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ANALYTIC APPROACH FOR LOVE WAVE DISPERSION IN AN INHOMOGENEOUS LAYER LYING OVER AN IRREGULAR POROUS HALF-SPACE

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Abstract. The present work examines the possibility of Love wave dispersion in an irregular interface of two different media expressively as an inhomogeneous layer and porous half-space. Dispersion equation has been computed in a compact form following the expansion of transformation method, perturbation technique, Whittaker equation and its derivative. The dispersion equation coincides with the classical equation of Love type wave which agrees with the pre-established results. The graphs are sketched to show the effect of porosity, irregularity and inhomogeneous parameters.

Keywords: Love wave, Irregularity, Porous, Perturbation technique, Dispersion equation.

AMS Subject Classification: 74A05, 74A10, 74B05, 74B20.

1. Introduction

Seismology is essentially accomplished on the solicitation of theory of mechanics in a continuous as well as discontinuous medium. In general, Seismologist applies the supposition and possible conditions of physics, mathematics, geology and engineering to examine the substantial characteristics beneath the earth's surface. The exploring area of seismologists is the earth's interior and its vibration applying mechanics of physics. Standing on the above concepts, this attempt will certainly help us to represent the earth's several characteristics. Love waves are transverse waves that vibrate the ground in the horizontal direction perpendicular to the direction in which the waves are traveling.

The concepts of propagation of waves during an earthquake is a very common issue for seismologist. It is well-known fact that in the earth's crust, pores plays an important role in some special type of rocks like igneous, metamorphic and sedimentary rocks. Seismic waves are the waves of energy that travel through the earth's crust, mantle and core. It is

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directly or indirectly a result of an earthquake and other natural phenomena. Earthquakes create distinct type of waves with different velocities.

A porous medium is specified by the division of total volume into a solid matrix and porous space which is usually completed by the different types of fluids. The porous medium occurs not only in sedimental rocks but also in oil reservoirs including fouling of the membrane. The understanding of effect of porosity and inhomogeneity is compulsory for the solution of the problem in geophysics, rock dynamics, rock sediments including earthquake engineering. The earth contains fluid saturated porous rocks on or below its surface in the form of sandstone and other sediments permeated by groundwater or oil. Therefore, the analysis of Love wave propagation in a porous medium has found a central interest for seismologist. Kundu et al.[9] discussed the propagation of Love wave a porous layer overlying an initially stressed half-space. L.L. ke et al.[10] discussed the propagation of Love wave in an inhomogeneous fluid saturated porous half-space. Son and Kang[11] studied the propagation of shear waves in a poroelastic layer constrained between two elastic layers. Seismic waves are usually generated by movements of earth's tectonic plates but may be caused by explosions and volcanoes Tuncay and Corapcioglu[13] investigated the body waves in a poroelastic media saturated by two immiscible Newtonian fluids. Biot's[2] theory of wave propagation in linear, elastic, saturated porous media are the basis of many velocities and attenuation analysis. Chattaraj and Samal[5] studied the propagation of Love waves in the fibre reinforced layer over a gravitating porous half-space. Gupta et al.[6] studied the torsional wave in an inhomogeneous layer over a fluid saturated porous half- space.

The study of effect of irregular boundaries on the propagation of Love waves in a porous medium has a prominent effect because of some natural phenomenon like an earthquake. Earthquake generated seismic wave confronts mountain basins, mountain roots and salt. Chattopadhyay et al.[4] studied the shear waves propagation in a viscoelastic medium with irregular boundaries. Singh et al.[12] contemplated the propagation of Love wave on an irregular surface under the effect of a rigid boundary. Ahmed and Dahab[1] studied the propagation of Love waves in an orthotropic granular layer under initial stress lying over a semi infinite granular medium. The study of surface wave in an inhomogeneous layer is not only of the hypothetical importance but also for a practical reason. The study of propagation of Love wave will assist us to understand the earth's interior during an earthquake and explosion. This analysis will be beneficial in oil exploration and mining. Wang and Zhang[15] derived the dispersion equation for propagating Love wave resting over a transversely isotropic half-space consisting of fluid saturation. Kundu et al.[14] examined the propagation of Love type wave with irregular rectangular boundaries lying over an orthotropic semi infinite medium. Gharoi et al.[8] examined the propagation of Love wave in a fluid saturated porous half-space which is influenced by rigid boundary and gravity.

In the present paper, the propagation of Love wave in an inhomogeneous layer lying over an irregular porous half-space is discussed carefully. The irregularity for propagation of Love wave is taken in the following manner

$$z = \varepsilon f(x), \quad f(x) = \begin{cases} 0 & |x| > a_1 \\ 2a_1 & |x| \leq a_1 \end{cases}, \quad \varepsilon = \frac{h}{2a_1} << 1 \tag{1}$$

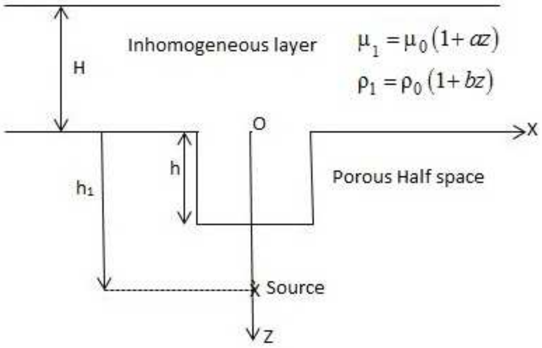

FIGURE 1. Geometry of the Problem

2. Formulation of the Problem

An inhomogeneous elastic layer of width H lying over a water saturated porous half-space with an irregular interface is introduced. It is considered that the shape of irregularity is rectangular whose length is $2a_1$ and depth h. For the propagation of Love wave, z axis is assumed vertically downwards and x axis is considered along the propagation of wave as exhibited in Figure 1. Origin O is considered at the interface of porous half- space and inhomogeneous layer. A source at depth h_1 from the origin is considered.

