.3. *Let L be a lattice and D_T be a symmetric bi-T-derivation on L. Then following condition is hold:*
(1) If D_T is an symmetric bi-T-derivation on L, then $D_T(x,y) = D_T(x,y) \vee (D_T(x \vee s, y) \wedge x)$

Proof: Let D_T be an isotone symmetric bi-T-derivation. Then,

$$\begin{aligned} D_T(x,y) &= D_T((x \vee s) \wedge x, y) \\ &= (D_T(x \vee s, y) \wedge x) \vee ((x \vee s) \wedge D_T(x,y)) \\ &= (D_T(x \vee s, y) \wedge x) \vee D_T(x,y). \end{aligned}$$

As, $D_T(x,y) \leq D_T(x \vee s, y) \leq (x \vee s)$. □

Theorem 3.5. *Let L be a lattice with greatest element 1 and D_T be a symmetric bi-T-derivation on L. Then followings are equivalent:*

(1) *D_T is isotone symmetric bi-T-derivation*
(2) $D_T(x,y) \vee D_T(s,y) \leq D_T(x \vee s, y)$
(3) $D_T(x,y) = x \wedge D_T(1,y)$
(4) $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$

Proof:

(1) (1) $\Rightarrow$ (2). We assume that D_T is a symmetric bi-T-derivation on L. Since $x \leq x \vee s$ and $s \leq x \vee s$, and so $D_T(x,y) \leq D_T(x \vee s, y)$ and $D_T(s,y) \leq D_T(x \vee s, y)$. Hence, $D_T(x,y) \vee D_T(s,y) \leq D_T(x \vee s, y)$
(2) (2) $\Rightarrow$ (1). Suppose that $D_T(x,y) \vee D_T(s,y) \leq D_T(x \vee s, y)$ and $x \leq s$. Then, we get $D_T(x,y) \leq D_T(x,y) \vee D_T(s,y) \leq D_T(x \vee s, y) = D_T(s,y)$. Therefore, D_T is an isotone symmetric bi-T-derivation on L.
(3) (1) $\Rightarrow$ (3). Suppose D_T is an isotone symmetric bi-T-derivation on L. Since, $D_T(x,y) \leq D_T(1,y)$, we have $D_T(x,y) \leq x \wedge D_T(1,y)$ by Theorem 3.1 (1). Using Proposition 3.3 and by $s = 1$, we get

$$\begin{aligned} D_T(x,y) &= (D_T(1,y) \wedge x) \vee D_T(x,y) \\ &= D_T(1,y) \wedge x. \end{aligned}$$

(4) (3) $\Rightarrow$ (4). Assume that $D_T(x,y) = x \wedge D_T(1,y)$, then $D_T(x \wedge s, y) = (x \wedge s) \wedge D_T(1,y) = x \wedge s \wedge D_T(1,y) = (x \wedge D_T(1,y)) \vee (s \wedge D_T(1,y)) = D_T(x,y) \wedge D_T(s,y)$
(5) (4) $\Rightarrow$ (1). Let $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$ and $x \leq s$. Then, $D_T(x,y) = D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$. Hence, $D_T(x,y) \leq D_T(s,y)$. □

Theorem 3.6. *Let L be a modular lattice and D_T be a symmetric bi-T-derivation on L. Then, followings are hold.*

(1) *If D_T is an isotone symmetric bi-T-derivation on L if and only if $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$*
(2) *If D_T is an isotone symmetric bi-T-derivation and $D_T(x,y) = x$, then $D_T(x \vee s, y) = D_T(x,y) \vee D_T(s,y)$.*

Proof:

(1) Let D_T be a symmetric bi-T-derivation on L. Since $x \wedge s \leq x$ and $x \wedge s \leq s$, then $D_T(x \wedge s, y) \leq D_T(x, y) \wedge D_T(s, y)$. Therefore,

$$\begin{aligned} D_T(x,y) \wedge D_T(s,y) &= (D_T(x,y) \wedge D_T(s,y)) \wedge (x \wedge s) \\ &= (D_T(x,y) \wedge s) \wedge (D_T(s,y) \wedge s) \\ &\leq (D_T(x,y) \wedge s) \vee (D_T(s,y) \wedge x) \\ &= D_T(x \wedge s, y). \end{aligned}$$

Conversely, let $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$ and $x \leq s$. Thus, $D_T(x,y) = D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$, and hence $D_T(x,y) \leq D_T(s,y)$ for all $x, y, s \in L$.

(2) Let D_T be a symmetric bi-T-derivation on L and $D_T(x,y) = x$. Then, by Proposition 3.3 and since L is a modular lattice, thus, $D_T(s,y) = (D_T(s,y) \vee D_T(x \vee s, y)) \wedge s = s \wedge D_T(x \vee s, y)$. Thus,

$$\begin{aligned} D_T(x,y) \vee D_T(s,y) &= D_T(x,y) \vee (s \wedge D_T(x \vee s, y)) \\ &= (D_T(x,y) \vee s) \wedge D_T(x \vee s, y) \\ &= (x \vee s) \wedge D_T(x \vee s, y) \\ &= D_T(x \vee s, y). \end{aligned}$$

□

Theorem 3.7. *Let L be a distributive lattice and D_T be a symmetric bi-T-derivation on L. Then, following conditions are hold.*

(1) *If D_T is an isotone symmetric bi-T-derivation on L, then $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$*
(2) *If D_T is an isotone symmetric bi-T-derivation on L if and only if $D_T(x \vee s, y) = D_T(x,y) \vee D_T(s,y)$.*

Proof:

(1) Since, D_T is an isotone symmetric bi-T-derivation and $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$. By Theorem 3.1 (1), we have

$$\begin{aligned} D_T(x,y) \wedge D_T(s,y) &= ((D_T(x,y) \wedge x) \wedge ((s \wedge D_T(s,y)) \\ &= (D_T(x,y) \vee s) \wedge (x \wedge D_T(s,y) \\ &\leq (D_T(x,y) \wedge s) \vee (x \wedge D_T(s,y) \\ &= D_T(x \wedge s, y). \end{aligned}$$

Therefore, $D_T(x \wedge s, y) = D_T(x,y) \wedge D_T(s,y)$ for all $x, y, s \in L$.

(2) Let D_T be an isotone symmetric bi-T-derivation. Then, using Theorem 3.1(A) and Proposition 3.3, we have

$$\begin{aligned} D_T(s,y) &= (D_T(s,y) \vee (s \wedge D_T(x \vee s, y)) \\ &= (D_T(s,y) \wedge s) \wedge (D_T(s,y) \vee D_T(x \vee s, y)) \\ &= s \wedge D_T(x \vee s, y). \end{aligned}$$

In similar way, $D_T(x,y) = x \wedge D_T(x \vee s, y)$. Thus,

$$\begin{aligned} D_T(x,y) \vee D_T(s,y) &= (x \wedge D_T(x \vee s, y)) \vee (s \wedge D_T(x \vee s, y)) \\ &= (x \vee s) \wedge D_T(x \vee s, y) \\ &= D_T(x \vee s, y). \end{aligned}$$

Conversely, let $D_T(x \vee s, y) = D_T(x, y) \vee D_T(s, y)$ and $x \leq s$, then obtained $D_T(s, y) = D_T(x \vee s, y) = D_T(x, y) \vee D_T(s, y)$, which imply $D_T(x, y) \leq D_T(s, y)$ for all $x, y, s \in L$. □

Acknowledgement The authors are thankful to the reviewers for their valuable suggestions which improved the quality and representation of the paper.

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