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NONEXISTENCE OF POSITIVE SOLUTIONS FOR A SYSTEMS OF COUPLED FRACTIONAL BVPS WITH p-LAPLACIAN

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Abstract. We investigate the nonexistence of positive solutions for a system of nonlinear Riemann-Liouville fractional differential equations with p-Laplacian two-point boundary value problem.

Keywords: Fractional order, Green's function, p-Laplacian, eigenvalue interval, nonexistence, cone.

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1. Introduction

In this paper, we consider the system of nonlinear Riemann-Liouville fractional order differential equations with p-Laplacian

$$\begin{aligned} D_{a^+}^{\beta_1}\big(\phi_p(D_{a^+}^{\alpha_1}u(t))\big) &= \phi_p(\lambda)f(t,u(t),v(t)),\ t\in(a,b),\\ D_{a^+}^{\beta_2}\big(\phi_p(D_{a^+}^{\alpha_2}v(t))\big) &= \phi_p(\mu)g(t,u(t),v(t)),\ t\in(a,b), \end{aligned} \tag{1}$$

satisfying the two-point boundary conditions,

$$\begin{aligned} \xi u(a)-\eta u'(a)=0,\ \gamma u(b)+\delta u'(b)=0,\ D_{a^+}^{\alpha_1}u(a)=0\\ \xi v(a)-\eta v'(a)=0,\ \gamma v(b)+\delta v'(b)=0,\ D_{a^+}^{\alpha_2}v(a)=0 \end{aligned} \tag{2}$$

where $\phi_p(s)=\mid s\mid^{p-2}s, p>1, \phi_p^{-1}=\phi_q, \frac{1}{p}+\frac{1}{q}=1$, ξ,η,γ,δ are positive real numbers, $1<\alpha_i\le 2,\ 0<\beta_i\le 1,\ f,g:[a,b]\times[0,\infty)\times[0,\infty)\to[0,\infty)$ are continuous functions, and $D_{a^+}^{\alpha_i},\ D_{a^+}^{\beta_i}$ for $i=1,2$ are the standard Riemann-Liouville fractional derivatives.

Differential equations with fractional order are a generalization of the ordinary differential equations to non-integer order. This generalization is not a mere mathematical curiosity but rather has interesting applications in many areas of science and engineering such as electrochemistry, control, porous media, electromagnetism, etc. There has been a significant development in the study of fractional differential equations in recent years. In this theory, the most applicable operator is the classical p-Laplacian, given by

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$\phi_p(u) = \mid u \mid^{p-2}, p > 1$. Furthermore, several kinds of the high-order boundary value problems of fractional equations have been studied. The nonexistence of positive solutions of boundary value problems associated with ordinary differential equations were studied by many authors [11, 15]. Recently, researchers are concentrating on the theory of fractional order boundary value problems [8]. In this paper we extend the work on fractional order BVP with p-Laplacian.

We shall give sufficient conditions on λ, μ, f and g such that the FBVP (1)-(2) has no positive solutions. By a positive solution of the FBVP (1)-(2), we mean a pair $(u, v) \in [a, b] \times [a, b]$ satisfying (1) and (2) with $u(t) \geq 0, v(t) \geq 0$ for all $t \in [a, b]$ and $(u, v) \neq (0, 0)$.

Throughout this paper we assume that following conditions hold:

(A1) The functions $f, g : [a, b] \times [0, \infty) \times [0, \infty) \to [0, \infty)$ are continuous,

(A2) each of

$$f_0^s = \lim_{u+v \to 0^+} \sup \frac{f(t, u, v)}{\phi_p(u + v)}, \ g_0^s = \lim_{u+v \to 0^+} \sup \frac{g(t, u, v)}{\phi_p(u + v)},$$
$$f_0^i = \lim_{u+v \to 0^+} \inf \frac{f(t, u, v)}{\phi_p(u + v)}, \ g_0^i = \lim_{u+v \to 0^+} \inf \frac{g(t, u, v)}{\phi_p(u + v)},$$
$$f_\infty^s = \lim_{u+v \to \infty} \sup \frac{f(t, u, v)}{\phi_p(u + v)}, \ g_\infty^s = \lim_{u+v \to \infty} \sup \frac{g(t, u, v)}{\phi_p(u + v)},$$
$$f_\infty^i = \lim_{u+v \to \infty} \inf \frac{f(t, u, v)}{\phi_p(u + v)}, \ g_\infty^i = \lim_{u+v \to \infty} \inf \frac{g(t, u, v)}{\phi_p(u + v)},$$

exists as positive real numbers.

This paper is organized as follows, In Section 2, we present the necessary definitions and properties from the fractional calculus and construct the Green's function bounds for the homogeneous FBVP. In Section 3, we prove some nonexistence results for the positive solutions with respect to a cone for our FBVP (1)-(2). Finally, In Section 4, as an application, we demonstrate our results with an example.

2. **Preliminaries and Green's function**

In this section, we present here the definitions, some lemmas from the theory of fractional calculus and established Green's function and bounds of a homogeneous boundary value problem that will be used to prove our main theorems.

Definition 2.1. *The (left-sided) fractional integral of order $\alpha > 0$ of a function $f : (0, \infty) \to \mathbb{R}$ is given by*

$$(I_{0^+}^\alpha f)(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha - 1} f(s) ds, \ t > 0,$$

provided the right-hand side is pointwise defined on $(0, \infty)$, where $\Gamma(\alpha)$ is the Euler gamma function defined by $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt, \ \alpha > 0$.