3. Solution for the upper layer

The fundamental equation of motion for inhomogeneous layer along x axis[3] is considered as follows

$$\begin{gathered} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2}, \ \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \end{gathered} \tag{2}$$

where $\sigma_{xx}, \sigma_{xy}, \sigma_{yz}, \sigma_{yx}, \sigma_{yy}, \sigma_{yz}, \sigma_{zx}, \sigma_{zy}$ and σ_{zz} are used to denote the incremental stress components. u_1, v_1 and w_1 are the displacement components along x, y and z axis respectively and ρ_1 symbolizes for density in the assumed inhomogeneous layer. Applying conventional condition of Love wave propagation, $u_1 = w_1 = 0$, $v_1 = v_1(x, z, t)$, the equation (2) becomes

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{3}$$

The stress-strain relations are defined as

$$\sigma_{xx} = \sigma_{yy} = \sigma_{xz} = \sigma_{zz} = 0, \ \sigma_{yx} = 2\mu_1 e_{xy} = \mu_1 \left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x} \right), \sigma_{yz} = 2\mu_1 e_{yz} = \mu_1 \left(\frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial y} \right)$$

The inhomogeneity in rigidity and density are considered as

$$\mu_1 = \mu_0 (1 + az), \ \rho_1 = \rho_0 (1 + bz)$$

where μ_0, ρ_0 denote the rigidity and density respectively at $z = 0$ and a, b are constants having dimensions inverse of its length. Using stress-strain relation and inhomogeneity parameter in equation (3), we get

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} + \frac{a}{1 + az} \frac{\partial v_1}{\partial z} = \frac{\rho_0 (1 + bz)}{\mu_0 (1 + az)} \frac{\partial^2 v_1}{\partial t^2} \tag{4}$$

The following analytical solution is considered for equation (4)

$$v_1(x,z,t) = V_1(x,z)\,e^{i\omega t} \tag{5}$$

The Fourier transformation for $V_1(x,z)$ is expressed as

$$\bar{V_1}(s,z) = \int_{-\infty}^{\infty} V_1(x,z)\,e^{ixs}ds \tag{6}$$

Therefore, the inverse Fourier transformation is defined as

$$V_1(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{V_1}(s,z)\,e^{-ixs}ds \tag{7}$$

Using equations (5) and (6) in equation (4), we get

$$\frac{d^2\bar{V_1}}{dz^2} + \frac{a}{1+az}\frac{d\bar{V_1}}{dz} + \left(\frac{\omega^2\rho_0(1+bz)}{\mu_0(1+az)} - s^2\right)\bar{V_1} = 0 \tag{8}$$

Substituting $\bar{V_1} = \frac{\phi(z,s)}{(1+az)^{\frac{1}{2}}}$ in equation (8), we get

$$\phi''(z,s) + \left[\frac{a^2}{4}\frac{1}{(1+az)^2} + \frac{\omega^2\rho_0(1+bz)}{\mu_0(1+az)}\right]\phi(z,s) = 0 \tag{9}$$

Using $\phi(z,s) = \psi(\eta,s)$ in equation (9), where $\eta = \frac{2p}{a}(1+az)$, $p = \sqrt{\left(s^2 - \frac{b}{a}\frac{\omega^2}{c_0^2}\right)}$

$$\psi''(\eta,s) + \left[\frac{1}{4\eta^2} + \frac{R}{\eta} - \frac{1}{4}\right]\psi(\eta,s) = 0 \tag{10}$$

where $R = \frac{\omega^2}{2pc_0^2}\frac{a-b}{a^2}$, and $c_0 = \sqrt{\frac{\mu_0}{\rho_0}}$, which is a Whittaker equation. From equation (10), the following equation is taken as

$$\psi(\eta,s) = B_1W_{R,0}(\eta) + C_1W_{-R,0}(-\eta) \tag{11}$$

Therefore, the solution for equation (8) is expressed as

$$\bar{V_1}(z,s) = \frac{B_1W_{R,0}(\eta) + C_1W_{-R,0}(-\eta)}{\left(\frac{a\eta}{2p}\right)^{\frac{1}{2}}}$$

Now the solution of equation (7) is

$$V_1(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{B_1W_{R,0}(\eta) + C_1W_{-R,0}(-\eta)}{\sqrt{\frac{a\eta}{2p}}}e^{-ixs}ds \tag{12}$$

where B_1 and C_1 are arbitrary constants.

4. Solution for the lower half-space

The constitutive equation of porous medium in absence of fluid and body force is considered as [2]

$$\begin{gathered}\frac{\partial\tau_{11}}{\partial x} + \frac{\partial\tau_{12}}{\partial y} + \frac{\partial\tau_{13}}{\partial z} + \frac{\partial\omega_2}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}u_2 + \rho_{12}U_1)\\ \frac{\partial\tau_{21}}{\partial x} + \frac{\partial\tau_{22}}{\partial y} + \frac{\partial\tau_{23}}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}v_2 + \rho_{12}U_2),\ \frac{\partial\tau_{31}}{\partial x} + \frac{\partial\tau_{32}}{\partial y} + \frac{\partial\tau_{33}}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{11}v_2 + \rho_{12}U_3)\\ \frac{\partial\tau}{\partial x} = \frac{\partial^2}{\partial t^2}(\rho_{12}u_2 + \rho_{22}U_1),\ \frac{\partial\tau}{\partial y} = \frac{\partial^2}{\partial t^2}(\rho_{12}v_2 + \rho_{22}U_2),\ \frac{\partial\tau}{\partial z} = \frac{\partial^2}{\partial t^2}(\rho_{12}v_2 + \rho_{22}U_3)\end{gathered} \tag{13}$$

where $\tau_{ij}\,(i,j = 1,2,3)$ and τ are used to denote the incremental stress component of the solid and liquid respectively. The terms $u = (u_2, v_2, w_2)$ and $U = (U_1, U_2, U_3)$ denote the displacement component vector of the solid and liquid part of the porous respectively. The symbols ρ_{11}, ρ_{12} and ρ_{22} denote the mass coefficient density for solid, liquid and their inertia coupling parameter while $\omega_j\,(j = 2)$ denotes the angular displacement vectors. Let us suppose that there is no relative motion between liquid and solid in the porous media. The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are related to the total mass density of solid with liquid ρ and mass densities ρ_s and ρ_f for solid and liquid respectively in the following manner as shown by Biot [3]

$$\rho_{11} + \rho_{12} = (1 - f)\,\rho_s,\ \rho_{12} + \rho_{22} = f\rho_f,\ \rho = \rho_s + f\,(\rho_f - \rho_s) \tag{14}$$

where f denotes porosity of the layer. In an anisotropic layer, the relation between stress and strain is defined as

$$\begin{aligned} \tau_{11} &= Ae_{11} + (A - 2N)\,(e_{22} + e_{33}) + Qe \\ \tau_{22} = Ae_{22} + (A - 2N)\,(e_{11} + e_{33}) + Qe,\ \tau_{33} &= Ae_{33} + (A - 2N)\,(e_{11} + e_{22}) + Qe \\ s_{12} = 2Ne_{12},\ s_{31} = 2Le_{31},\ s_{23} &= 2Le_{23},\ s = Qe + Me' \end{aligned} \tag{15}$$

where $e = \nabla \times u$ and $e' = \nabla \times U$ are the respective dilation, which are opposite in sign. The Lame's coefficients are denoted by A, N and L, M denote the total pressure exerted on the liquid. Let Q be the measure of coupling between the volume change of the solid and liquid. Also, the strain component e_{ij} is defined as

$$e_{ij} = \frac{(u_{i,j} + u_{j,i})}{2}$$

The angular displacements are given by

$$\omega_2 = \frac{1}{2}\left(\frac{\partial u_2}{\partial z} - \frac{\partial w_2}{\partial x}\right)$$

For the Love wave propagation along the x- axis are $u_2 = 0 = w_2$, $v_2 = v_2\,(x, z, t)$. Using the above conventional condition, the equation of motion given by equation (13) can be expressed as

$$N\frac{\partial^2 v_2}{\partial x^2} + L\frac{\partial^2 v_2}{\partial z^2} = \frac{\partial^2}{\partial t^2}\,(\rho_{11}v_2 + \rho_{12}U_2)\,,\ \frac{\partial^2}{\partial t^2}\,(\rho_{12}v_2 + \rho_{22}U_2) = 0 \tag{16}$$

Eliminating the component of liquid displacement v_2 from equation (16), we get

$$N\frac{\partial^2 v_2}{\partial x^2} + L\frac{\partial^2 v_2}{\partial z^2} = d'\frac{\partial^2 v_2}{\partial t^2} \tag{17}$$

where $d' = \rho_{11} - \left(\frac{\rho_{12}^2}{\rho_{22}}\right)$. Considering the non dimensional parameter $\gamma'_{11} = \frac{\rho_{11}}{\rho}$, $\gamma'_{12} = \frac{\rho_{12}}{\rho}$ and $\gamma'_{22} = \frac{\rho_{22}}{\rho}$, equation (17) reduces to

$$\frac{N}{L}\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} = \frac{d}{c_{\beta_2}^2}\frac{\partial^2 v_2}{\partial t^2} \tag{18}$$

where $d = \gamma'_{11} - \left(\frac{\gamma_{12}'^2}{\gamma'_{22}}\right)$, $c_\beta = \sqrt{\frac{L}{\rho}}$. The harmonic solution of equation (18) is assumed as

$$v_2\,(x, z, t) = V_2\,(x, z)\,e^{i\omega t} \tag{19}$$

Imposing Fourier transformation on equation (19), we get

$$\bar{V_2}\,(s, z) = \int_{-\infty}^{\infty} V_2\,(x, z)\,e^{ixs}dx \tag{20}$$

Now applying inverse Fourier transformation on equation (20), we get

$$v_2\,(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{V_2}\,(s, z)\,e^{-ixs}ds \tag{21}$$