Definition 2.2. *The Riemann-Liouville fractional derivative of order $\alpha \geq 0$ for a function $f : (0, \infty) \to \mathbb{R}$ is given by*

$$(D_{0^+}^\alpha f)(t) = \Big(\frac{d}{dt}\Big)^n (I_{0^+}^{n-\alpha} f)(t) = \frac{1}{\Gamma(n - \alpha)} \Big(\frac{d}{dt}\Big)^n \int_0^t \frac{f(s)}{(t - s)^{\alpha - n + 1}} ds, \ t > 0,$$

where $n = [\alpha] + 1$, provided that the right-hand side is pointwise defined on $(0, \infty)$.

The notation $[\alpha]$ *stands for the largest integer not greater than* α. *We also denote the Riemann-Liouville fractional derivative of* f *by* $D_{0+}^{\alpha}f(t)$. *If* $\alpha = m \in \mathbb{N}$ *then* $D_{0+}^{m}f(t) = f^{(m)}(t)$ *for* $t > 0$, *and if* $\alpha = 0$ *then* $D_{0+}^{0}f(t) = f(t)$ *for* $t > 0$.

Lemma 2.1 ([10]). *a) If* $\alpha > 0,\ \beta > 0$ *and* $f \in L^p(0,1), (1 \le p \le \infty)$, *then the relation* $(I_{0+}^{\alpha}I_{0+}^{\beta}f)(t) = (I_{0+}^{\alpha+\beta}f)(t)$ *is satisfied at almost every point* $t \in (0,1)$. *If* $\alpha + \beta > 1$, *then the above relation holds at any point of* $[0,1]$.
b) If $\alpha > 0$ *and* $f \in L^p(0,1), (1 \le p \le \infty)$, *then the relation* $(D_{0+}^{\alpha}I_{0+}^{\alpha}f)(t) = f(t)$ *holds almost everywhere on* $(0,1)$.
c)If $\alpha > \beta > 0$ *and* $f \in L^p(0,1), (1 \le p \le \infty)$, *then the relation* $(D_{0+}^{\beta}I_{0+}^{\alpha}f)(t) = (I_{0+}^{\alpha-\beta}f)(t)$ *holds almost everywhere on* $(0,1)$.

Lemma 2.2 ([10]). *Let* $\alpha > 0$ *and* $n = [\alpha] + 1$ *for* $\alpha \notin N$ *and* $n = \alpha$ *for* $\alpha \in N$; *that is, n is the smallest integer greater than or equal to* α. *Then, the solutions of the fractional differential equation* $D_{0+}^{\alpha}u(t) = 0,\ 0 < t < 1$, *are*

$$u(t) = c_1t^{\alpha-1} + c_2t^{\alpha-2} + ... + c_nt^{\alpha-n},\ 0 < t < 1,$$

where $c_1, c_2, ..., c_n$ *are arbitrary real constants.*

Lemma 2.3 ([10]). *Let* $\alpha > 0$, n *be the smallest integer greater than or equal to* α $(n-1 < \alpha \le n)$ *and* $y \in L^1(0,1)$. *The solutions of the fractional equation* $D_{0+}^{\alpha}u(t) + y(t) = 0,\ 0 < t < 1$, *are*

$$u(t) = \frac{-1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}y(s)ds + c_1t^{\alpha-1} + ... + c_nt^{\alpha-n},\ 0 < t < 1,$$

where $c_1, c_2, ..., c_n$ *are arbitrary real constants.*

Proof. By Lemma 2.1 b), the equation $D_{0+}^{\alpha}u(t) + y(t) = 0$ can be written as

$$D_{0+}^{\alpha}u(t) + D_{0+}^{\alpha}(I_{0+}^{\alpha}y)(t) = 0 \text{ or } D_{0+}^{\alpha}(u + I_{0+}^{\alpha}y)(t) = 0.$$

By using Lemma 2.2, the solutions for the above equation are

$$\begin{aligned} u(t) + I_{0+}^{\alpha}y(t) &= c_1t^{\alpha-1} + ... + c_nt^{\alpha-n} \Leftrightarrow \\ u(t) &= -I_{0+}^{\alpha}y(t) + c_1t^{\alpha-1} + ... + c_nt^{\alpha-n} \\ &= \frac{-1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}y(s)ds + c_1t^{\alpha-1} + ... + c_nt^{\alpha-n}, 0 < t < 1, \end{aligned}$$

where $c_1, c_2, ..., c_n$ are arbitrary real constants. □

Let $G_1(t,s)$ be the Green's function of a homogeneous boundary value problem

$$-D_{a^+}^{\alpha_1}u(t) = 0,\ t \in (a,b) \tag{3}$$

$$\xi u(a) - \eta u'(a) = 0,\ \gamma u(b) + \delta u'(b) = 0 \tag{4}$$

Lemma 2.4. *Let* $d = \xi\gamma(b-a) + \xi\delta + \gamma\eta > 0$. *If* $y \in C[a,b]$, *then the fractional order BVP*

$$D_{a^+}^{\alpha_1}u(t) + y(t) = 0,\ a < t < b, \tag{5}$$

with (4), *has a unique solution,*

$$u(t) = \int_a^b G_\lambda(t,s)y(s)ds$$

where $G_\lambda(t,s)$ *is the Green's function for the BVP* (3)-(4) *and is given by*

$$G_\lambda(t,s)=\begin{cases}\Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big] & t\le s,\\ \Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big]-\frac{(t-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}, & s\le t,\end{cases} \tag{6}$$