Using equation (19) and equation (20) in equation (18), it is obtained that

$$\frac{d^2\bar{V_2}}{dz^2} = q^2\bar{V_2} \tag{22}$$

where $q^2 = d\nu\left[\frac{s^2}{d} - \frac{\omega^2}{c_\beta^2}\right]$, $\nu = \frac{N}{L}$. From equation (22), we have

$$\bar{V_2} = A_1e^{-qz} + A_2e^{qz}$$

where A_1, A_2 are arbitrary constants. The displacement component is bounded as $z \to \infty$, therefore, the second term of displacement component becomes unbounded. Thus, the approximate solution is taken as

$$\bar{V_2} = A_1 e^{-qz} \tag{23}$$

Using equation (21) and equation (23), we get

$$v_2(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A_1 e^{-qz} e^{-ixs} ds \tag{24}$$

where A_1 is an arbitrary constant. Introducing the perturbation method in A_1, B_1 and C_1 in the following manner such that

$$\begin{gathered} A_1 \equiv A_{11} + \epsilon A_{12}, B_1 \equiv B_{11} + \epsilon B_{12}, C_1 \equiv C_{11} + \epsilon C_{12} \\ e^{\pm \epsilon \nu f} \equiv 1 \pm \epsilon \nu f, (1 \pm \epsilon\theta)^{R'} = 1 \pm \epsilon \theta R' \end{gathered} \tag{25}$$

Following equation (12) and equation (25), we get

$$V_1(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{(B_{11} + \epsilon B_{12}) W_{R,0}(\eta) + (C_{11} + \epsilon C_{12}) W_{-R,0}(-\eta)}{\left(\frac{a\eta}{2p}\right)^{\frac{1}{2}}} e^{-ixs} ds \tag{26}$$

$$V_2(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[(A_{11} + \epsilon A_{12}) e^{-qz} + \frac{2}{q} e^{qz} e^{-qh_1}\right] e^{-ixs} ds \tag{27}$$

where 2^{nd} term in integrand appears for an existence of the source in the half-space.

5. Boundary conditions

(i) The surface is free of stress at $z = -H$, therefore it requires that

$$\mu_0 (1 - aH) \frac{\partial v_1}{\partial z} = 0$$

(ii) The displacement component is continuous at $z = \epsilon f(x)$, therefore it requires that

$$v_1 = v_2$$

(iii) At $z = \epsilon f(x)$

$$\mu_0 \left(\epsilon f' \frac{\partial v_1}{\partial x} - \frac{\partial v_1}{\partial z}\right) = \epsilon f' N \frac{\partial v_2}{\partial x} - L \frac{\partial v_2}{\partial z}$$

The Fourier transform of $f(x)$ is acknowledged as

$$f(\alpha) = \int_{-\infty}^{\infty} f(x) e^{i\alpha x} dx,\ f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\alpha) e^{-i\alpha x} d\alpha,\ f'(x) = -\frac{i}{2\pi}\int_{-\infty}^{\infty} \alpha f(\alpha) e^{-i\alpha x} d\alpha \tag{28}$$

Using boundary conditions (i) and the equation (26) and (27), we get

$$B_{11} T_1 + C_{11} T_2 = 0,\ B_{12} T_1 + C_{12} T_2 = 0 \tag{29}$$

where

$$T_1 = e^{-\frac{\eta}{2}} \eta^R \Big[\eta\left(-\frac{1}{2} + \frac{R}{\eta}\right)\left(1 - \frac{(R - .5)^2}{\eta}\right) - \left(\frac{1}{2} - 3\frac{(R - .5)^2}{\eta}\right)\Big]$$

$$T_2 = e^{\frac{\eta}{2}} (-\eta)^R \Big[\eta\left(-\frac{1}{2} + \frac{R}{\eta}\right)\left(1 - \frac{(R - .5)^2}{\eta}\right) - \left(\frac{1}{2} - 3\frac{(R - .5)^2}{\eta}\right)\Big]$$

Applying boundary conditions (ii) and (iii) in equation (26) and equation (27), it is expressed as

$$B_{11}T_3 + C_{11}T_4 - A_{11} = \frac{2e^{-qh_1}}{q},\ B_{12}T_3 + C_{12}T_4 - A_{12} = R_1(\chi),$$

$$B_{11}T_5 + C_{11}T_6 + n_2A_{11} = \left(\frac{2p}{a}\right)^2 \frac{Lq}{2\mu_0 p}\frac{2e^{-qh_1}}{q},\ B_{12}T_5 + C_{12}T_6 + n_2A_{12} = R_2(\chi),\ \text{where}$$

$$R_1(\chi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-\frac{q}{a}A_{11} + B_{11}T_7 + C_{11}T_8 + \frac{2e^{-qh_1}}{a}\right]_{s=\chi-\alpha} f(\alpha)d\alpha \tag{30}$$

$$R_2(\chi) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Big[\left(\frac{2p}{a}\right)^2\frac{Lq}{2\mu_0 p}\{A_{11}\left(\left(\frac{q}{a}-\frac{5}{2}\right)+s\alpha N\right) + \frac{2e^{-qh_1}}{a}\left(1+\frac{5}{2}\frac{a}{q}(1+s\alpha N)\right)\}$$
$$-B_{11}(T_9 - s\alpha\mu_0) - C_{11}(T_{10} - s\alpha\mu_0)\Big]_{s=\chi-\alpha} f(\alpha)d\alpha$$

where the values of T_i,$(i = 2,...10)$ are given in appendices(See Appendix A). Solving simultaneous equation (30), we get

$$A_{11} = \frac{T_2D_2 - T_1D_3}{\Delta(\chi)},\ B_{11} = \frac{T_2D_1}{\Delta(\chi)},\ C_{11} = -\frac{T_1D_1}{\Delta(\chi)}\ B_{12} = \frac{T_2(n_2R_1(k) + R_2(\chi))}{\Delta(\chi)}$$

$$C_{12} = -\frac{T_1(n_2R_1(\chi) + R_2(\chi))}{\Delta(\chi)},\ A_{12} = \frac{T_1\Big[R_1(\chi)T_6 - R_2(\chi)T_4\Big] - T_2\Big[R_1(\chi)T_5 - R_2(\chi)T_2\Big]}{\Delta(\chi)}$$

where $\Delta(\chi) = T_2D_4 - T_1D_5$, the values of D_i,$(i = 1,..5)$ are given in appendix.(See Appendix A)

Using the above expressions, the displacement component in the inhomogeneous layer is expressed as

$$V_1(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{(B_{11}+\epsilon B_{12})W_{R,0}(\eta) + (C_{11}+\epsilon C_{12})W_{-R,0}(-\eta)}{\left(\frac{a\eta}{2p}\right)^{\frac{1}{2}}}e^{-ixs}ds \tag{31}$$

$$V_1(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\Big[D_1 + (n_2R_1(\chi)) + R_2(\chi)\Big]\Big[T_2W_{R,0}(\eta) - T_1W_{-R,0}(-\eta)\Big]}{\left(\frac{a\eta}{2p}\right)^{\frac{1}{2}}}e^{-ixs}ds \tag{32}$$

Now from equation (1) and equation (28), one may observe that

$$f(\alpha) = 4a_1\frac{\sin a_1\alpha}{\alpha} \tag{33}$$

Using equation (30) and equation (33), it gives that

$$n_2R_1(\chi) + R_2(\chi) = \frac{\left(\frac{2p}{a}\right)^2\frac{Lq}{\mu_0 2p}\frac{2e^{-qh_1}}{q}}{\pi}\int_{-\infty}^{\infty}\Big[\psi(k-\alpha) + \psi(k+\alpha)\Big]4a_1\frac{\sin a_1\alpha}{\alpha}d\alpha \tag{34}$$

The equation (34) implies that

$$n_2R_1(\chi) + R_2(\chi) = \frac{4a_1\left(\frac{2p}{a}\right)^2\frac{Lq}{\mu_0 2p}\frac{2e^{-qh_1}}{q}}{\pi}\int_{-\infty}^{\infty}\Big[\psi(k-\alpha) + \psi(k+\alpha)\Big]\frac{\sin a_1\alpha}{\alpha}d\alpha \tag{35}$$

where $\psi(k-\alpha) + \psi(k+\alpha) = \frac{T_1D_6 + T_2D_7}{\Delta\chi}$. D_6, D_7 are defined in Appendix (See Appendix A).

The asymptotic formula used by Chattopadhyay et al. [4] as follows

$$\int_{-\infty}^{\infty}\psi(k-\alpha) + \psi(k+\alpha)\frac{\sin a_1\alpha}{\alpha} \equiv \frac{\pi}{2}2g(\chi) = \pi g(\chi) \tag{36}$$

Using equation (36) in equation (35), we get

$$n_2R_1(\chi) + R_2(\chi) = \left(\frac{2p}{a}\right)^2\frac{Lq}{\mu_0 2p}\frac{2e^{-qh_1}}{q}\frac{2h}{\epsilon}g(\chi) \tag{37}$$

In light of equation (32) and equation (37), the displacement component in an inhomogeneous layer for the small value of ϵ is given by

$$V_1(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\left(\frac{2p}{a}\right)^2\frac{Lq}{\mu_0 2p}\frac{2e^{-qh_1}}{q}\left(\frac{2p}{a}\right)^{\frac{1}{2}}}{\Delta(\chi)\left(1-hg(\chi)\right)}\frac{T_2W_{R,0}(\eta)-T_1W_{-R,0}(-\eta)}{\eta^{\frac{1}{2}}}e^{-ixs}ds \tag{38}$$

where $g(\chi)=\frac{T_1D_6+T_2D_7}{\Delta(\chi)}$. The integral given in equation (38) will depend on the contribution of the poles of the integrand. Thus, the dispersion equation of Love wave is obtained as

$$\Delta(\chi)\Big[1-hg(\chi)\Big]=0 \tag{39}$$

Equation (39) implies that

$$\frac{T_1}{T_2}=\frac{D_4-hD_7}{D_5+hD_6} \tag{40}$$

Using the values of above terms in equation (40), it is obtained as

$$\frac{e^{pH}}{e^{-pH}}=\frac{NU}{DE} \tag{41}$$

where

$$NU=(1-aH)^{-R}R_{22}\Big[D_4-hD_7\Big],\ DE=(1-aH)^{R}R_{11}\Big[D_5+hD_6\Big]$$

Equating real part from equation (41), it is obtained as

$$\tan\left[\sqrt{\left(\frac{b}{a}\frac{c^2}{c_0^2}-1\right)}kH\right]=\frac{L_1L_3-L_2L_4}{L_2^2+L_4^2} \tag{42}$$

where

$$R_{11}=\frac{2p}{a}(1-aH)\left(-\frac{1}{2}+\frac{R}{\frac{2p}{a}(1-aH)}\right)\left(1-\frac{\left(R-\frac{1}{2}\right)^2}{\frac{2p}{a}(1-aH)}\right)-\frac{1}{2}\left(1+\frac{3\left(R+\frac{1}{2}\right)^2}{\frac{2p}{a}(1-aH)}\right)$$

$$R_{22}=\frac{2p}{a}(1-aH)\left(\frac{1}{2}-\frac{R}{\frac{2p}{a}(1-aH)}\right)\left(1-\frac{\left(R+\frac{1}{2}\right)^2}{\frac{2p}{a}(1-aH)}\right)-\frac{1}{2}\left(1+\frac{3\left(R+\frac{1}{2}\right)^2}{\frac{2p}{a}(1-aH)}\right)$$

The values of L_i,($i=1,..4$) are given in appendix(See Appendix A).