Proof. Assume that $u\in C^{[\alpha_1]+1}[a,b]$ is a solution of fractional order BVP (5),(4) and is uniquely expressed as $I^{\alpha_1}_{a^+}D^{\alpha_1}_{a^+}u(t)=-I^{\alpha_1}_{a^+}y(t)$, so that

$$u(t)=-\int_a^t\frac{(t-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}y(s)ds+c_1(t-a)^{\alpha_1-1}+c_2(t-a)^{\alpha_1-2}.$$

Using the boundary condition (4), we can determine c_1 and c_2 as

$$c_1=\frac{\xi}{d}\Big[\frac{\gamma}{\Gamma(\alpha_1)}\int_a^b(b-s)^{\alpha_1-1}y(s)ds+\frac{\delta}{\Gamma(\alpha_1-1)}\int_a^b(b-s)^{\alpha_1-2}y(s)ds\Big]$$

and

$$c_2=\frac{\eta}{d}\Big[\frac{\gamma}{\Gamma(\alpha_1)}\int_a^b(b-s)^{\alpha_1-1}y(s)ds+\frac{\delta}{\Gamma(\alpha_1-1)}\int_a^b(b-s)^{\alpha_1-2}y(s)ds\Big].$$

Hence, the unique solution of (3)-(4) is

$$\begin{aligned}u(t)&=\frac{-1}{\Gamma(\alpha_1)}\int_a^t(t-s)^{\alpha_1-1}y(s)ds+\frac{\xi(t-a)^{\alpha_1-1}}{d}\Big[\frac{\gamma}{\Gamma(\alpha_1)}\int_a^b(b-s)^{\alpha_1-1}y(s)ds\\&\quad+\frac{\delta}{\Gamma(\alpha_1-1)}\int_a^b(b-s)^{\alpha_1-2}y(s)ds\Big]+\frac{\eta(t-a)^{\alpha_1-2}}{d}\Big[\frac{\gamma}{\Gamma(\alpha_1)}\int_a^b(b-s)^{\alpha_1-1}y(s)ds\\&\quad+\frac{\delta}{\Gamma(\alpha_1-1)}\int_a^b(b-s)^{\alpha_1-2}y(s)ds\Big]\\&=\int_a^t\Big[\frac{-(t-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\Big(\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big)\Big(\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big)\Big]y(s)ds\\&\quad+\int_t^b\Big(\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big)\Big(\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big)y(s)ds\\&=\int_a^bG_\lambda(t,s)y(s)ds,\ \text{where } G_\lambda(t,s)\text{ is given in (6).}\end{aligned}$$

□

Lemma 2.5. *Let* $1<\alpha_1\le 2, 0<\beta_1\le 1$. *Then the boundary value problem of the fractional differential equation*

$$D^{\beta_1}_{a^+}\Big(\phi_p\big(D^{\alpha_1}_{a^+}u(t)\big)\Big)+y(t)=0,\ a<t<b, \tag{7}$$

with (2) *has a unique solution,*

$$u(t)=\int_a^bG_\lambda(t,s)\phi_q\Big(\int_a^s\frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}y(\tau)d\tau\Big)ds$$

where $G_\lambda(t,s)$ *is defined as* (6).

Proof. An equivalent integral equation for (7) is given by

$$\phi_p\Big(D^{\alpha_1}_{a^+}u(t)\Big)=-\int_a^t\frac{(t-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}y(\tau)d\tau+c_1(t-a)^{\beta_1-1}.$$

By $D^{\alpha_1}_{a^+}u(a) = 0$, we have $c_1 = 0$. So, $D^{\alpha_1}_{a^+}u(t) + \phi_q\big(\int_a^t \frac{(t-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}y(\tau)d\tau\big) = 0$. Thus, the boundary value problem (7) is equivalent to the following problem:

$$D^{\alpha_1}_{a^+}u(t) + \phi_q\Big(\int_a^t \frac{(t-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}y(\tau)d\tau\Big) = 0,\ a < t < b,$$
$$\xi u(a) - \eta u'(a) = 0,\ \gamma u(b) + \delta u'(b) = 0.$$

Lemma 2.4 implies that boundary value problem (7) has a unique solution,
$u(t) = \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}y(\tau)d\tau\Big)ds.$ □

Lemma 2.6. *The Green's function $G_\lambda(t,s)$ defined by* (6) *is continuous on $[a,b]\times[a,b]$. Assume that $\eta > \big(\frac{\alpha_1-1}{2-\alpha_1}\big)(b-a)\xi$, then $G_\lambda(t,s)$ also has the following properties:*

(i) $G_\lambda(t,s) \geq 0$, *for all* $(t,s)\in[a,b]\times[a,b]$
(ii) $G_\lambda(t,s) \leq G_\lambda(s,s)$, *for all* $(t,s)\in[a,b]\times[a,b]$,
(iii) $G_\lambda(t,s) \geq m_1 G_\lambda(s,s)$, *for all* $(t,s)\in[a,b]\times[a,b]$, *where* $m_1 = \min\{\lambda_1,\lambda_2\}$.