6. Particular cases

Case 1: When the porous half space has no irregularity, that is, $h=0$, then equation (42) reduces to

$$\frac{e^{pH}}{e^{-pH}}=\frac{(1-aH)^{-R}R_{22}D_4}{(1-aH)^{R}R_{11}D_5} \tag{43}$$

The equation (43) represents the dispersion equation of Love wave in an inhomogeneous layer lying over a porous half-space when there is no irregularity.
Case 2: When irregularity component is absent and neglecting inhomogeneity parameter, that is, when $h=0$, $a\to 0$ and $b\to 0$, then equation (43) reduces to

$$\tan\left(\left(\sqrt{\frac{c^2}{c_0^2}-1}\right)kH\right)=\frac{L}{\mu_0}\frac{q}{\sqrt{\left(\frac{c^2}{c_0^2}-1\right)}} \tag{44}$$

The equation (44) represents the dispersion equation of Love wave in a homogeneous layer lying over a porous half-space when there is no irregularity at the interface.
Case 3: When irregularity component is absent and neglecting inhomogeneity parameter in a nonporous half-space, that is, when $h=0$, $a\to 0$, $b\to 0$ and $d\to 1$ and $N=L=\mu_1$ then equation (44) takes the form

$$\tan\left(\sqrt{\left(\frac{c^2}{c_0^2}-1\right)}kH\right)=\frac{\mu_1}{\mu_0}\frac{\sqrt{1-\frac{c^2}{c_\beta^2}}}{\sqrt{\frac{c^2}{c_0^2}-1}} \tag{45}$$

The equation (45) represents the dispersion equation of Love wave in a homogeneous layer lying over a nonporous half-space when there is no irregularity at the interface.

7. Graphical Observation

For graphical observation, the data has been taken from Gubbins [7] that is $\mu_0 = 74.5 \times 10^9 N/m^2$, $\rho_0 = 3293 Kg/m^3$, $N = 2.774 \times 10^9 N/m^2$, $L = 1.318 \times 10^9 N/m^2$ and $\rho = 3535 Kg/m^3$.

Figure 2 is sketched to show the effect of the size of irregularity on phase velocity for a porous half-space. It is observed that the phase velocity increases with a small increment of $\frac{h}{H}$. It is noticed that phase velocity decreases when wave number increases.

Figure 3 is designed to express the importance of size of irregularity in nonporous half-space. Phase velocity increases with increment of $\frac{h}{H}$. It is examined that in case of nonporous half-space phase velocity increases for large variation with a comparison of porous half-space. It is clear that phase velocity decreases when dimensionless wave number increases.

Figure 4 renders the effect of inhomogeneous parameter aH for porous half-space. When the value of inhomogeneous parameter increases, phase velocity increases.

Figure 5 represents the effect of inhomogeneous parameter aH for nonporous half-space.The phase velocity increases when the value of aH increases. But it can be noticed that phase velocity varies more rapidly in non porous half-space compared to porous half-space.

Figure 6 **and Figure** 7 signify the effect of inhomogeneous parameter bH in case of porous and nonporous half-space respectively. When the inhomogeneous parameter increases phase velocity decreases in both the cases. But it is examined that phase velocity in porous half-space decrease more than nonporous half-space.

Figure 8 indicates the effect of the porosity in respect of wave number and phase velocity. It is seen that increment of porosity gives the decrements in phase velocity. Also, phase velocity decreases when wave number increases.

Figure 9 is drawn in respect of porosity parameter and phase velocity to show the variation of kH. It is verified that phase velocity decreases with the increment of kH. For stable size of irregularity, an increase of beam of inhomogeneous layer, phase velocity decreases.

Finally, it can be concluded from all the figures that

(i) Phase velocity decreases with the increment of wave number and porosity parameter.

(ii) Inhomogeneous parameter, porosity, shape of irregularity have a considerable effect in phase velocity.

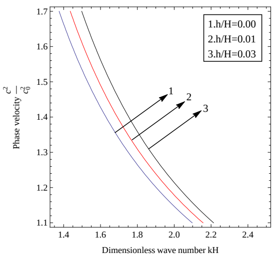

Figure 2. Variation of phase velocity with respect to wave number for different value of $\frac{h}{H}$ when $d = 0.74$

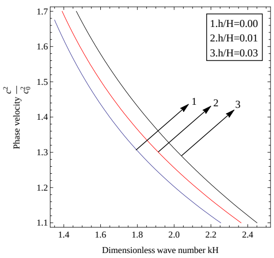

Figure 3. Variation of phase velocity with respect to wave number for different value of $\frac{h}{H}$ when $d = 1$

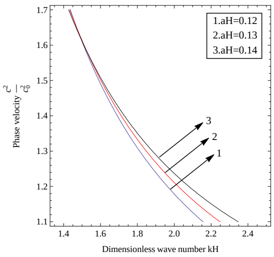

Figure 4. Variation of phase velocity with respect to wave number for different value of aH when $d = 0.74$

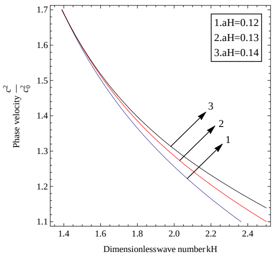

Figure 5. Variation of phase velocity with respect to wave number for different value of aH when $d = 1$

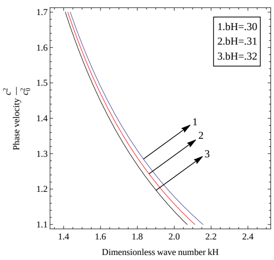

Figure 6. Variation of phase velocity with respect to wave number for different value of bH when $d = 0.74$