Proof. The Green's function $G_\lambda(t,s)$ is given (6).
For $a \leq t \leq s \leq b$,

$$G_\lambda(t,s) = \Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big] \geq 0$$

For $a \leq s \leq t \leq b$

$$\begin{aligned} G_\lambda(t,s) &= \Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big] - \frac{(t-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} \\ &\geq \Big[\Big(\frac{(t-a)^{\alpha_1-2}(\eta+\xi)}{d}\Big)\Big(\frac{\gamma}{\Gamma(\alpha_1)} + \frac{\delta(b-bs)^{-1}}{\Gamma(\alpha_1-1)}\Big)b^{\alpha_1-1} - \frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)}\Big](1-s)^{\alpha_1-1} \geq 0. \end{aligned}$$

Hence, the inequality (i) is proved.
For $a \leq t \leq s \leq b$
$\frac{\partial G_\lambda(t,s)}{\partial t} = \Big[\frac{(\alpha_1-2)\eta(t-a)^{\alpha_1-3}+(\alpha_1-1)\xi(t-a)^{\alpha_1-2}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big] \geq 0.$
Therefore $G_\lambda(t,s)$ is increasing in t, which implies $G_\lambda(t,s) \leq G_\lambda(s,s)$.
For $a \leq s \leq t \leq b$

$$\begin{aligned} \frac{\partial G_\lambda(t,s)}{\partial t} &= \Big[\frac{(\alpha_1-2)\eta(t-a)^{\alpha_1-3}+(\alpha-1)\xi(t-a)^{\alpha_1-2}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big] \\ &\quad - \frac{(\alpha_1-1)(t-s)^{\alpha_1-2}}{\Gamma(\alpha_1)} \\ &\leq \frac{1}{d\Gamma(\alpha_1)}\Big[\Big((\alpha_1-2)\eta(b-a)^{\alpha_1-3} + (\alpha_1-1)\xi(b-a)^{\alpha_1-2}\Big)\Big(\gamma(b-s)+(\alpha_1-1)\delta\Big) \\ &\quad - d(\alpha_1-1)\Big](b-s)^{\alpha_1-2} \leq 0. \end{aligned}$$

Therefore $G_\lambda(t,s)$ is decreasing in t, for $s\in[a,b]$ which implies that $G_\lambda(t,s) \leq G_\lambda(s,s)$. Hence the inequality (ii) is proved. Now we establish the inequality (iii).
For $a \leq t \leq s \leq b$

$$\begin{aligned} \frac{G_\lambda(t,s)}{G_\lambda(s,s)} &= \frac{\Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big]}{\Big[\frac{\eta(s-a)^{\alpha_1-2}+\xi(s-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)} + \frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big]} \\ &\geq \frac{4\xi\gamma\delta\big[(\alpha_1-2)(b-a)\xi + (\alpha_1-1)\eta\big]}{\big[(\alpha_1-1)\xi\delta + \xi\gamma(a+b) - \gamma\eta\big]^2 + 4\xi\gamma\big[(\eta-a\xi)(\gamma b + \delta(\alpha_1-1))\big]} = \lambda_1. \end{aligned}$$

For $a \leq s \leq t \leq b$

$$\frac{G_\lambda(t,s)}{G_\lambda(s,s)} = \frac{\Big[\frac{\eta(t-a)^{\alpha_1-2}+\xi(t-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big]-\frac{(t-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}}{\Big[\frac{\eta(s-a)^{\alpha_1-2}+\xi(s-a)^{\alpha_1-1}}{d}\Big]\Big[\frac{\gamma(b-s)^{\alpha_1-1}}{\Gamma(\alpha_1)}+\frac{\delta(b-s)^{\alpha_1-2}}{\Gamma(\alpha_1-1)}\Big]}$$
$$\geq \frac{4\xi\eta\gamma\delta\big[(\alpha_1-2)(b-a)\xi+(\alpha_1-1)\eta\big]}{\big[(\alpha_1-1)\xi\delta+\xi\gamma(a+b)-\gamma\eta\big]^2+4\xi\gamma\big[(\eta-a\xi)(\gamma b+\delta(\alpha_1-1))\big]} = \lambda_2.$$

□

We can also formulate the same results as Lemma 2.4-2.6 above for the following FBVP with p-laplacian

$$-D_{a^+}^{\alpha_2} v(t) = 0,\ t \in (a,b) \tag{8}$$

$$\xi v(a) - \eta v'(a) = 0,\ \gamma v(b) + \delta v'(b) = 0 \tag{9}$$

the results of the Green's function $G_\mu(t,s)$ and constant m_2 for the homogeneous BVPs corresponding to the fractional differential equation (8)-(9) and define in a similar manner as $G_\lambda(t,s)$.

Remark: Consider the following

$G_\lambda(t,s) \geq mG_\lambda(s,s)$ and $G_\mu(t,s) \geq mG_\mu(s,s)$ for all $(t,s) \in [a,b]\times[a,b]$, where $m = \min\{m_1, m_2\}$.