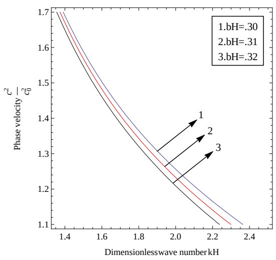

Figure 7. Variation of phase velocity with respect to wave number for different value of bH when $d = 1$

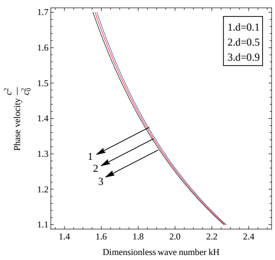

Figure 8. Variation of phase velocity with respect to wave number for different value of porosity

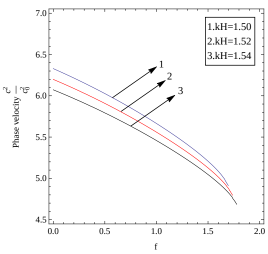

Figure 9. Variation of porosity with respect to porosity for different value of kH

8. Conclusion

The propagation of Love wave in an irregular inhomogeneous layer lying over a porous half-space is analyzed carefully. The dispersion equation and displacement components are observed mathematically. The numerical results are deliberated through figures by plotting graphs between phase velocity and dimensionless wave number. Also, it is observed that the phase velocity is highly influenced by porosity, length of irregularity and inhomogeneous parameter. The analysis of this aspects of propagation of Love wave in an irregular medium has been introduced in this paper. Hence it contributes valuable entropy for classification of some structural material that exists in construction sector and oil exploration which is not only advantageous to seismologist but also to geophysicist, mathematicians including civil engineers.

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9. Appendix A

$$L_1 = w_1w_5+w_1w_6+w_2w_5-w_2w_6,\ L_2 = w_1w_6+w_2w_5+w_3w_8+w_4w_7,\ L_3 = w_1w_5-w_1w_6-w_2w_5-w_2w_6$$

$$L_4 = w_1w_6+w_2w_5-w_3w_8-w_4w_7,\ w_1 = k_4+aHRk_3,\ w_2 = k_3-aHRk_4,\ w_3 = k_2-aHRk_1,\ w_4 = k_1-aHRk_2$$

$$w_5 = z_7-ahk_7,\ w_6 = z_9-ahk_8,\ w_7 = z_{11}-ahk_5,\ w_8 = z_{13}-ahk_6,\ k_1 = \frac{2\gamma}{a}(1-aH)\left(-\frac{1}{2}-l_2\right)(1-l_2)+\frac{3}{2}l_1$$

$$k_2 = \frac{2\gamma}{a}(1-aH)\left(-\frac{1}{2}-l_2\right)l_1-\frac{1}{2}(1-3l_2),\ k_3 = \frac{2\gamma}{a}(1-aH)\left(\frac{1}{2}+l_2\right)(1+l_2)-\frac{3}{2}l_1$$

$$k_4 = \frac{2\gamma}{a}(1-aH)\left(\frac{1}{2}+l_2\right)l_1-\frac{1}{2}(1-3l_2),\ l_1 = \frac{a}{2\gamma(1-aH)}\left(R^2-.5^2\right), l_2 = \frac{aR}{2\gamma(1-aH)}$$

$$k_5 = \frac{5}{2}z_{12}-\left(\frac{5}{2}+\frac{2q}{a}\right)z_{17}-2d_3,\ k_6 = \frac{5}{2}z_{14}-\left(\frac{5}{2}+\frac{2q}{a}\right)z_{13}-2d_4,\ k_7 = -\frac{5}{2}z_8+\left(\frac{5}{2}+\frac{2q}{a}\right)z_7+2d_1$$

$$k_8 = -\frac{5}{2}z_{10}+\left(\frac{5}{2}+\frac{2q}{a}\right)z_9+2d_2,\ d_1 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}z_3-z_6, d_2 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}z_1-z_5, d_3 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}z_4+z_6$$

$$d_4 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}z_2-z_5,\ z_1 = \frac{1}{2}\left(1-\frac{a}{2\gamma}\right)-j_1,\ z_2 = \frac{1}{2}\left(1-\frac{a}{2\gamma}\right)+j_1,\ z_3 = \frac{1}{2}\frac{a}{2\gamma}\left(R^2-.5^2\right)+j_2$$

$$z_4 = \frac{1}{2}\frac{a}{2\gamma}\left(R^2-.5^2\right)+j_2,\ z_5 = -\left(R+\frac{\gamma}{a}\right)j_3-\frac{\gamma}{a}-\frac{\gamma}{a}\left(R^2-.5^2\right),\ z_6 = \frac{2\gamma}{a}\left(R+\frac{\gamma}{a}\right)+\frac{\gamma}{a}\left(\frac{2\gamma}{a}-R\right)+j_4\left(R+\frac{\gamma}{a}\right)$$

$$z_7 = \left(\frac{2\gamma}{a}\right)\frac{Lq}{2\gamma\mu_0}\left(R^2-.5^2\right)+j_3,\ z_8 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}\left(\frac{a}{2\gamma}\right)\left(R^2-.5^2\right)j_3,\ z_9 = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}\left(1-\frac{a}{2\gamma}\right)+j_4$$

$$z_{10} = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}\left(1-\frac{a}{2\gamma}\right)-j_4,\ z_{11} = -\left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}j_2-j_3,\ z_{12} = -\left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}j_2+z_3$$