Let the Banach space $\mathcal{B} = \mathcal{E}\times\mathcal{E}$, where $\mathcal{E} = \{u : u \in C[a,b]\}$ be endowed with the norm $\|(u,v)\| = \|u\| + \|v\|$, for $(u,v) \in \mathcal{B}$ and $\|u\| = \max_{t\in[a,b]} \mid u(t) \mid$. Define a cone $\mathcal{P} \subset \mathcal{B}$ by

$$\mathcal{P} = \left\{(u,v) \in \mathcal{B} :\ u(t) \geq 0,\ v(t) \geq 0\ \forall t \in [a,b] \text{ and } \min_{t\in[a,b]}\{u(t)+v(t)\} \geq m\|(u,v)\|\right\},$$

where $m = \min\{m_1, m_2\}$.

It is well known that the system of fractional order boundary value problem (1)-(2) is equivalent to

$$\begin{aligned} u(t) &= \lambda \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds \\ v(t) &= \mu \int_a^b G_\mu(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau,u(\tau),v(\tau))d\tau\Big)ds \end{aligned}.$$

We define the operators $T_\lambda, T_\mu : \mathcal{P} \to \mathcal{E}$ as

$$\begin{aligned} T_\lambda(u,v)(t) &= \lambda \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds \\ T_\mu(u,v)(t) &= \mu \int_a^b G_\mu(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau,u(\tau),v(\tau))d\tau\Big)ds \end{aligned}$$

and an operator $T : \mathcal{P} \to \mathcal{B}$ as

$$T(u,v) = \big(T_\lambda(u,v), T_\mu(u,v)\big),\ \text{for } (u,v) \in \mathcal{B}. \tag{10}$$

It is clear that the existence of a positive solution to the system (1)-(2) is equivalent to the existence of fixed points of the operator T.

Lemma 2.7. *$T : \mathcal{P} \to \mathcal{P}$ is completely continuous*

Proof. By using standard arguments, we can easily show that, the operator T is completely continuous and we need only to prove $T(\mathcal{P}) \subset \mathcal{P}$, we have

$$\min_{t\in[a,b]} T_\lambda(u,v)(t) = \min_{t\in[a,b]} \lambda \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds$$
$$\geq m\lambda \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds \geq m\|T_\lambda(u,v)\|.$$

Similarly, $\min_{t\in[a,b]} T_\mu(u,v)(t) \geq m\|T_\mu(u,v)\|$. Therefore,

$$\min_{t\in[a,b]} \Big\{T_\lambda(u,v)(t) + T_\mu(u,v)(t)\Big\} \geq m\|T_\lambda(u,v)\| + m\|T_\mu(u,v)\|$$
$$= m\big(\|T_\lambda(u,v)\| + \|T_\mu(u,v)\|\big) = m\|\big(T_\lambda(u,v), T_\mu(u,v)\big)\| = m\|T(u,v)\|.$$

Hence, $T(\mathcal{P}) \subset \mathcal{P}$. Let $(u,v) \in \mathcal{P}$ and $\epsilon > 0$ be given. By the continuity of f and g, there exists $\delta > 0$ such that $\mid f(t,u,v) - f(t,u',v') \mid < \epsilon, \mid g(t,u,v) - g(t,u',v') \mid < \epsilon$, whenever $\mid u-u' \mid < \delta$, $\mid v - v' \mid < \delta$ for all $t \in [a,b]$.

$$\mid T_\lambda(u,v)(t) - T_\lambda(u',v')(t) \mid = \lambda \int_a^b G_\lambda(t,s) \mid f(s,u,v) - f(s,u',v') \mid ds$$
$$\leq \epsilon\lambda \int_a^b G_\lambda(t,s)ds.$$

Thus, $\| T_\lambda(u,v)(t) - T_\lambda(u',v')(t) \| \leq \epsilon\lambda \int_a^b G_\lambda(t,s)ds$. In a similar manner $\| T_\mu(u,v)(t) - T_\mu(u',v')(t) \| \leq \epsilon\mu \int_a^b G_\mu(t,s)ds$ and T is continuous. Now, let $\{(u_n,v_n)\}$ be a bounded sequence in $\mathcal{P}$. Since f and g are continuous, there exists $N > 0$ such that $\mid f(t,u_n,v_n) \mid \leq N$, $\mid g(t,u_n,v_n) \mid \leq N$ for all $u_n, v_n \in [0,\infty)$. Then, for each $t \in [a,b]$ and for each n,

$$\mid T_\lambda(u_n,v_n)(t) \mid = \mid \lambda \int_a^b G_\lambda(t,s) f(s,u_n,v_n)ds \mid$$
$$\leq \lambda \int_a^b \mid G_\lambda(b,s) \mid\mid f(s,u_n,v_n) \mid ds$$
$$\leq N\lambda \int_a^b G_\lambda(b,s)ds.$$

In a similar manner $\mid T_\mu(u_n,v_n)(t) \mid \leq N\mu \int_a^b G_\mu(b,s)ds$. By choosing successive subsequences, there exists a subsequence $\{T(u_{n_j}, v_{n_j})\}$ which converges uniformly on $[a,b]$. Hence T is completely continuous. □

3. Main result

In this, we shall give sufficient conditions on λ, μ, f and g such that positive solutions with respect to a cone for our problem (1)-(2) exist.

For convenience of the reader, we denote

$$A = \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}d\tau\Big)ds; B = \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)}d\tau\Big)ds,$$
$$C = m^2\int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)}d\tau\Big)ds; D = m^2\int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)}d\tau\Big)ds.$$

Theorem 3.1. *Assume that (A1) and (A2) hold. If $f_0^s, f_\infty^s, g_0^s, g_\infty^s < \infty$ then there exist positive constants λ_0, μ_0 such that for every $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$, the boundary value problem (1)-(2) has no positive solution.*

Proof. Since $f_0^s, f_\infty^s < \infty$, which are finite, we deduce that there exist $M_1', M_1'', r_1, r_1' > 0, r_1 < r_1'$ such that

$$f(t,u,v) \leq \phi_p(M_1')\phi_p(u+v), \forall u,v \geq 0, u+v \in [0,r_1]$$
$$f(t,u,v) \leq \phi_p(M_1'')\phi_p(u+v), \forall u,v \geq 0, u+v \in [r_1',\infty).$$

We consider $M_1 = \Big\{M_1', M_1'', \max_{r_1 \leq u+v \leq r_1'} \frac{f(t,u,v)}{\phi_p(u+v)}\Big\} > 0$, then we obtain

$$f(t,u,v) \leq \phi_p(M_1)\phi_p(u+v), \forall u,v \geq 0.$$

Since $g_0^s, g_\infty^s < \infty$, which are finite, we deduce that there exist $M_2', M_2'', r_2, r_2' > 0, r_2 < r_2'$ such that

$$g(t,u,v) \leq \phi_p(M_2')\phi_p(u+v), \forall u,v \geq 0, u+v \in [0,r_2]$$
$$g(t,u,v) \leq \phi_p(M_2'')\phi_p(u+v), \forall u,v \geq 0, u+v \in [r_2',\infty].$$

We consider $M_2 = \Big\{M_2', M_2'', \max_{r_2 \leq u+v \leq r_2'} \frac{g(t,u,v)}{\phi_p(u+v)}\Big\} > 0$, then we obtain

$$g(t,u,v) \leq \phi_p(M_2)\phi_p(u+v), \forall u,v \geq 0.$$

We define $\lambda_0 = \frac{1}{2M_1A}, \mu_0 = \frac{1}{2M_2B}$. We shall show that for every $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$, FBVP (1)-(2) has a no positive solution.

Let $\lambda \in (0,\lambda_0)$ and $\mu \in (0,\mu_0)$. We suppose that (1)-(2) has a positive solution $(u(t),v(t)), t \in [a,b]$. Then by using lemma 2.6, we obtain

$$\begin{aligned}
u(t) &= \big(T_\lambda(u,v)\big)(t) = \lambda \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds \\
&\leq \lambda \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau,u(\tau),v(\tau))d\tau\Big)ds \\
&\leq \lambda M_1 \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} \phi_p\big(u(\tau)+v(\tau)\big)d\tau\Big)ds \\
&\leq \lambda M_1 \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} d\tau\Big)ds\big(\parallel u \parallel + \parallel v \parallel\big) \\
&= \lambda M_1 A \parallel (u,v) \parallel, \ \forall t \in [a,b].
\end{aligned}$$

Therefore, we conclude

$$\parallel u \parallel \leq \lambda M_1 A \parallel (u,v) \parallel < \lambda_0 M_1 A \parallel (u,v) \parallel = \frac{1}{2} \parallel (u,v) \parallel \tag{11}$$

In a similar manner, we obtain

$$\begin{aligned}
v(t) &= \big(T_\mu(u,v)\big)(t) = \mu \int_a^b G_\mu(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau,u(\tau),v(\tau))d\tau\Big)ds \\
&\leq \mu \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau,u(\tau),v(\tau))d\tau\Big)ds \\
&\leq \mu M_2 \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} \phi_p\big(u(\tau)+v(\tau)\big)d\tau\Big)ds \\
&\leq \mu M_2 \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} d\tau\Big)ds\big(\parallel u \parallel + \parallel v \parallel\big) \\
&= \mu M_2 B \parallel (u,v) \parallel, \ \forall t \in [a,b].
\end{aligned}$$

Therefore, we conclude

$$\| v \| \leq \mu M_2 B \| (u,v) \| < \mu_0 M_2 B \| (u,v) \| = \frac{1}{2} \| (u,v) \| . \tag{12}$$

Hence, by (11) and (12), we conclude

$$\| (u,v) \| = \| u \| + \| v \| < \frac{1}{2} \| (u,v) \| + \frac{1}{2} \| (u,v) \| = \| (u,v) \|$$

which is a contradiction, so the FBVP (1)-(2) has no positive solution. $\square$

Theorem 3.2. *Assume that* (A1) *and* (A2) *hold. If* $f_0^i, f_\infty^i > 0$ *and* $f(t,u,v) > 0$ *for all* $t \in [a,b], u \geq 0, v \geq 0, u+v > 0$, *then there exists a positive constant* $\widetilde{\lambda}_0$ *such that for every* $\lambda > \widetilde{\lambda}_0$ *and* $\mu > 0$, *the boundary value problem* (1)-(2) *has no positive solution.*