$$z_{13} = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}j_1+j_4, z_{14} = \left(\frac{2\gamma}{a}\right)^2\frac{Lq}{2\gamma\mu_0}j_1-j_4,\ j_1 = \frac{aR}{2\gamma}+\frac{a}{2\gamma}\left(R^2-.5^2\right)\left(R-\frac{\gamma}{a}\right)$$

$$j_2 = \frac{a}{2\gamma}\left(R^2 - .5^2\right) - \frac{aR}{2\gamma}\left(R - \frac{\gamma}{a}\right) - \left(R + \frac{\gamma}{a}\right), j_3 = \left(R + \frac{\gamma}{a}\right)\left(\frac{2\gamma}{a} - R\right) - \frac{3}{2}\left(R^2 - .5^2\right)$$

$$j_4 = -\left(R + \frac{\gamma}{a}\right)\left(R^2 - .5^2\right) - \frac{\gamma}{a} + \frac{3}{2}R, R_1 = \frac{5}{2}\left(\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P} T_4 - T_6\right)$$

$$D_1 = 2\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P}\frac{2e^{-qh_1}}{q} + M_2, D_2 = \frac{2e^{-qh_1}}{q}\Big[\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P} T_3 - T_5\Big], D_3 = \frac{2e^{-qh_1}}{q}\Big[\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P} T_4 - T_7\Big]$$

$$D_4 = \left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P} T_3 + T_5, D_5 = \left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P} T_4 + T_6, D_6 = R_4 + R_5 + R_6 \; D_7 = R_1 + R_2 + R_3$$

$$R_2 = 2\Big[2\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P}\frac{a}{q}\frac{2e^{-qh_1}}{a}\Big[\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2p} D_{11} - D_{12}\Big]\Big], R_3 = \frac{q}{a}\left(2 + \frac{5}{2}\frac{a}{q}\right)\left(\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2p} T_4 + T_6\right)$$

$$R_4 = \frac{-5}{2}\left(\left(\frac{2p}{a}\right)^2 \frac{Lq}{mu_0 2p} T_3 - T_5\right), R_5 = 4\left(\frac{2p}{a}\right)^2 \frac{Lq}{\mu_0 2P}\frac{a}{q}\frac{2e^{-qh_1}}{a}\left(\left(\frac{2p}{a}\right)^2 \frac{Lq}{mu_0 2p} D_{13} - D_{14}\right)$$

$$R_6 = \frac{2}{a}\left(2 + \frac{5}{2}\frac{a}{q}\right)\left(\left(\frac{2p}{a}\right)^2 \frac{Lq}{mu_0 2p} T_3 + T_5\right)$$

$$D_{11} = \frac{1}{2}\left(1 + \frac{2p}{a}\left(R + \frac{1}{2}\right)^2\right) - \left(\frac{p}{a} - R\right) - \frac{a}{2p}\left(R + \frac{1}{2}\right)^2\left(1 - \frac{p}{a} - R\right)$$

$$D_{12} = \frac{p}{a}\left(\frac{2p}{a} + \left(R + \frac{1}{2}\right)^2\right) + \frac{2p}{a}\left(\frac{p}{a} - R\right) - \frac{p}{a} + T_6\left(R - \frac{p}{a}\right)$$

$$D_{13} = \frac{1}{2}\left(1 - \frac{a}{2p}\left(R - \frac{1}{2}\right)^2\right) - \left(\frac{p}{a} - R\right) - \frac{a}{2p}\left(R - \frac{1}{2}\right)^2\left(1 + \frac{p}{a} + R\right), T_3 = 1 - \frac{a}{2p}\left(R - \frac{1}{2}\right)^2,$$

$$D_{14} = \frac{2p}{a}\left(R - \frac{p}{a}\right) - \frac{p}{a} - \frac{p}{a}\left(\frac{2p}{a} - \left(R - \frac{1}{2}\right)^2\right) + T_5\left(\frac{p}{a} - R\right), T_4 = \left(\frac{p}{a} - R\right) + \frac{a}{2p}\left(R - \frac{1}{2}\right)^2\left(1 + \frac{p}{a} + R\right)$$

$$T_5 = \left(\frac{2p}{a} - \left(R - \frac{1}{2}\right)^2\right)\left(R - \frac{p}{a}\right) - \frac{p}{a} + \frac{3}{2}\left(R - \frac{1}{2}\right)^2$$

$$T_7 = \frac{1}{2}\left(1 - \frac{a}{2P}(R - .5)^2\right) - \left(\frac{P}{a} - R\right) - \frac{a}{2P}(R - .5)^2\left(1 + R + \frac{P}{a}\right)$$

$$T_6 = \left(\frac{2p}{a} + \left(R + \frac{1}{2}\right)^2\right)\left(\frac{p}{a} - R\right) - \frac{p}{a} - \frac{3}{2}\left(R + \frac{1}{2}\right)^2$$

$$T_8 = \frac{1}{2}\left(1 + \frac{a}{2P}(R + .5)^2\right) - \left(\frac{P}{a} - R\right) - \frac{a}{2P}(R + .5)^2\left(1 - R - \frac{P}{a}\right)$$

$$T_9 = \frac{2P}{a}\left(R - \frac{P}{a}\right) - \frac{P}{a} - \frac{P}{a}\left(\frac{2P}{a} - (R - .5)^2\right) + T_5\left(R - \frac{P}{a}\right)$$

$$T_{10} = \frac{2P}{a}\left(\frac{P}{a} - R\right) + \frac{P}{a}\left(\frac{2P}{a} + (R + .5)^2\right) + T_6\left(R - \frac{P}{a}\right)$$

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