Proof. Since $f_0^i, f_\infty^i > 0$, we deduce that there exist $M_3', M_3'', r_3, r_3' > 0, r_3 < r_3'$ such that

$$f(t,u,v) \geq \phi_p(M_3')\phi_p(u+v), \forall u,v \geq 0, u+v \in [0,r_3]$$
$$f(t,u,v) \geq \phi_p(M_3'')\phi_p(u+v), \forall u,v \geq 0, u+v \in [r_3',\infty).$$

We consider $M_3 = \Big\{ M_3', M_3'', \max_{r_3 \leq u+v \leq r_3'} \frac{f(t,u,v)}{\phi_p(u+v)} \Big\} > 0$, then we obtain

$$f(t,u,v) \geq \phi_p(M_3)\phi_p(u+v), \forall u,v \geq 0.$$

We define $\widetilde{\lambda}_0 = \frac{1}{M_3 C}$. We shall show that, $\lambda > \widetilde{\lambda}_0$ and $\mu > 0$ for every FBVP (1)-(2) has a no positive solution.

Let $\lambda > \widetilde{\lambda}_0$ and $\mu > 0$. We suppose that (1)-(2) has a positive solution $(u(t), v(t)), t \in [a,b]$. Then by using lemma 2.6, we obtain

$$\begin{aligned}
u(t) = \big(T_\lambda(u,v)\big)(t) &= \lambda \int_a^b G_\lambda(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau, u(\tau), v(\tau))d\tau\Big)ds \\
&\geq \lambda \int_a^b m G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} f(\tau, u(\tau), v(\tau))d\tau\Big)ds \\
&\geq \lambda m M_3 \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} \phi_p\big(u(\tau)+v(\tau)\big)d\tau\Big)ds \\
&\geq \lambda m^2 M_3 \int_a^b G_\lambda(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_1-1}}{\Gamma(\beta_1)} d\tau\Big)ds\big(\| u \| + \| v \| \big) \\
&= \lambda M_3 C \| (u,v) \|, \ \forall t \in [a,b].
\end{aligned}$$

Then, we conclude $\| u \| \geq \lambda M_3 C \| (u,v) \| > \widetilde{\lambda}_0 M_3 C \| (u,v) \| = \| (u,v) \|$ and so, $\| (u,v) \| = \| u \| + \| v \| \geq \| u \| > \| (u,v) \|$, which is a contradiction. Therefore the boundary value problem (1)-(2) has no positive solution. $\square$

Theorem 3.3. *Assume that* (A1) *and* (A2) *hold. If* $g_0^i, g_\infty^i > 0$ *and* $g(t,u,v) > 0$ *for all* $t \in [a,b], u \geq 0, v \geq 0, u+v > 0$, *then there exists a positive constant* $\widetilde{\mu}_0$ *such that for every* $\mu > \widetilde{\mu}_0$ *and* $\lambda > 0$, *the boundary value problem* (1)-(2) *has no positive solution.*

Proof. Since $g_0^i, g_\infty^i > 0$, we deduce that there exist $M_4', M_4'', r_4, r_4' > 0, r_4 < r_4'$ such that

$$g(t,u,v) \geq \phi_p(M_4')\phi_p(u+v), \forall u,v \geq 0, u+v \in [0,r_4]$$
$$g(t,u,v) \geq \phi_p(M_4'')\phi_p(u+v), \forall u,v \geq 0, u+v \in [r_4',\infty).$$

We consider $M_4 = \Big\{ M_4', M_4'', \max_{r_4 \leq u+v \leq r_4'} \frac{g(t,u,v)}{\phi_p(u+v)} \Big\} > 0$, then we obtain

$$g(t,u,v) \geq \phi_p(M_4)\phi_p(u+v), \forall u,v \geq 0.$$

We define $\widetilde{\mu}_0 = \frac{1}{M_4 D}$. We shall show that, for every $\mu > \widetilde{\mu}_0$ and $\lambda > 0$ the FBVP (1)-(2) has a no positive solution.

Let $\mu > \widetilde{\mu}_0$ and $\lambda > 0$. We suppose that (1)-(2) has a positive solution $(u(t), v(t)), t \in [a, b]$. Then by using lemma 2.6, we obtain

$$\begin{aligned}
v(t) = \big(T_\mu(u,v)\big)(t) &= \mu \int_a^b G_\mu(t,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau, u(\tau), v(\tau))d\tau\Big)ds \\
&\geq \mu \int_a^b mG_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} g(\tau, u(\tau), v(\tau))d\tau\Big)ds \\
&\geq \mu M_4 \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} \phi_p\big(u(\tau)+v(\tau)\big)d\tau\Big)ds \\
&\geq \mu m^2 M_4 \int_a^b G_\mu(s,s)\phi_q\Big(\int_a^s \frac{(s-\tau)^{\beta_2-1}}{\Gamma(\beta_2)} d\tau\Big)ds\big(\parallel u \parallel + \parallel v \parallel\big) \\
&= \mu M_4 D \parallel (u,v) \parallel, \ \forall t \in [a,b].
\end{aligned}$$

Then, we conclude $\parallel v \parallel \geq \mu M_4 D \parallel (u,v) \parallel > \widetilde{\mu}_0 M_4 D \parallel (u,v) \parallel = \parallel (u,v) \parallel$ and so, $\parallel (u,v) \parallel = \parallel u \parallel + \parallel v \parallel \geq \parallel v \parallel > \parallel (u,v) \parallel$, which is a contradiction. Therefore the boundary value problem (1)-(2) has no positive solution. □

Theorem 3.4. *Assume that (A1) and (A2) hold. If $f_0^i, f_\infty^i, g_0^i, g_\infty^i > 0$ and $f(t,u,v) > 0, g(t,u,v) > 0$ for all $t \in [a,b], u \geq 0, v \geq 0, u+v > 0$, then there exists a positive constants $\widetilde{\lambda}_0$ and $\widetilde{\mu}_0$ such that, for every $\lambda > \widetilde{\lambda}_0$ and $\mu > \widetilde{\mu}_0$, the boundary value problem (1)-(2) has no positive solution.*

Proof. From the assumptions of the theorem, we deduced that there exist $M_3, M_4 > 0$ such that

$$f(t,u,v) \geq \phi_p(M_3)\phi_p(u+v), g(t,u,v) \geq \phi_p(M_4)\phi_p(u+v)$$

for all $t \in [a,b]$ and $u, v \geq 0$. We define $\hat{\lambda_0} = \frac{1}{2M_3 C}, \hat{\mu_0} = \frac{1}{2M_4 D}$. Then, for every $\lambda > \hat{\lambda_0}$ and $\mu > \hat{\mu_0}$, problem (1)-(2) has a positive solution $(u(t), v(t)), t \in [a,b]$. In a similar manner to that used in the proofs of theorem 3.2 and 3.3, we obtain

$$\parallel u \parallel \geq \lambda M_3 C \parallel (u,v) \parallel, \parallel v \parallel \geq \mu M_4 D \parallel (u,v) \parallel$$

and so

$$\begin{aligned}
\parallel (u,v) \parallel &= \parallel u \parallel + \parallel v \parallel \geq \lambda M_3 C \parallel (u,v) \parallel + \mu M_4 D \parallel (u,v) \parallel \\
&> \hat{\lambda_0} M_3 C \parallel (u,v) \parallel + \hat{\mu_0} M_4 D \parallel (u,v) \parallel \\
&= \frac{1}{2} \parallel (u,v) \parallel + \frac{1}{2} \parallel (u,v) \parallel = \parallel (u,v) \parallel,
\end{aligned}$$

which is a contradiction. Therefore, the boundary value problem (1)-(2) has no positive solution. □

4. Example

In this section, we demonstrate our result with an example.

Let $a = 0, b = 1, \alpha_1 = \frac{3}{2}, \alpha_2 = \frac{5}{4}, \beta_1 = \beta_2 = \frac{1}{2}, \xi = \gamma = \delta = \frac{1}{2}, \eta = 1, p = 2$. We consider the system of fractional order differential equations with p-Laplacian operator

$$\begin{aligned}
D_{0^+}^{0.5}\Big(\phi_p\big(D_{0^+}^{1.5}u(t)\big)\Big) &= \phi_p(\lambda) f(t, u(t), v(t)), \ t \in (0,1) \\
D_{0^+}^{0.5}\Big(\phi_p\big(D_{0^+}^{1.25}v(t)\big)\Big) &= \phi_p(\mu) g(t, u(t), v(t)), \ t \in (0,1)
\end{aligned} \tag{13}$$

with the boundary conditions

$$\begin{aligned} &\frac{1}{2}u(0) - u'(0) = 0, \frac{1}{2}u(1) + \frac{1}{2}u'(1) = 0, D_{0^+}^{1.5}u(1) = 0, \\ &\frac{1}{2}v(0) - v'(0) = 0, \frac{1}{2}v(1) + \frac{1}{2}v'(1) = 0, D_{0^+}^{1.25}v(1) = 0, \end{aligned} \tag{14}$$

where the functions f and g are given by

$$\begin{aligned} f(t, u, v) &= \frac{\sqrt[2]{t}(u + v)[800(u + v) + 1](9 + \sin v)}{u + v + 1}, \\ g(t, u, v) &= \frac{\sqrt{1 - t}(u + v)[400(u + v) + 1](18 + \cos u)}{u + v + 1}. \end{aligned}$$

By a simple calculation, we obtain $m = 0.32, f_0^s = f_0^i = 9,\ f_\infty^s = 8000, f_\infty^i = 6400, g_0^s = g_0^i = 19, g_\infty^s = 7600, g_\infty^i = 6800, A = 0.1645,\ B = 0.05546,\ C = 0.0168448, D = 0.0009343$. We obtain $f_0^s = 9, g_0^s = 19, f_\infty^s = 8000, g_\infty^s = 7600$, we can apply theorem 3.1 then we conclude that there exist $\lambda_0, \mu_0 > 0$ such that, for every $\lambda \in (0, \lambda_0)$ and $\mu \in (0, \mu_0)$, the boundary value problem (13)-(14) has no positive solution.

We obtain $f_0^i = 9, f_\infty^i = 6400$, we can apply theorem 3.2 then there exists $\lambda_0 > 0$ such that, for every $\lambda > \tilde{\lambda_0}$ and $\mu > 0$, the boundary value problem (13)-(14) has no positive solution. We obtain $g_0^i = 19, g_\infty^i = 6800$, we can apply theorem 3.3 then there exists $\mu_0 > 0$ such that, for every $\mu > \tilde{\mu_0}$ and $\lambda > 0$, the boundary value problem (13)-(14) has no positive solution.

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